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# Analysis of the Decapole Systematic Error in the Dipoles and of the Correctors

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# Analysis of the Decapole Systematic Error in the Dipoles and of the Correctors

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The equations of motion in the presence of a decapole term are

$$x'' + K_H x = \frac{b_4}{\rho}\left(x^4 - 6x^2y^2 + y^4\right) \qquad (1a)$$

$$y'' + K_V y = -4\frac{b_4}{\rho}\left(x^3y - xy^3\right) \qquad (1b)$$

where $\rho$ is the bending radius in the dipoles and $b_4$ is the strength of the decapole term.

We can calculate the first-order contribution to the tune-shift vs. betatron amplitude and off-momentum value, by separating the free betatron oscillations from the closed-orbit deviation

$$x = \eta\delta + \tilde{x} \quad , \quad y = \tilde{y}$$

$\tilde{x}, \tilde{y}$ free betatron oscillations

$\eta$, dispersion (only in the horizontal plane)

$\delta$, off-momentum value

~~Here The term in $y^4$ in eq. (1a) can be ignored since it does not give any first order contribution to tune-shift. Then subtracting from (1a and b) the equations for the closed orbit~~

$$\tilde{x}'' + K_H \tilde{x} = \frac{b_4}{\rho}\left(\tilde{x}^4 + 4\eta\delta\tilde{x}^3 + 6\eta^2\delta^2\tilde{x}^2 + \right.$$

$$+ 4\eta^3\delta^3\tilde{x} - 6\tilde{x}^2\tilde{y}^2 - 12\eta\delta\tilde{x}\tilde{y}^2 -$$

$$\left. - 6\eta^2\delta^2\tilde{y}^2 + \tilde{y}^4\right) \qquad (2a)$$

$$\tilde{y}'' + K_V \tilde{y} = -4\frac{b_4}{\rho}\left(\tilde{x}^3\tilde{y} + 3\eta\delta\tilde{x}^2\tilde{y} + 3\eta^2\delta^2\tilde{x}\tilde{y} + \right.$$

$$\left. + \eta^3\delta^3\tilde{y} - \tilde{x}\tilde{y}^3 - \eta\delta\tilde{y}^3\right) \qquad (2b)$$

The tune-shifts are

$$\Delta Q_H = -\frac{1}{4\pi\rho}\oint \beta_H b_4\left(\tilde{x}^3 + 4\eta\delta\tilde{x}^2 + 6\eta^2\delta^2\tilde{x} + 4\eta^3\delta^3 + \right. \qquad (3a)$$

$$\left. - 6\tilde{x}\tilde{y}^2 - 12\eta\delta\tilde{y}^2\right) ds$$

where we have ignored the last two terms at the r.h. side of eq. 2a since they do not give contribution to first-order

$$\Delta Q_V = \frac{1}{4\pi\rho}\oint 4\beta_V b_4\left(\tilde{x}^3 + 3\eta\delta\tilde{x}^2 + 3\eta^2\delta^2\tilde{x} + \eta^3\delta^3 + \right.$$

$$\left. - \tilde{x}\tilde{y}^2 - \eta\delta\tilde{y}^2\right) ds \qquad (3b)$$

where the integrals are taken over the full circumference of the ring -

To first-order, the tune-shifts are averaged over several betatron oscillations, thus the terms in $\tilde{x}^4$ and $\tilde{x}^3$ in both eqs (3a, b) give in average zero contribution. ~~Similarly~~

Again in first-order, we can set

$$\tilde{x}^2 = \frac{\varepsilon_H}{\pi} \beta_H \cos^2 \psi_H$$

$$\tilde{y}^2 = \frac{\varepsilon_V}{\pi} \beta_V \cos^2 \psi_V$$

~~Hence~~ Finally

$$\Delta Q_H = - \frac{\delta^3}{\pi \rho} \oint b_4 \beta_H \eta^3 \, ds \; +$$

$$- \frac{\delta \, \varepsilon_H}{\pi \rho} \oint b_4 \beta_H^2 \eta \cos^2 \psi_H \, ds \; + \qquad (4a)$$

$$+ \frac{3 \delta \varepsilon_V}{\pi \rho} \oint b_4 \beta_H \beta_V \eta \cos^2 \psi_V \, ds$$

and

$$\Delta Q_V = \frac{\delta^3}{\pi\rho} \oint b_4 \beta_V \eta^3 \, ds \; +$$

$$\frac{\delta \varepsilon_V}{\pi\rho} \oint b_4 \beta_V^2 \eta \cos^2\psi_V \, ds \; +$$

$$+ \; \frac{3\delta \varepsilon_H}{\pi\rho} \oint b_4 \beta_V \beta_H \eta \cos^2\psi_H \, ds \qquad (4b)$$

