

Analysis of the Decapole Systematic Error in the Dipoles and of the Correctors

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Analysis of the Decapole Systematic Error
in the Dipoles and of the Correctors

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(1)

The equations of motion in the presence of a decapole term are

$$x'' + \kappa_H x = \frac{b_4}{\rho} (x^4 - 5x^2y^2 + y^4) \quad (1a)$$

$$y'' + \kappa_H y = -4 \frac{b_4}{\rho} (x^3y - xy^3) \quad (1b)$$

where ρ is the bending radius in the dipoles and b_4 is the strength of the decapole term.

We can calculate the first-order contribution to the tune-shift vs. betatron amplitude and off-momentum value, by separating the free betatron oscillations from the closed-orbit deviation

$$x = \eta \delta + \tilde{x}, \quad y = \tilde{y}$$

\tilde{x}, \tilde{y} free betatron oscillations

η , dispersion (only in the horizontal plane)

δ , off-momentum value

~~The term in y^4 in eq (1a) can be ignored since it does not give any first-order contribution to tune-shift - then subtracting from (1a and b) the equations for the closed orbit~~

$$\begin{aligned} \ddot{\tilde{x}} + K_H \tilde{x} = \frac{b_4}{\rho} \left(\tilde{x}^4 + 4\eta\delta\tilde{x}^3 + \cancel{6}\eta^2\delta^2\tilde{x}^2 + \right. \\ \left. + 4\eta^3\delta^3\tilde{x} - 6\tilde{x}\tilde{y}^2 - 12\eta\delta\tilde{x}\tilde{y}^2 - \right. \\ \left. - 6\eta^2\delta^2\tilde{y}^2 + \tilde{y}^4 \right) \quad (2a) \end{aligned}$$

$$\begin{aligned} \ddot{\tilde{y}} + K_V \tilde{y} = -\frac{b_4}{\rho} \left(\tilde{x}^3\tilde{y} + 3\eta\delta\tilde{x}^2\tilde{y} + 3\eta^2\delta^2\tilde{x}\tilde{y} + \right. \\ \left. + \eta^3\delta^3\tilde{y} - \tilde{x}\tilde{y}^3 - \eta\delta\tilde{y}^3 \right) \quad (2b) \end{aligned}$$

where tune-shift are

$$\begin{aligned} \Delta Q_H = -\frac{1}{4\pi\rho} \oint \beta_H b_4 \left(\tilde{x}^3 + 4\eta\delta\tilde{x}^2 + \cancel{6}\eta^2\delta^2\tilde{x} + 4\eta^3\delta^3 + \right. \\ \left. - 6\tilde{x}\tilde{y}^2 - 12\eta\delta\tilde{y}^2 \right) ds \quad (3a) \end{aligned}$$

where we have ignored the last two terms at the r.h. side of eq. 2a since they do not give contribution to first-order

$$\begin{aligned} \Delta Q_V = \frac{1}{4\pi\rho} \oint \beta_V b_4 \left(\tilde{x}^3 + 3\eta\delta\tilde{x}^2 + 3\eta^2\delta^2\tilde{x} + \eta^3\delta^3 + \right. \\ \left. - \tilde{x}\tilde{y}^2 - \eta\delta\tilde{y}^2 \right) ds \quad (3b) \end{aligned}$$

where the integrals are taken over the full circumference of the ring.

To first-order, the tune-shifts are averaged over several betatron oscillations, thus the terms in \tilde{x}^2 and \tilde{x}^3 in eqs (3a & 3b) give in average zero contribution. ~~Similarly~~

Again in first-order, we can set

$$\tilde{x}^2 = \frac{\varepsilon_H}{\pi} \beta_H \cos^2 \psi_H$$

$$\tilde{y}^2 = \frac{\varepsilon_V}{\pi} \beta_V \cos^2 \psi_V$$

~~Thus~~ Finally

$$\begin{aligned} \Delta Q_H = & - \frac{\delta^3}{\pi \rho} \oint b_4 \beta_H \eta^3 ds + \\ & - \frac{\delta \varepsilon_H}{\pi \rho} \oint b_4 \beta_H \eta \cos^2 \psi_H ds + \\ & + \frac{3 \delta \varepsilon_V}{\pi \rho} \oint b_4 \beta_H \beta_V \eta \cos^2 \psi_V ds \end{aligned} \quad (4a)$$

and

$$\begin{aligned}
\Delta Q_v = & \frac{\delta^3}{\pi \rho} \oint b_4 \beta_v \eta^3 ds + \\
& + \frac{\delta \epsilon_v}{\pi \rho} \oint b_4 \beta_v^2 \eta \cos^2 \psi_v ds + \\
& + \frac{3\delta \epsilon_H}{\pi \rho} \oint b_4 \beta_v \beta_H \eta \cos^2 \psi_H ds \quad (4b)
\end{aligned}$$

