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BROOKHAVEN NATIONAL LABORATORY

RHIC/RF Technical Note No. 10

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Introduction

The shunt impedance of the rf accelerator cavities is measured using a beadpull test set. The method is based upon the field perturbation technique developed by Slater.[1] The perturbation of the electromagnetic fields caused by a metal or ceramic sphere as it moves along the axis of the cavity causes a small change in the resonant frequency of the cavity. The magnitude of this frequency shift, which can be measured to high accuracy, is proportional to the value of the square of the electric and magnetic field at the point of perturbation. A beadpull test set was developed and results were compared to computer models of a test cavity.

Computational

The SUPERFISH subset of the Poisson codes were used to calculate the shunt impedance and Q of the test cavity. SUPERFISH calculates either TE or TM modes of axisymmetric cylindrical cavities based on the boundary conditions. The SUPERFISH results were used as a method of comparison for the data obtained from the beadpull test. SUPERFISH is quite accurate when dealing with the fundamental mode and with perfectly axisymmetrical cavities. However, when the cavity symmetry is broken by either tuners, amplifiers, or mode dampers, the code results are no longer exact and another method is needed to measure the field distribution. The objective of the beadpull is to supplement our computational tools in determining shunt impedance and field patterns. This is especially true for transverse modes in asymmetric cavities.

The boundary conditions that delineated the physical regions inside the cavity was the input file for AUTOMESH. AUTOMESH creates a file called TAPE73 which is used for the LATTICE program. LATTICE fills the defined regions with an irregular triangular mesh that fits the boundaries of the cavity. SUPERFISH is the eigenvalue solver which calculates the resonant frequency and determines the magnetic field values throughout the mesh problem. The cavity boundary is shown in Appendix 1. The SUPERFISH output routine, SF01, accepts the input file and prints information about the properties of the cavity. All quantities appearing in the output of SUPERFISH are normalized by assuming that the average electric field on the axis is 1MV/meter. The SF01 output file contains the value of E_z at points on the axis, the values of moment integrals of E_z along the axis, and tables for each segment on which power and frequency calculations were sought.The SUPERFISH output postprocessor (Appendix 2) also provides us with the data we need to calculate R/Q. R is the shunt impedance and Q is the ratio of the time-average energy stored at a resonant frequency to the energy dissipated in one period of this frequency. According to SUPERFISH calculations, the shunt impedance and quality factor,Q, for the small scale model was 0.394543MΩ and 5716, respectively. The R/Q value was calculated as 69.02 ohms. This value of R/Q could now be used as a comparison in the bead perturbation test.

BEAD PERTURBATION TEST

Theory

The perturbation theory demonstrates that the introduction of a small body into a resonant electromagnetic cavity will change the frequency by an amount that is related to the size, shape, and material of the object and the values of the electric and magnetic fields of the cavity. There are two important conditions that must be remembered when applying the perturbation theory.[2] One, is that the theory is only applicable for infinitesimal perturbations, and strictly speaking cannot be applied for finite distortions. Second, the formulas about to be derived will not hold when any distortion is made from a situation not originally satisfying the boundary conditions. This is due to the fact that the perturbation formula is based on the unperturbed electric and magnetic fields satisfying the boundary conditions prior to the time when the perturbation is introduced. In practice, this method may be used as long as the perturbation is small and does not distort the fields patterns in the cavity.

The frequency shift due to the introduction of a perturbing sphere in an electromagnetic resonant cavity is derived by Papas[3] as:

$$\frac{-\delta\omega_0}{\omega_0} = \frac{3\mu_0 \frac{\mu_1-\mu_0}{\mu_1+2\mu_0} H_0 H_0^* + 3\epsilon_0 \frac{\epsilon_1-\epsilon_0}{\epsilon_1+2\epsilon_0} E_0 E_0^*}{4U} \Delta V$$

where ω_o = resonant frequency

$\epsilon_1 = \epsilon_o \epsilon_r$

$\epsilon_o = 8.85 * 10^{-12}$ (F/m)

V = volume of the bead (m^3)

U = total stored energy in cavity (J)

A perturbation consisting of a dielectric sphere introduced into the cavity affects only electric fields, and so one can neglect the magnetic field contributions. Therefore, for a ceramic sphere, the equation above becomes:

$$\frac{-\delta\omega_0}{\omega_0}=\frac{3\epsilon_0\frac{\epsilon_1-\epsilon_0}{\epsilon_1+2\epsilon_0}E_0E_0^*}{4U}\Delta V$$

Since a perfectly conducting metal can be considered as having $\mu_1=0$ and $\epsilon_1=\infty$, the frequency shift produced by a metal sphere is:

$$\frac{-\delta\omega_0}{\omega_0}=\frac{-\frac{3}{2}\mu_0H_0H_0^*+3\epsilon_0E_0E_0^*}{4U}\Delta V$$

It should be noted that the frequency shift is proportional to the magnitude of the field strength and differs in sign for electric and magnetic fields. In order to determine magnetic field properties, it is necessary to use both dielectric and metallic spheres of equal size and subtract the data sets. Solving for the electric field of the ceramic sphere gives:

$$E=\left[\frac{4U}{3\epsilon_0\Delta V}\frac{\epsilon_r+2}{\epsilon_r-1}\frac{\delta\omega_0}{\omega_0}\right]^{1/2}$$

For a cavity of length l at power P,

$$R=\frac{\left(\frac{1}{l}\int_0^l E\,dz\right)^2}{2P}$$

Since $Q = \frac{2\pi\,(\text{stored energy})}{\text{energy loss/cycle}} = \frac{\omega_0 W}{P}$

then R/Q for the ceramic sphere can be found with the following equation:

$$\frac{R}{Q}=\frac{2}{3\epsilon_0\omega_0^2 l\Delta V}\frac{\epsilon_r+2}{\epsilon_r-1}\left(\int_0^l\sqrt{\delta\omega_0}\,dl\right)^2$$

Setup

The beadpull test on the cavity was done once with a metal sphere and again with a dielectric sphere. The sphere was brought through the cavity fields on a nylon monofilament which was suspended in a loop over four small reels. The tension in the monofilament was kept very taut in order to minimize sagitta and the amplitude of vertical vibrations. A very important fact to note is that this monofilament does not affect the fields in the cavity. One of the reels was driven by a motor in order to keep the velocity of the sphere constant. The bead would move through the entire gap, stop, and then return to its original position along the same center axis. The resonant frequency of the cavity was found using a network analyzer. In the case of our test cavity, the resonant frequency was found to be 141.39Mhz. This frequency is entered into a signal generator whose output is coupled to the RF cavity. The pickup signal from the other end of the cavity is amplified and fed into a mixer with the signal from the generator. The diagram of the bead perturbation setup is shown in Appendix 3. The beadpull test set uses a double balanced mixer (DBM) as a phase error detector to phase lock the synthesized frequency source to the cavity.[2] This is done by comparing a sample of the drive signal to the pickup from the cavity. The two signals entering the mixer are adjusted, with a phase shifter, so that the reading on the output of the mixer is 0 volts DC when there is no perturbation to the cavity. The output is then used to drive the voltage controlled oscillator (VCO) to maintain a zero phase error at the DBM. The voltage required to do this is proportional to the frequency shift of the cavity due to the perturbation introduced. This voltage is read by an oscilloscope via Labview. This voltage is directly proportional to the frequency to which the VCO in the signal generator was driven to maintain zero phase error at the mixer. In order to directly relate this change in voltage with a shift in frequency, a calibration sequence programmed a set of three frequency steps (5kHz, 10kHz, 15kHz) to the signal generator. See Figure 1 for an example of the calibration sequence of the metal sphere. The resulting oscilloscope reading is then stored in memory.