The term in $\delta^3$ has an equivalent in the second-order contribution from the sextupoles. By properly arranging the sextupoles in families, this term should be compensated for by the sextupole strength alone (an exercise to be done, though). We will neglect therefore the term in $\delta^3$ in both eq-s (4a and b). For the remaining terms, it is sufficient to take

$$\langle \cos^2\psi_V \rangle = \langle \cos^2\psi_H \rangle = 1/2$$

and

$$\Delta Q_H = -\frac{\delta \varepsilon_H}{2\pi\rho} \oint b_4 \beta_H^2 \eta \, ds + \frac{3\delta \varepsilon_V}{2\pi\rho} \oint b_4 \beta_H \beta_V \eta \, ds \qquad (5a)$$

$$\Delta Q_V = -\frac{\delta \varepsilon_V}{2\pi\rho} \oint b_4 \beta_V^2 \eta \, ds + \frac{3\delta\varepsilon_H}{2\pi\rho} \oint b_4 \beta_V \beta_H \eta \, ds \quad (5b)$$

A. Contribution from Dipole error (systematic)

$$b_4 = -4.7 \times 10^{-4} / \text{in}^4$$

$$\begin{cases} \Delta Q_H = -b_4 \langle \eta \beta_H^2 \rangle \delta\varepsilon_H + 3 b_4 \langle \eta \beta_H \beta_V \rangle \delta\varepsilon_V & (6a) \\ \Delta Q_V = 3 b_4 \langle \eta \beta_H \beta_V \rangle \delta\varepsilon_H - b_4 \langle \eta \beta_V^2 \rangle \delta\varepsilon_V & (6b) \end{cases}$$

where $\langle \ldots \rangle$ is the average value over a single dipole

$$\langle \eta \beta_H^2 \rangle = 862.1 \text{ m}^3$$

$$\langle \eta \beta_V^2 \rangle = 682.6 \text{ m}^3$$

$$\langle \eta \beta_H \beta_V \rangle = 505.3 \text{ m}^3$$

| | | $\beta_H$ | $\beta_V$ | $\eta$ |
|---|---|---|---|---|
| first end | dipole | 38.3 m | 11.8 m | 1.31 m |
| middle | dipole | 23.0 | 23.0 | 1.07 |
| second end | dipole | 11.1 | 40.3 | 0.80 |
| QF | | 45.0 | 10.0 | 1.45 |
| QD | | 10.0 | 45.0 | 0.80 |

Beam parameters from RHIC CD

Gold – 100 GeV
After 10 hours with $1.1 \times 10^9$

$\varepsilon_N = 30\pi$ mm.mrad (95% of beam, H and V)

For a $6\sigma_{H,V}$ good-field criterion

$$\boxed{\varepsilon_H = \varepsilon_V = 1.8\pi \text{ mm}\cdot\text{mrad}}$$

Also the rf-bucket size with design rf system

$$\boxed{\delta = \pm 0.27\%}$$

| | $\delta\varepsilon_H$ | $\delta\varepsilon_V$ |
|---|---|---|
| $\Delta Q_H$ | $-0.0158$ | $0.0278$ |
| $\Delta Q_V$ | $0.0278$ | $-0.0125$ |

There is clearly a need for correction!

Two families (1) before $Q_F$ (2) before $Q_D$ —

Tune-shift contribution from correctors

$$\Delta Q_H = -\frac{ML}{2\pi\rho}\left(b_{4F}\langle\beta_H^2\eta\rangle_F + b_{4D}\langle\beta_H^2\eta\rangle_D\right)\delta\varepsilon_H + \frac{3ML}{2\pi\rho}\left(b_{4F}\langle\beta_H\beta_V\eta\rangle_F + b_{4D}\langle\beta_H\beta_V\eta\rangle_D\right)\delta\varepsilon_V \quad (7a)$$

$$\Delta Q_D = \frac{3ML}{2\pi\rho}\left(b_{4F}\langle\beta_H\beta_V\eta\rangle_F + b_{4D}\langle\beta_H\beta_V\eta\rangle_D\right)\delta\varepsilon_H - \frac{ML}{2\pi\rho}\left(b_{4F}\langle\beta_V^2\eta\rangle_F + b_{4D}\langle\beta_V^2\eta\rangle_D\right)\delta\varepsilon_V \quad (7b)$$