The term in δ^3 has an equivalent in the second-order contribution from the sextupoles. By properly arranging the sextupoles in families, this term should be compensated for by the sextupole strength alone (an exercise to be done, though). We will neglect therefore the term in δ^3 in both eqs (4a and b). For the remaining terms, it is sufficient to take

$$\langle \cos^2 \psi_v \rangle = \langle \cos^2 \psi_H \rangle = 1/2$$

and

$$\Delta Q_H = - \frac{\delta \epsilon_H}{2\pi \rho} \oint b_4 \beta_H^2 \eta ds + \frac{3\delta \epsilon_v}{2\pi \rho} \oint b_4 \beta_H \beta_v \eta ds \quad (5a)$$

(5)

$$\Delta Q_v = - \frac{\delta \varepsilon_v}{2\pi\rho} \oint b_4 \beta_v^2 \eta \, ds + \frac{3\delta \varepsilon_H}{2\pi\rho} \oint b_4 \beta_v \beta_H \eta \, ds \quad (51)$$

A. Contribution from Dipole error (systematic)

$$b_4 = -4.7 \times 10^{-4} / \text{in}^4$$

$$\left\{ \begin{array}{l} \Delta Q_H = -b_4 \langle \eta \beta_H^2 \rangle \delta \varepsilon_H + 3b_4 \langle \eta \beta_H \beta_v \rangle \delta \varepsilon_v \quad (6a) \\ \Delta Q_v = 3b_4 \langle \eta \beta_H \beta_v \rangle \delta \varepsilon_H - b_4 \langle \eta \beta_v^2 \rangle \delta \varepsilon_v \quad (6b) \end{array} \right.$$

where $\langle \dots \rangle$ is the average value over a single dipole

$$\langle \eta \beta_H^2 \rangle = 862.1 \, \text{m}^3$$

$$\langle \eta \beta_v^2 \rangle = 682.6 \, \text{m}^3$$

$$\langle \eta \beta_H \beta_v \rangle = 505.3 \, \text{m}^3$$

(6)

	β_H	β_V	η
first end	38.3 m	11.8 m	1.31 m
middle	23.0	23.0	1.07
second end	11.1	40.3	0.80
dipole			
before QF	45.0	10.0	1.45
before QD	10.0	45.0	0.80

Beam Parameters from RHIC CD

Gold - 100 GeV

After 10 hours with 1.1×10^9

$$\epsilon_N = 30 \pi \text{ mm.mrad} \quad (95\% \text{ of beam, H and V})$$

For a $6\sigma_{H,V}$ good-field criterion

$$\epsilon_H = \epsilon_V = 1.8 \pi \text{ mm.mrad}$$

Also the rf-bucket size with design rf system

$$\delta = \pm 0.27\%$$

$\delta \epsilon_H$ $\delta \epsilon_V$ ΔQ_H

- 0.0158

0.0278

 ΔQ_V

0.0278

- 0.0125

There is clearly a need for correction!

Two families (1) before Q_F (2) before Q_D -

Two-shift contribution from correctors

$$\Delta Q_H = - \frac{ML}{2\pi\rho} \left(b_{4F} \langle \beta_H^2 \eta \rangle_F + b_{4D} \langle \beta_H^2 \eta \rangle_D \right) \delta \epsilon_H +$$

$$+ \frac{3ML}{2\pi\rho} \left(b_{4F} \langle \beta_H \beta_V \eta \rangle_F + b_{4D} \langle \beta_H \beta_V \eta \rangle_D \right) \delta \epsilon_V \quad (7a)$$

$$\Delta Q_D = \frac{3ML}{2\pi\rho} \left(b_{4F} \langle \beta_H \beta_V \eta \rangle_F + b_{4D} \langle \beta_H \beta_V \eta \rangle_D \right) \delta \epsilon_H +$$

$$- \frac{ML}{2\pi\rho} \left(b_{4F} \langle \beta_V^2 \eta \rangle_F + b_{4D} \langle \beta_V^2 \eta \rangle_D \right) \delta \epsilon_V \quad (7b)$$