Calculations

The output data from Labview was arranged in two columns, consisting of time(position) and voltage(frequency) data, and entered into MATHCAD where the remaining calculations

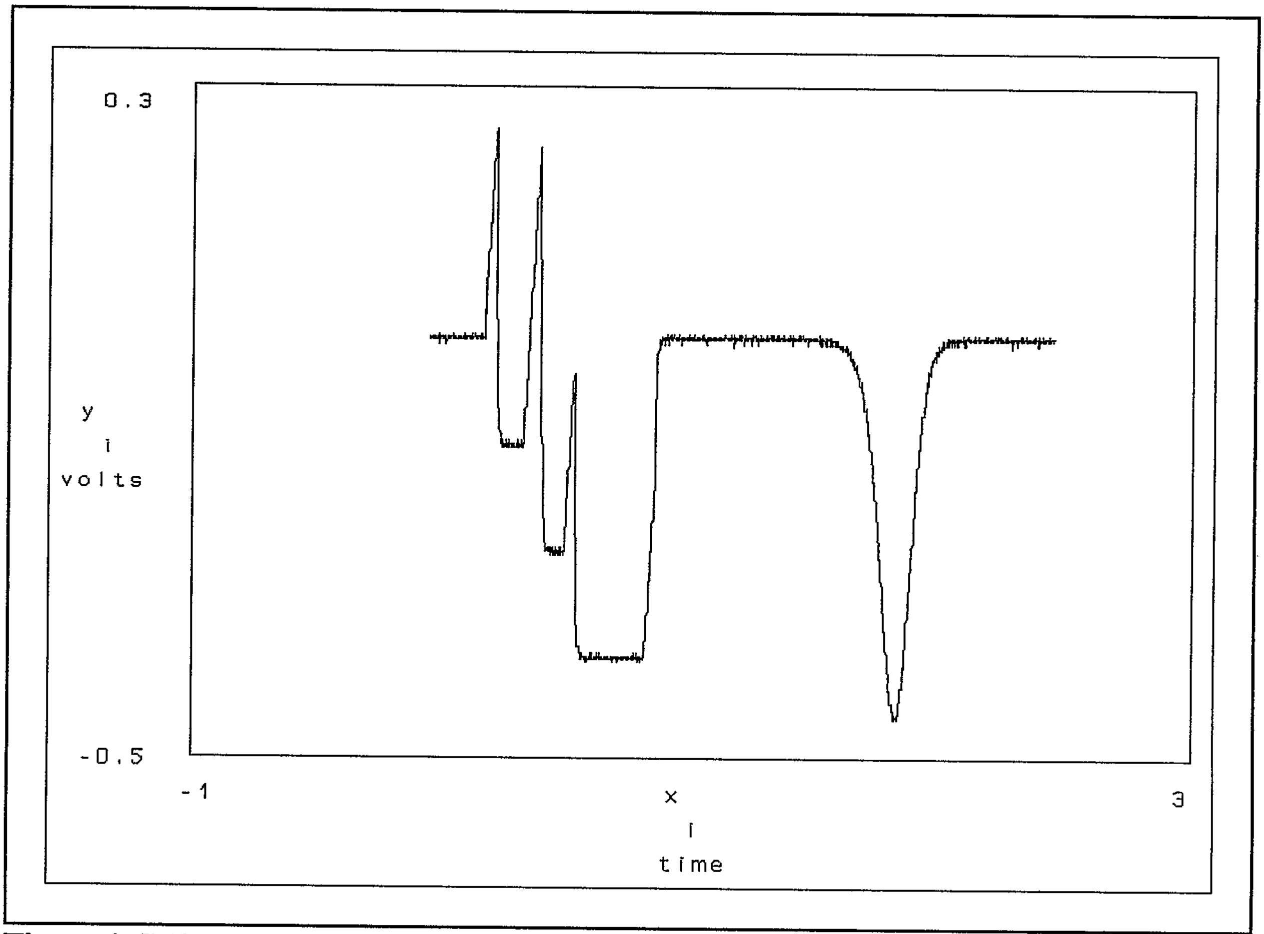


Figure 1 Calibration Sequence of Metal Sphere

were done. See Figure 2 for the graph of output data for the metal sphere.

The value of ϵ_r for the ceramic sphere was found to be 9.7. In the case of the metal sphere, $(\epsilon_r+2)/(\epsilon_r-1)$ drops out of the equation. The beadpull was done on axis for the fundamental mode and it was assumed that the magnetic field is zero in the gap. Thus the equation for $\delta\omega_0/\omega_0$ of the metal sphere reduces to:

$$\frac{\delta\omega_0}{\delta\omega_0} = \frac{3\epsilon_0 E_0 E_0^*}{4U}\Delta V$$

The perturbing volume, ΔV, for the ceramic sphere equals the volume of the entire sphere minus the volume of the hole. However, the volume of the hole in the metal sphere was