There are four terms to be cancelled with only two parameters ( $b_{4F}$ and $b_{4D}$ ). For $\Delta Q_H$

$$-\frac{ML}{2\pi\rho}\left(b_{4F}\langle\beta_H^2\eta\rangle_F + b_{4D}\langle\beta_H^2\eta\rangle_D\right) = b_4\langle\eta\beta_H^2\rangle \quad (8)$$

$$\frac{3ML}{2\pi\rho}\left(b_{4F}\langle\beta_H\beta_V\eta\rangle_F + b_{4D}\langle\beta_H\beta_V\eta\rangle_D\right) = -3b_4\langle\eta\beta_H\beta_V\rangle \quad (9)$$

and for $\Delta Q_V$

$$\frac{3ML}{2\pi\rho}\left(b_{4F}\langle\beta_H\beta_V\eta\rangle_F + b_{4D}\langle\beta_H\beta_V\eta\rangle_D\right) = -3b_4\langle\eta\beta_H\beta_V\rangle \quad (10)$$

$$-\frac{ML}{2\pi\rho}\left(b_{4F}\langle\beta_V^2\eta\rangle_F + b_{4D}\langle\beta_V^2\eta\rangle_D\right) = b_4\langle\eta\beta_V^2\rangle \quad (11)$$

M number of correctors per family
L length of each corrector

Observe that condition (9) and (10) are identical. Therefore there are really only three conditions to be satisfied with only two parameters.
Chose only (8) and (9)

$$\langle \beta_H^2 \eta \rangle_F = 2936.\ m^3$$

$$\langle \beta_H^2 \eta \rangle_D = 80.$$

$$\langle \beta_H \beta_V \eta \rangle_F = 652.5$$

$$\langle \beta_H \beta_V \eta \rangle_D = 360.$$

$$\langle \beta_V^2 \eta \rangle_F = 145.$$

$$\langle \beta_V^2 \eta \rangle_D = 1620.$$

$$\left\{ \begin{array}{l} 2936\left(\frac{ML}{2\pi\rho}\frac{b_{4F}}{b_4}\right) + 80\left(\frac{ML}{2\pi\rho}\frac{b_{4D}}{b_4}\right) = -862.1 \\ 652.5\left(\frac{ML}{2\pi\rho}\frac{b_{4F}}{b_4}\right) + 360\left(\frac{ML}{2\pi\rho}\frac{b_{4D}}{b_4}\right) = -505.3 \end{array} \right.$$

$M = 144$

$L = 0.5\,m$

$\rho = 244\ m$

The required corrector strengths are

$$\begin{cases} b_{4F} = -5.7\ b_4 = -26.8 \times 10^{-4} / in^4 \\ b_{4D} = -19.5\ b_4 = -91.7 \times 10^{-4} / in^4 \end{cases}$$

There is though a residual tune shift given by

$$\Delta Q_D = -\frac{ML}{2\pi\rho}\left( b_{4F} \langle \beta_V^2 \eta \rangle_F + b_{4D} \langle \beta_V^2 \eta \rangle_D \right) \delta \varepsilon_V$$

+ dipole contribution $(-0.0125)$

$$= 0.0155 \quad \text{(too large !)}$$

Instead of (8) and (9) make use of (10) and (11)

$$\begin{cases} 652.5 \left( \frac{ML}{2\pi\rho} \frac{b_{4F}}{b_4} \right) + 360 \left( \frac{ML}{2\pi\rho} \frac{b_{4D}}{b_4} \right) = -505.2 \\ 145. \left( \frac{ML}{2\pi\rho} \frac{b_{4F}}{4} \right) + 1620. \left( \frac{ML}{2\pi\rho} \frac{b_{4D}}{b_4} \right) = -682.6 \end{cases}$$

From these the required corrector strengths are

$$\begin{cases} b_{4F} = -12.1\ b_4 = -56.9\times 10^{-4}/\text{in}^4 \\ b_{4D} = -7.9\ b_4 = -37.1\times 10^{-4}/\text{in}^4 \end{cases}$$

~~The~~ residual now is

$$\Delta Q_H = -\frac{ML}{2\pi\rho}\left(b_{4F}\langle\beta_H^2\eta\rangle_F + b_{4D}\langle\beta_H^2\eta\rangle_D\right)\eta\,\varepsilon_H$$

$$+ \text{dipole contribution } (-0.058)$$

$$= 0.0155 \quad (\text{too large})$$

It seems the only solution is to shim the dipoles to eliminate the $b_4$ error. Unless one wants to make use of correctors also in the insertions as a third family.