There are four terms to be cancelled with only two parameters (b_{4F} and b_{40}) - For ΔQ_H

$$-\frac{ML}{2\pi P} (b_{4F} \langle \beta_H^2 \eta \rangle_F + b_{40} \langle \beta_H^2 \eta \rangle_0) = b_4 \langle \eta \beta_H^2 \rangle \quad (8)$$

$$\frac{3ML}{2\pi P} (b_{4F} \langle \beta_H \beta_V \eta \rangle_F + b_{40} \langle \beta_H \beta_V \eta \rangle_0) = -3b_4 \langle \eta \beta_H \beta_V \rangle \quad (9)$$

and for ΔQ_V

$$\frac{3ML}{2\pi P} (b_{4F} \langle \beta_H \beta_V \eta \rangle_F + b_{40} \langle \beta_H \beta_V \eta \rangle_0) = -3b_4 \langle \eta \beta_H \beta_V \rangle \quad (10)$$

$$-\frac{ML}{2\pi P} (b_{4F} \langle \beta_V^2 \eta \rangle_F + b_{40} \langle \beta_V^2 \eta \rangle_0) = b_4 \langle \eta \beta_V^2 \rangle \quad (11)$$

M number of correctors per family
L length of each corrector

Observe that condition (9) and (10) are identical -
Therefore there are really only three conditions to be satisfied with only two parameters -
Choose only (8) and (9)

$$\langle \beta_H^2 \eta \rangle_F = 2935. \text{ m}^3$$

$$\langle \beta_H^2 \eta \rangle_D = 80$$

$$\langle \beta_H \beta_V \eta \rangle_F = 652.5$$

$$\langle \beta_H \beta_V \eta \rangle_D = 360.$$

$$\langle \beta_V^2 \eta \rangle_F = 145.$$

$$\langle \beta_V^2 \eta \rangle_D = 1620.$$

$$\left\{ \begin{array}{l} 2936 \left(\frac{ML}{2\pi\rho} \frac{b_{4F}}{b_4} \right) + 80 \left(\frac{ML}{2\pi\rho} \frac{b_{4D}}{b_4} \right) = - \cancel{852.1} 852.1 \\ 652.5 \left(\frac{ML}{2\pi\rho} \frac{b_{4F}}{b_4} \right) + 360 \left(\frac{ML}{2\pi\rho} \frac{b_{4D}}{b_4} \right) = - 505.3 \end{array} \right.$$

$$M = 144$$

$$L = 0.5 \text{ m}$$

$$\rho = 244 \text{ m}$$

The required corrector strengths are

$$\begin{cases} b_{4F} = -5.7 b_4 = -25.8 \times 10^{-4} / \text{in}^4 \\ b_{40} = -19.5 b_4 = -91.7 \times 10^{-4} / \text{in}^4 \end{cases}$$

There is though a residual tune shift given by

$$\begin{aligned} \Delta Q_s &= -\frac{ML}{2\pi p} (b_{4F} \langle \beta_v^2 \eta \rangle_F + b_{40} \langle \beta_v^2 \eta \rangle_0) \delta \varepsilon_v \\ &\quad + \text{dipole contribution } (-0.0125) \\ &= \frac{0.025}{155} \quad (\text{too large!}) \end{aligned}$$

Instead of (8) and (9) make use of (10) and (11)

$$\begin{cases} 652.5 \left(\frac{ML}{2\pi p} \frac{b_{4F}}{b_4} \right) + 360 \left(\frac{ML}{2\pi p} \frac{b_{40}}{b_4} \right) = -505.2 \\ 145 \left(\frac{ML}{2\pi p} \frac{b_{4F}}{4} \right) + 1520 \left(\frac{ML}{2\pi p} \frac{b_{40}}{b_4} \right) = -682.6 \end{cases}$$

From these the required corrector strengths are

$$\begin{cases} b_{4F} = -12.1 b_4 = -56.9 \times 10^{-4} / \text{in}^4 \\ b_{4D} = -7.9 b_4 = -37.1 \times 10^{-4} / \text{in}^4 \end{cases}$$

~~This~~ residual now is

$$\begin{aligned} \Delta Q_H &= -\frac{ML}{2\pi f} \left(b_{4F} \langle \beta_H^2 \eta \rangle_F + b_{4D} \langle \beta_H^2 \eta \rangle_D \right) \eta \varepsilon_H \\ &\quad + \text{dipole contribution } (-0.058) \\ &= 0.0155 \text{ (too large)} \end{aligned}$$

It seems the only solution is to
 add the dipoles to eliminate the b_4 error.
 Unless one wants to make use of correctors
 also in the insertions as a third family -