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Proposed Criterion for the Dynamic Aperture

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It has been pointed out by F. Dell and H. Hahn that the tracking studies that have been done, have been asking for more stable aperture than is required. This note attempts to state requirements for the stable aperture in a more precise way.

The dynamic aperture requirement may be stated by specifying a 4-dimensional surface in x, x', y, y' space such that any particle whose initial coordinates are within this surface is required to be stable. This 4-dimensional surface may be called the safety surface, because this surface is considerably outside the 4-dimensional surface that contains most of the beam. Stability is required in this larger region to add an extra measure of safety.

Intra beam scattering assumes a particle distribution in x, x', y, y' that is given by

$$\rho(x, x', y, y') = \frac{1}{\pi^2 \bar{\varepsilon}_x \bar{\varepsilon}_y} \exp\left(- \frac{\varepsilon_x(x, x')}{\bar{\varepsilon}_x} - \frac{\varepsilon_y(y, y')}{\bar{\varepsilon}_y} \right)$$

where

$$\bar{\varepsilon}_x = 2 \sigma_x^2 / \beta_x ,$$

$$\varepsilon_x(x, x') = \gamma_x x^2 + 2 \alpha_x x x' + \beta_x x'^2 ,$$

$$\gamma_x = (1 + \alpha_x^2) / \beta_x$$

$\bar{\varepsilon}_x$ is the average value of $\varepsilon_x(x, x')$;

$$\bar{\varepsilon}_x = \int dx\, dx'\, dy\, dy'\ \rho(x, x', y, y')\ \varepsilon_x(x, x').$$

The surface of constant $\rho(x, x', y, y')$ is given by

$$\varepsilon_x(x, x') + \varepsilon_y(y, y') = \text{Const.}$$

when $\bar{\varepsilon}_x = \bar{\varepsilon}_y = \bar{\varepsilon}$, or

$$\mathcal{E}_T(x, x', y, y') = \text{const},$$

where $\mathcal{E}_T = \mathcal{E}_x + \mathcal{E}_y$.

The ~~surfa~~ 4-dimensional surface that contains 90% of the beam is

$$\mathcal{E}_T(x, x', y, y') = 4\bar{\mathcal{E}} = 8\sigma_x^2/\beta_x$$

On this surface the maximum x is given by

$$\mathcal{E}_x = 4\bar{\mathcal{E}} = 8\sigma_x^2/\beta_x = (\sqrt{8}\,\sigma_x)^2/\beta_x,$$

or $$x_{max} = \sqrt{8}\,\sigma_x = 2.8\,\sigma_x.$$

~~The 90%~~ This may be compared with the usual 95% emittance $6\sigma_x^2/\beta_x = 3\bar{\mathcal{E}}$. ~~the~~ The 2-dimensional surface

$$\mathcal{E}_x(x, x') = 3\bar{\mathcal{E}}, \quad x_{max} = \sqrt{6}\,\sigma_x = 2.5\,\sigma_x$$

contains 95% of the beam.

The safety surface may now be proposed as the surface

$$\varepsilon_T(x, x', y, y') = 18\bar{\varepsilon} = 36\sigma_x^2/\beta_x .$$

Particles whose initial coordinates are inside this surface are required to be stable.

Note that the safety surface is considerably outside the 90% surface $\varepsilon_T = 4\bar{\varepsilon}$.

On this surface the maximum x is given by

$$\varepsilon_x = 18\bar{\varepsilon} = 36\sigma_x^2/\beta_x = (6\sigma_x)^2/\beta_x$$

or $$x_{max} = 6\sigma_x .$$

If one starts at x, x', y, y' where $\varepsilon_x = \varepsilon_y$, then the maximum x is given by

$$\varepsilon_x = 9\bar{\varepsilon} = 18\sigma_x^2/\beta_x = (6\sigma_x/\sqrt{2})^2/\beta_x$$

or $$x_{max} = 6\sigma_x/\sqrt{2} = 4.3\ \sigma_x$$

It may be noted, that, in the same spirit, the dynamic aperture is not just a number but is also a 4-dimensional surface in x, x', y, y', such that any particle, whose initial coordinates are inside this surface, is stable, and particles outside this surface are unstable.