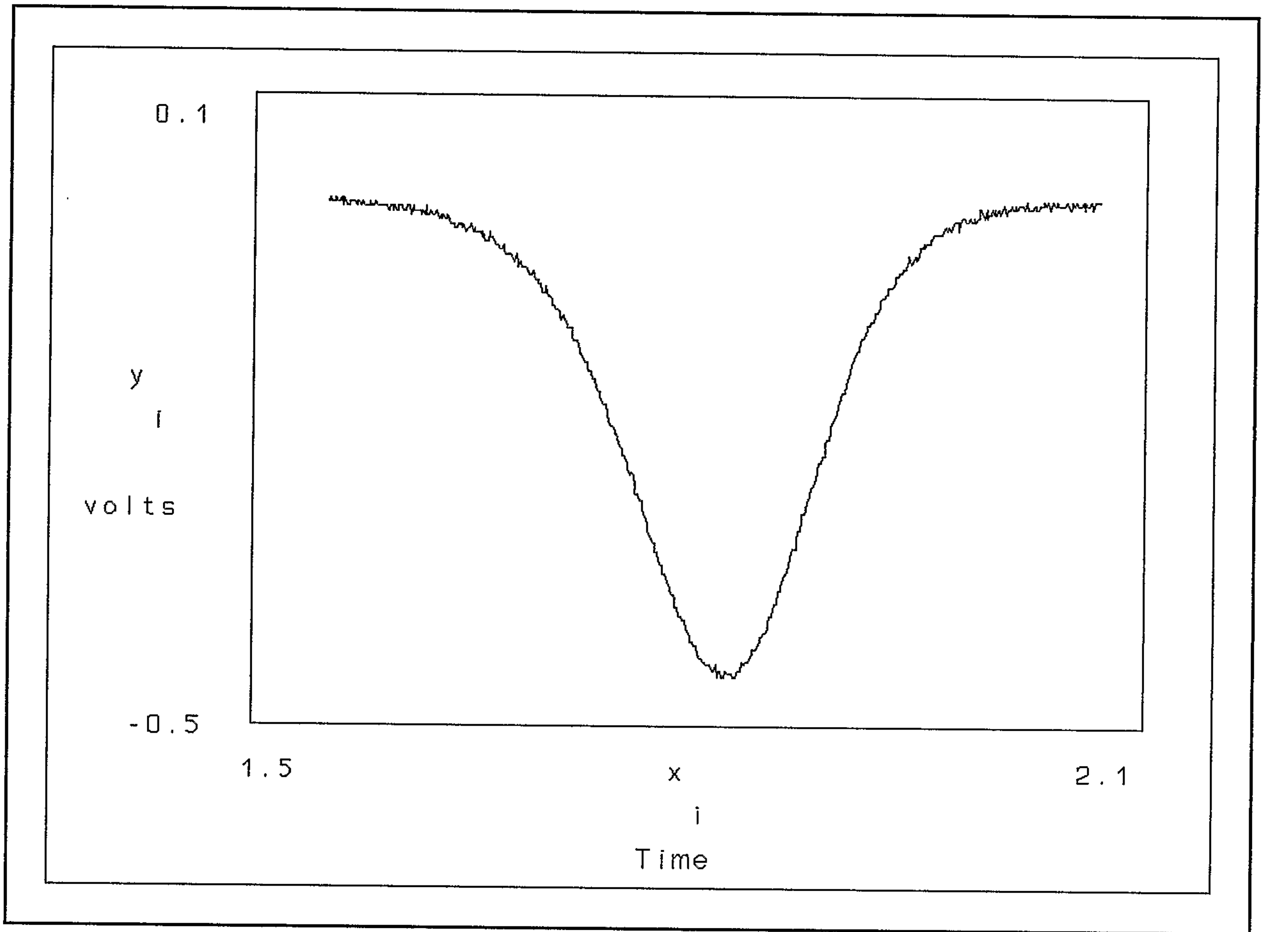


Figure 2 Output Data for Metal Sphere

neglected because the electric field goes to zero inside a hollow metal sphere. The length, l, of the cavity, refers to the length of all the field regions over which the integration is performed. Since the data is taken as a function of time, and must be converted to dimensions of length, l was found by taking the number of seconds per reading set on the scope and multiplying it by the number of readings in the beadpull data, and then multiplying it by the speed in which the sphere was moving. The accuracy of this measurement is maintained by using a stepper motor with a constant and highly repeatable number of steps per second.

As can be seen from the MATHCAD calculations in Appendix 4, the R/Q value for the ceramic sphere was 70.137Ω. Similar calculations were performed for the metal sphere resulting in an R/Q value of 70.22Ω. Therefore, the metal sphere differs from the reference value of 69.02Ω in SUPERFISH by only 1.7%, while the ceramic sphere data differs by 1.6% which demonstrates that the bead perturbation results are in excellent agreement with our computational results. If the magnetic field in the gap was non-zero, a value of R/Q less than SUPERFISH would have resulted. A non-zero magnetic field could be caused by a beadpull that is not exactly on axis, thereby causing a reduction in the measured frequency shift.

The quality factor, Q, of the system was found on a network analyzer. Two methods were employed to find the value of Q. A Smith Chart(Appendix 5) was utilized to find the unloaded Q of the cavity. The unloaded Q_0 includes only those losses in the cavity alone. A log magnitude scale(Appendix 6), on the other hand, was used to find the loaded Q_L, which includes all sources of dissipation. The relation between the various Q's is:

$$\frac{1}{Q_L}=\frac{1}{Q_0}+\frac{\beta}{Q_0}$$

Since the cavity was found to be undercoupled, the value of $\beta << 1$. From this, the value of Q_L should be comparable with that of Q_0, but slightly smaller. Usually it is the value of Q_0 that is used to find the shunt impedance, but since our measured value of Q_0 was very similar to Q_L but still smaller, we took the average of the two(1,891) as our overall value of Q. As a result, the shunt impedance of the cavity was found to be 133kΩ. This departure from the theoretical value is due to additional losses in the test cavity that are not taken into account by SUPERFISH. These losses are due to the cavity's rf joints and to the substitution of brass for copper in many sections of the cavity which lowers the conductivity. For the actual 26MHz accelerating cavity, the measured Q is $\approx$85% of the theoretical value and a closer agreement between calculated and measured shunt impedance is expected.

Conclusion

R/Q of an rf cavity was measured using the bead perturbation technique and compared with computational (SFISH) results. The agreement, to within 2% confirms the reliability of the approach for measuring R/Q of high Q structures. Future work will include automation of the data reduction utilizing Labview, a program development application which uses a graphical programming language to create programs in block diagram form. Another task will be designing a low-pass filter on the output of the DBM in order to reduce noise to provide a better signal to noise ratio in larger cavities where the frequency shifts to be measured are smaller.

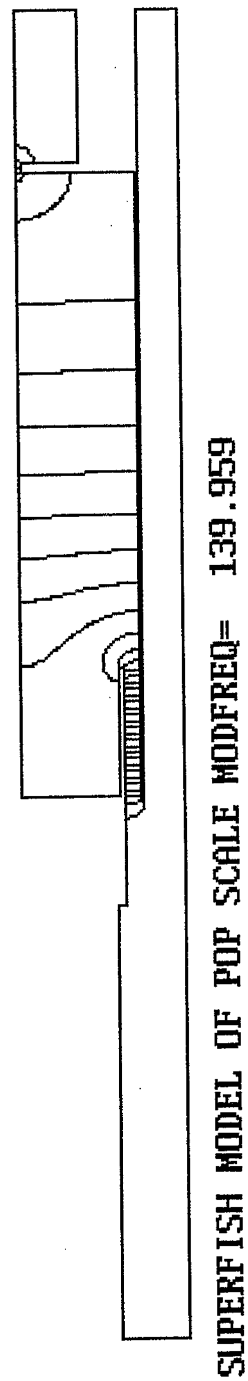
References

1. S.C. Slater, Microwave Electronics, March 1950.

2. C.H. Papas "On Perturbation Theory of Electromagnetic Cavity Resonators" California Institute of Technology, March 1954,Pasadena, California.

3. L.C. Maier Jr. "Field Strength Measurements in Resonant Cavities" Massachusetts Institute of Technology, November 2, 1949.

4. James Rose, "Radio Frequency Quadrupole Accelerator", Polytechnic University, June 1990.



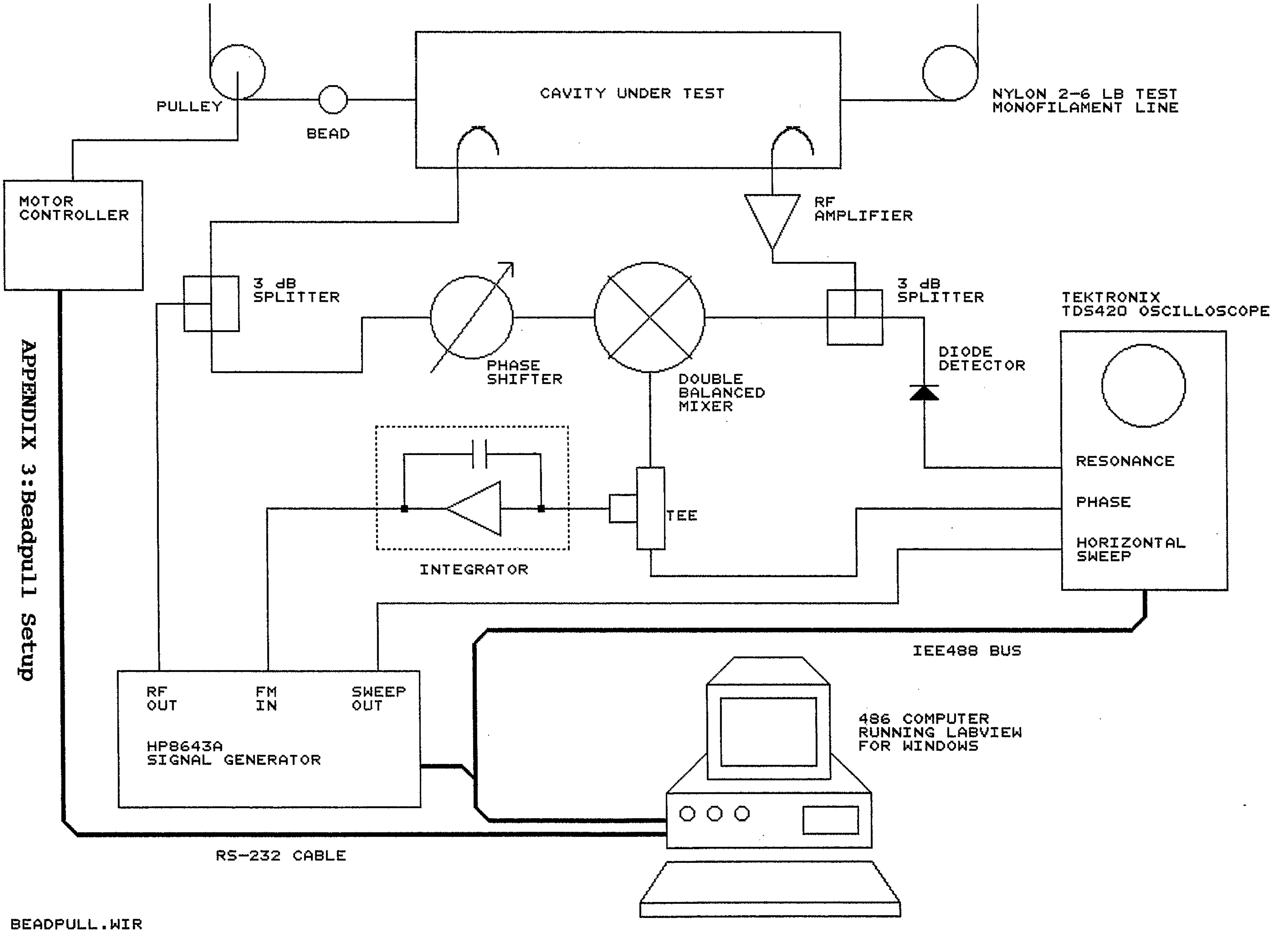
APPENDIX 1: Cavity Boundary with Field Lines

```
LIST    3365    3410      07-14-93 17:04  OUTSF1.LIS
Full cavity length [2L]  =  122.7354 cm        Diameter =   15.2400 cm
Mesh problem length [L]  =   61.3677 cm
Full drift-tube gap [2g] =   40.0000 cm
Beta                     = 0.5729926      Proton energy =   206.567 MeV
Frequency [f] (starting value = 139.000) =             139.958710 MHz
Eo normalization factor (CON(74)=ASCALE) for  1.000 MV/m =   90136.5
Stored energy [U] for mesh problem only =              3101.72217 mJ
Power dissipation [P] for mesh problem only =           477180.28 W
Q (2.0*pi*f(Hz)*U(J)/P(W)) =                                 5716
Transit time factor [T] =                                 0.33053
Shunt impedance [Z] mesh problem only, ((Eo*L)**2/P) =    0.78922 Mohm
Shunt impedance per unit length [Z/L] =                     1.286 Mohm/m
Effective shunt impedance per unit length [Z/L*T*T] =       0.140 Mohm/m
Magnetic field on outer wall =                              23926 A/m
Hmax for wall and stem segments at z= 53.60,r=  2.06 cm =   75036 A/m
Emax for wall and stem segments at z= 24.91,r=  2.00 cm =  156.513 MV/m

   Beta        T         Tp        S        Sp       g/L         Z/L
0.57299262  0.33053  0.18483  0.94014  0.06382  0.325904    1.286049


ISEG    zbeg      rbeg      zend      rend   Emax*epsrel   Power     df/dz    df/dr
        (cm)      (cm)      (cm)      (cm)     (MV/m)       (W)        (MHz/mm)
Command                                    St: d8kMpcwT Keys: arrows X=exit ?=Help
```

```
LIST    3388    3410      07-14-93 17:04  OUTSF1.LIS
Wall-----------------------------------------------------------------------------Wall
  1    0.0000    0.0000    0.0000   3.1686   0.0000        0.0000    0.0000    0.0000
  2    0.0000    3.1686   20.0000   3.1686   0.1433        0.0018    0.0000    0.0000
  3   20.0000    3.1686   20.0000   2.7241   0.3567        0.0010    0.0000    0.0000
  4   20.0000    2.7241   30.9690   2.7243 105.4048     8765.9063    0.0002    5.8327
  5   30.9690    2.7243   30.9690   3.0162  94.6344     1169.9579    0.4215    0.0000
  6   30.9690    3.0162   25.0000   3.0162  38.9049    26456.7930    0.0000   -0.1528
  7   25.0000    3.0162   25.0000   7.6200   1.3865    12684.8428   -0.1172    0.0000
  8   25.0000    7.6200   61.3677   7.6200  30.6048    99416.7813    0.0000   -0.4806
  9   61.3677    7.6200   61.3677   4.7333   0.5631    15241.3057   -0.1410    0.0000
 10   61.3677    4.7333   54.3150   4.7333   1.5684    47118.2891    0.0000   -0.4334
 11   54.3150    4.7333   54.3150   7.4150  24.7467    13812.3125   -0.0652    0.0000
 12   54.3150    7.4150   53.7710   7.4150  37.0998     1993.1312    0.0000    0.2884
 13   53.7710    7.4150   53.7710   2.0638  24.2711    29881.9375   -0.2080    0.0000
 14   53.7710    2.0638   32.0000   2.0638  41.1049   202982.7340    0.0000   -1.3085
 15   32.0000    2.0638   31.5000   2.0002  51.4568     2928.4282    0.0159    0.1252
 16   31.5000    2.0002   24.8247   2.0002 156.5135    14723.4805    0.0000    7.9571
 17   24.8247    2.0002   24.8247   1.8414 153.7094        2.6982    0.4522    0.0000
 18   24.8247    1.8414   61.3677   1.8414  65.8864        1.7341    0.0000    0.1138
 19   61.3677    1.8414   61.3677   0.0000   0.0000        0.0000    0.0000    0.0000
Wall------------------------------------ Total =  477180.3130 --------------Wall

Command     *** End-of-file ***            St: d8kMpcwT Keys: arrows X=exit ?=Help
```

APPENDIX 2: SUPERFISH Output

APPENDIX 3: Beadpull Setup



CERAMIC BEAD

$A := \mathrm{READPRN}(cerpull)$

$Measure := A^{<1>}$:Voltage Matrix

$Calibration := 4.09167 \cdot 10^{4} \cdot 2 \cdot \pi \cdot Measure$:Calibration to Frequency

$Sqrtvolt := \overrightarrow{\sqrt{|Calibration|}}$:Square root of each Frequency

$sumsqrt := \sum \left(Sqrtvolt \cdot .72870 \cdot 10^{-4} \right)$:Sum of the square roots

$sumsqrt = 8.172$

$Radcer := \frac{.1245 \cdot 2.54}{100}$ $Radcer = 0.003$:Radius of the Ceramic Bead (m)

$Fullbeadvol := \frac{4 \cdot \pi \cdot Radcer^{3}}{3}$ $Fullbeadvol = 1.325 \cdot 10^{-7}$:Volume of entire Bead (m)

$Radhole := \frac{.04 \cdot 2.54}{100}$ $Volhole := \pi \cdot Radhole^{2} \cdot Radcer$ $Volhole = 1.026 \cdot 10^{-8}$:Volume of hole

$Newbeadvol := Fullbeadvol - Volhole$ $Newbeadvol = 1.222 \cdot 10^{-7}$:Corrected volume of Bead

$length := \frac{50}{1000} \cdot \frac{100}{5000} \cdot 1108 \cdot .072870$ $length = 0.081$:Length of gap

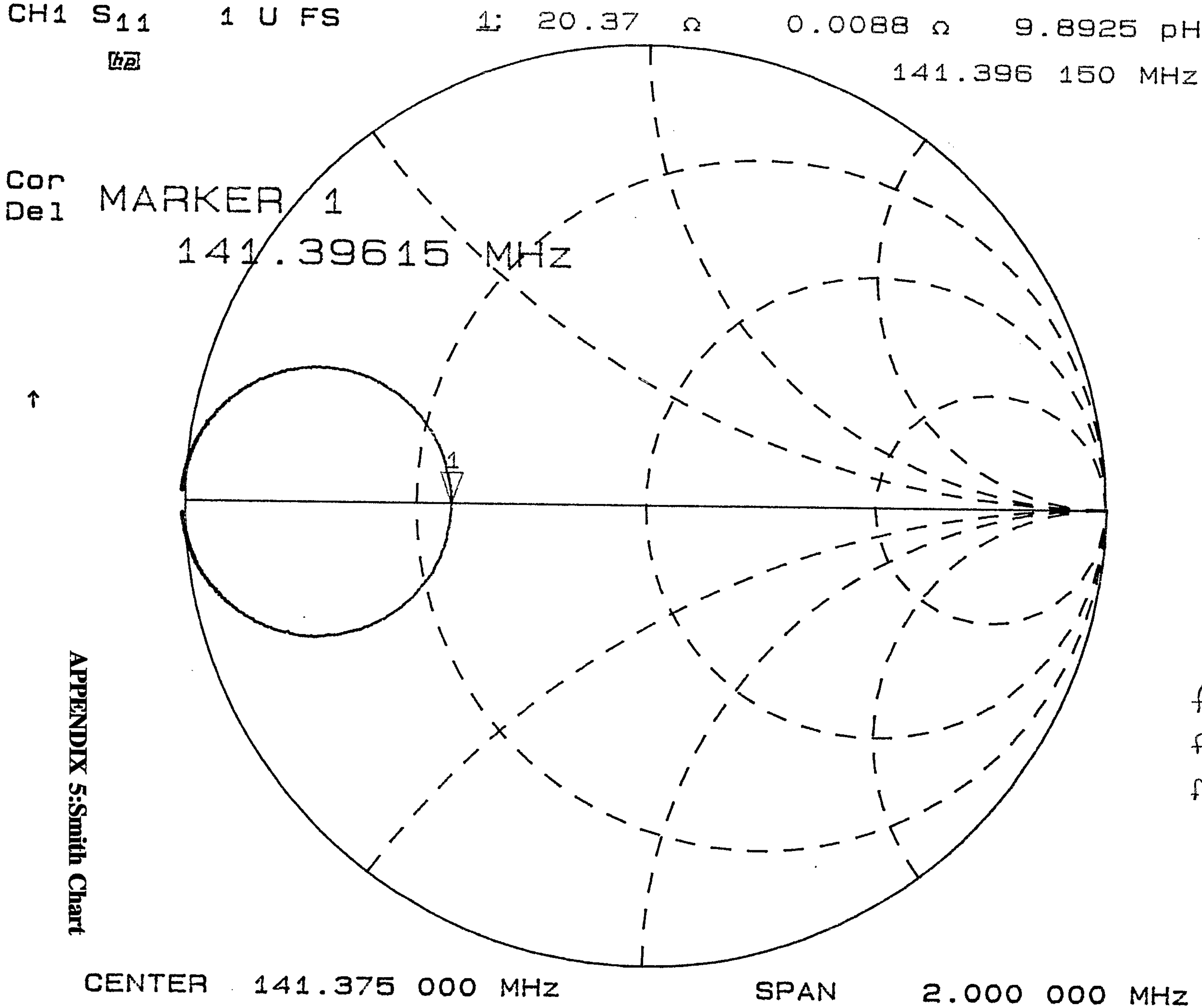
$$Respermeter := \frac{4}{6 \cdot 8.85 \cdot 10^{-12} \cdot \left(2 \cdot \pi \cdot 141.39 \cdot 10^{6}\right)^{2} \cdot Newbeadvol \cdot length} \cdot \frac{11.7}{8.7} \cdot sumsqrt^{2}$$:R/Q in ohms/meter

$Respermeter = 868.682$

$Shunt := Respermeter \cdot length$

$Shunt = 70.137$:R/Q in ohms

APPENDIX 5:Smith Chart

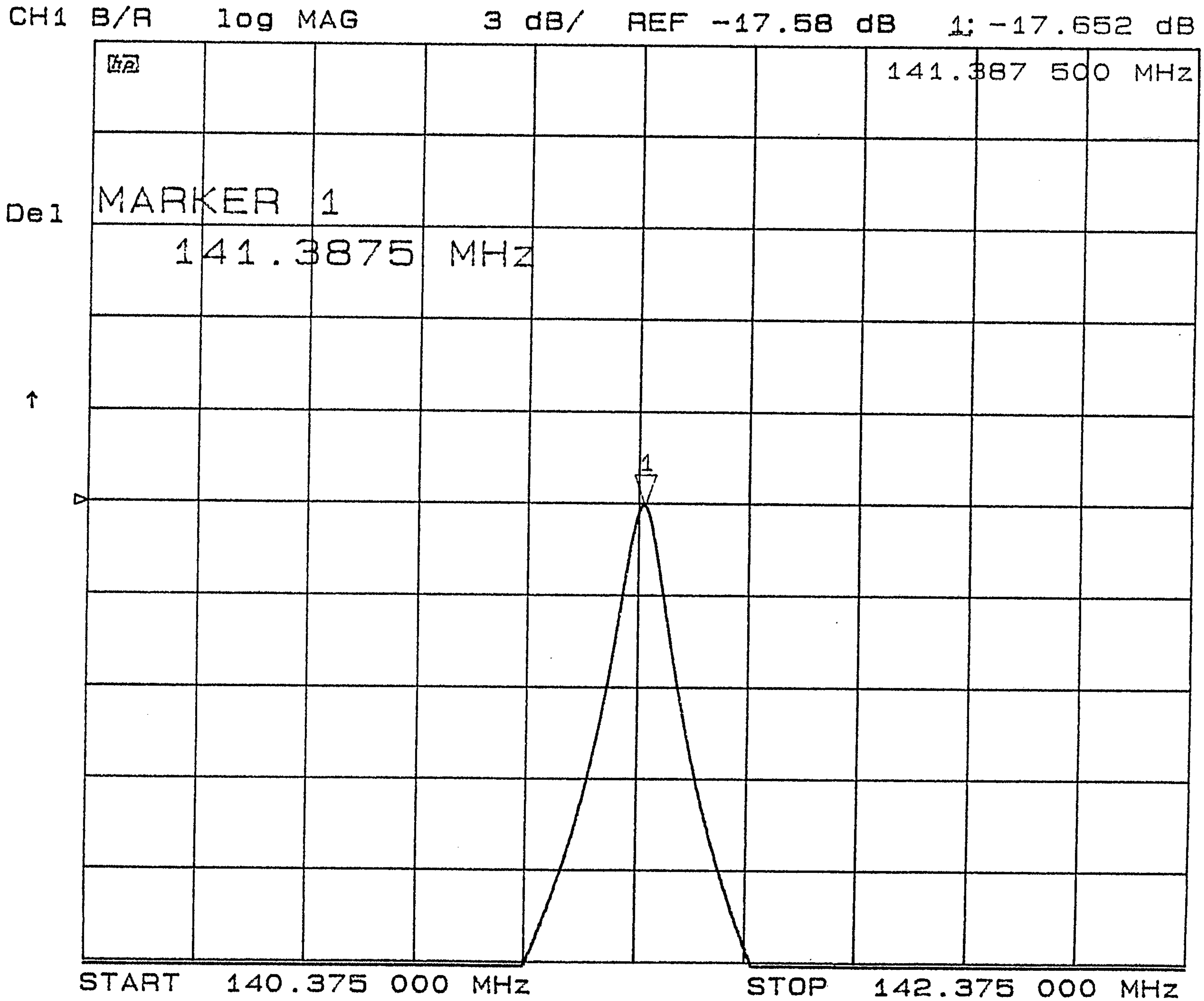


f_0 = 141.39615 MHz

f_2 = 141.43415 MHz

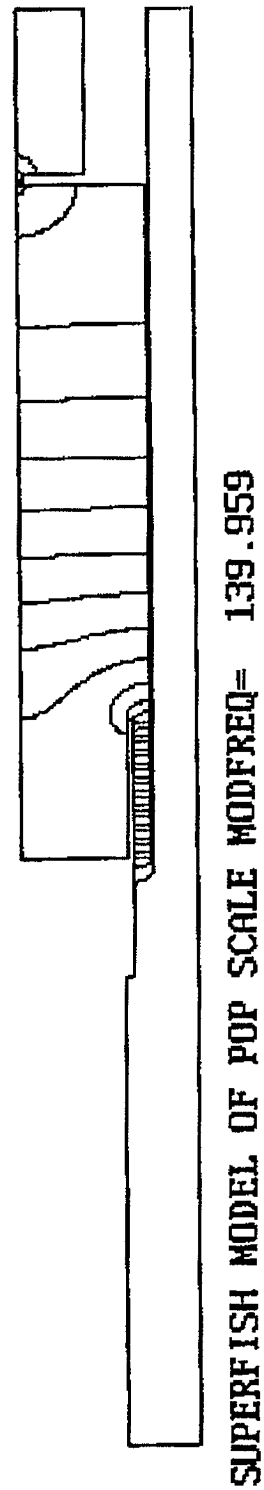
f_1 = 141.35835 MHz

Q = 1865.3846

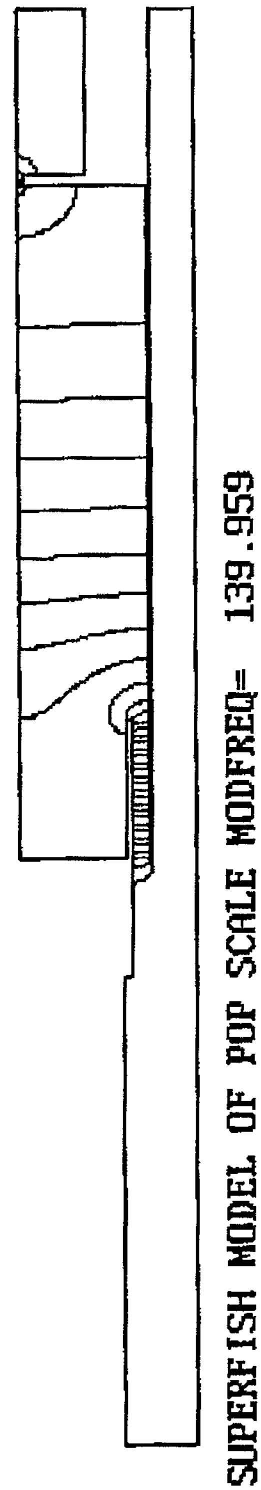


$f_0 = 141.3875$ MHz
$f_2 = 141.4226$ MHz
$f_1 = 141.349$ MHz

$Q_L = 1917.0848$



APPENDIX 1: Cavity Boundary with Field Lines



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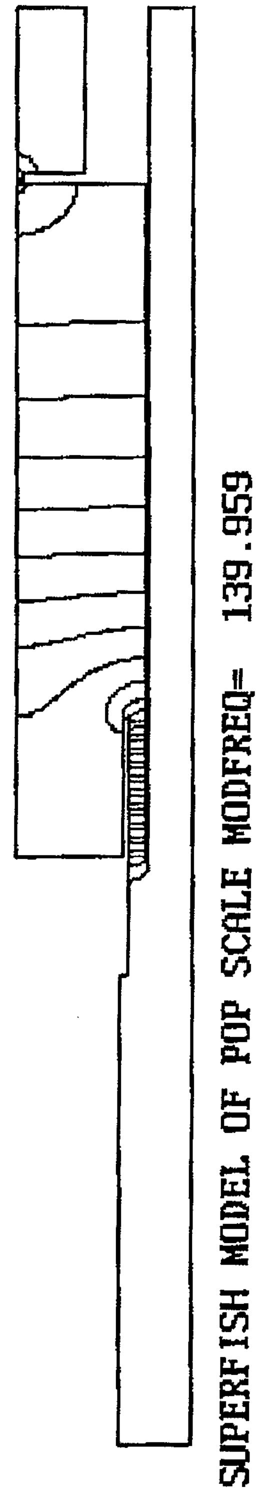

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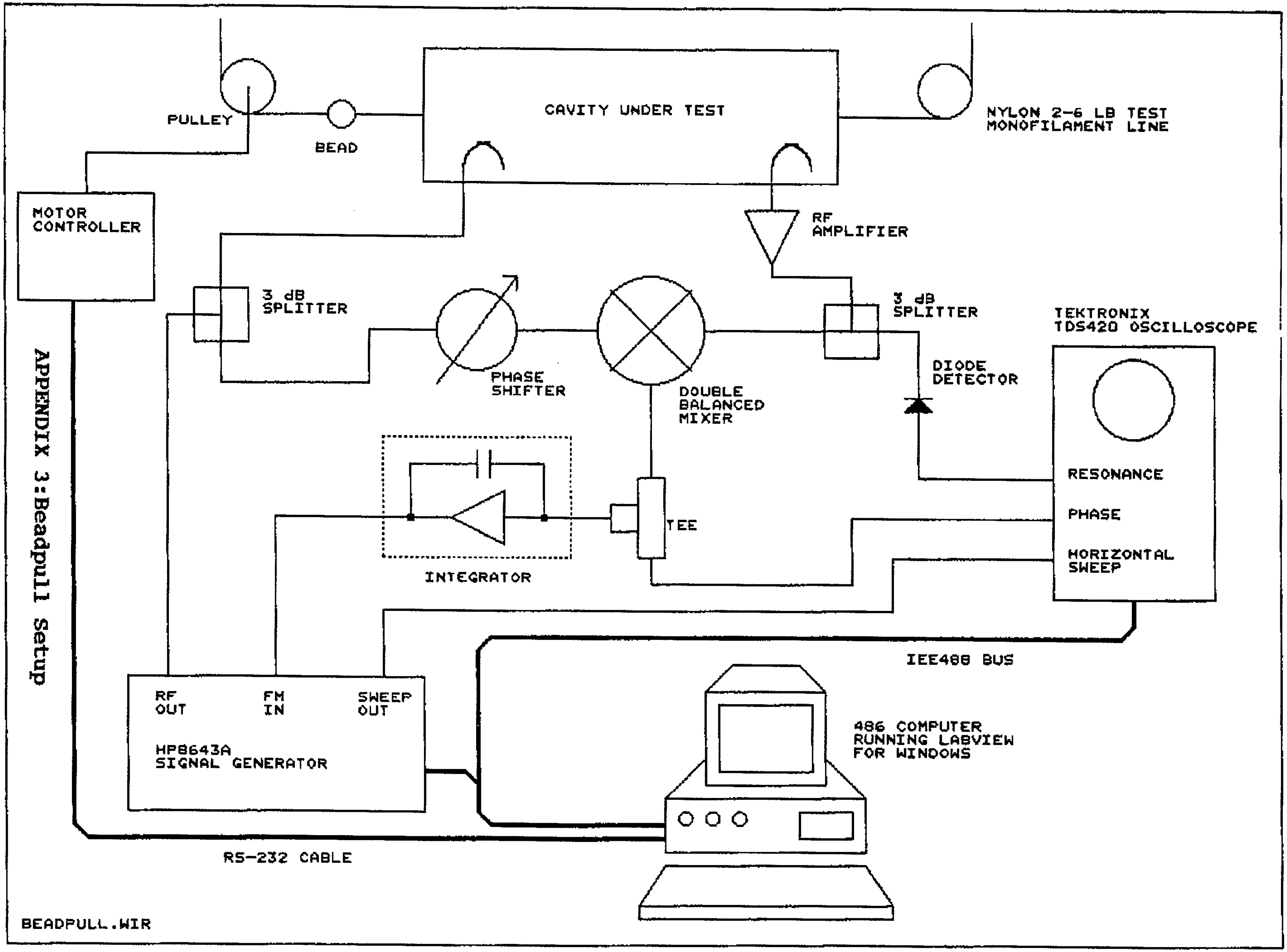
APPENDIX 3: Beadpull Setup



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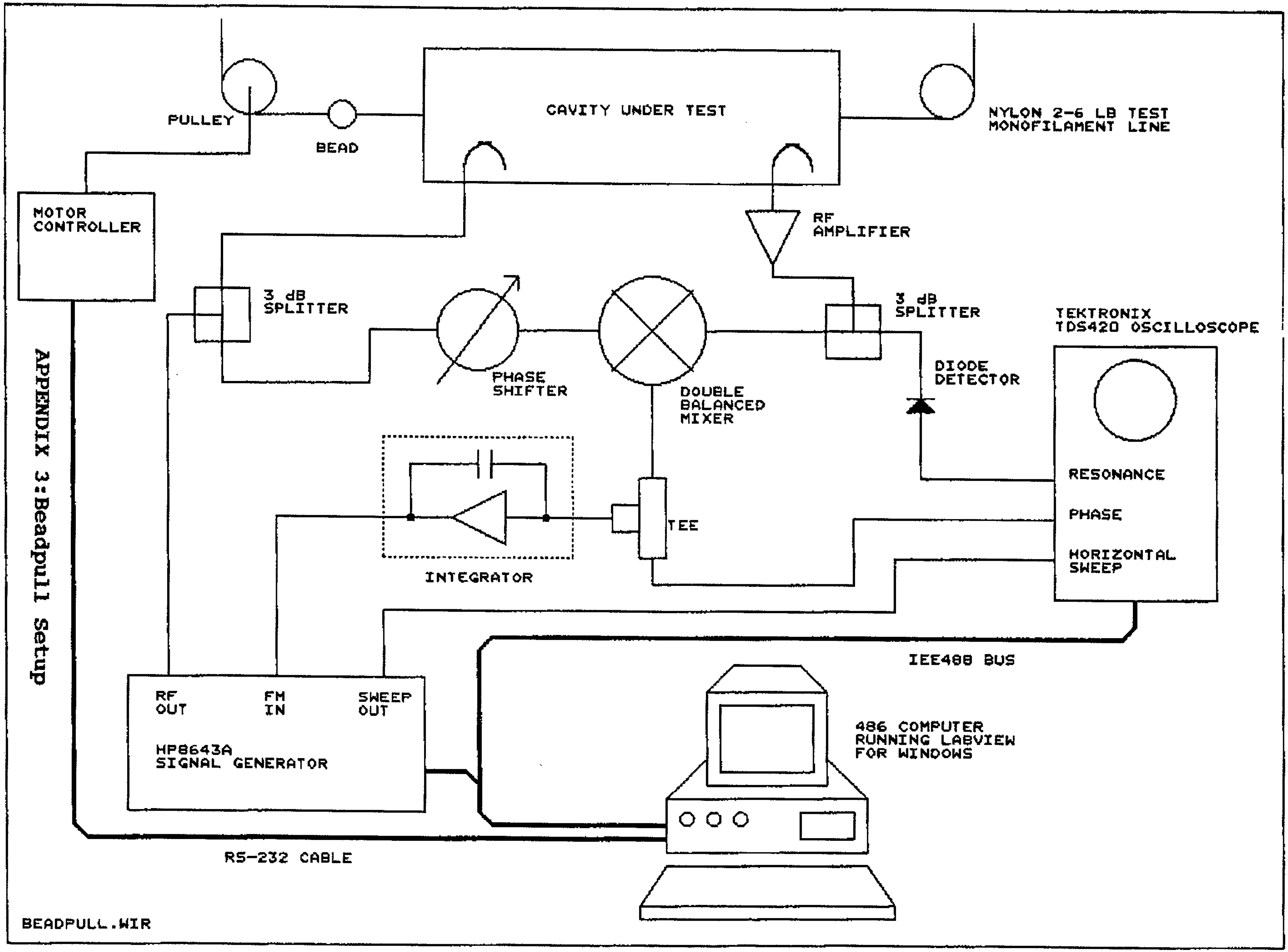

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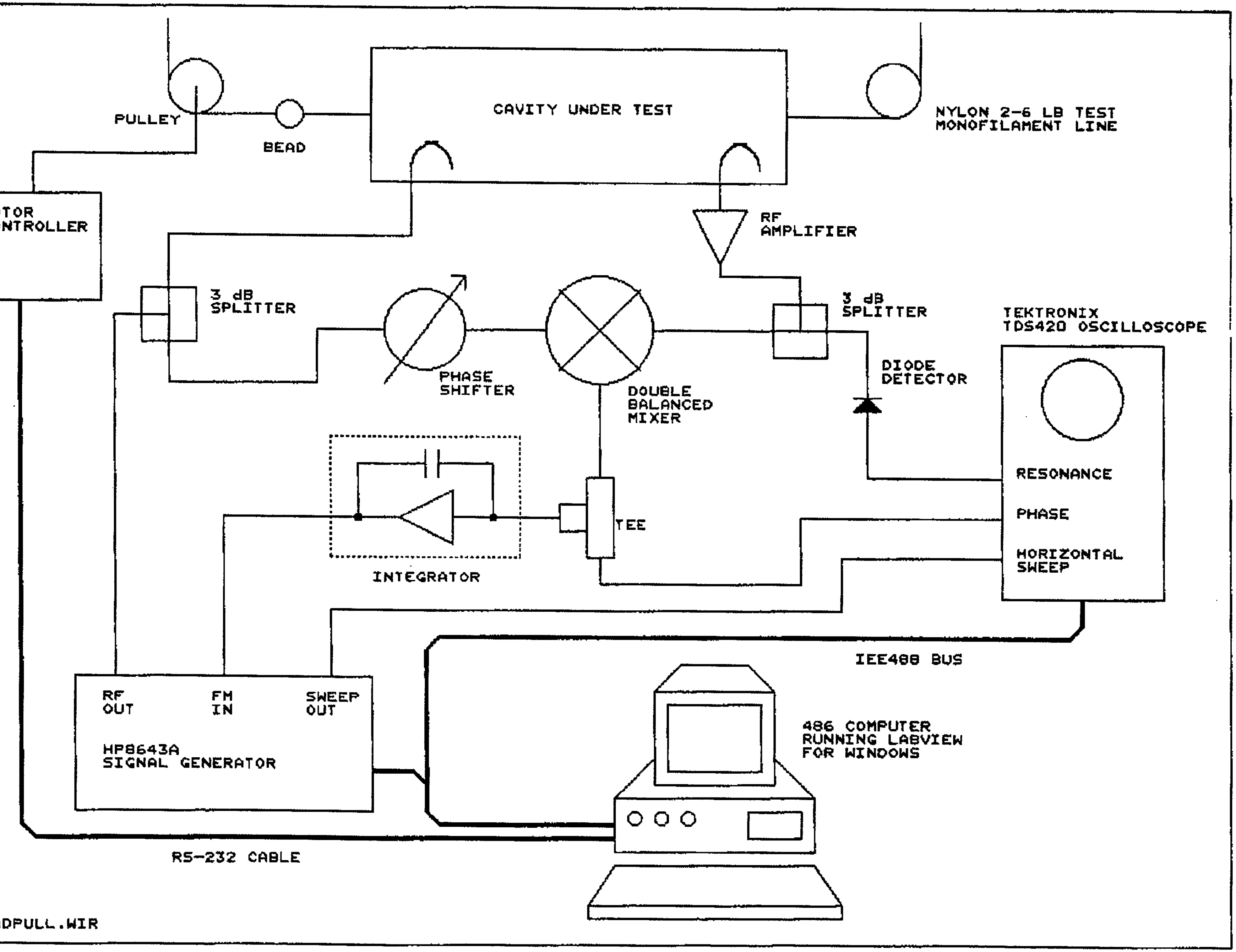

Full cavity length [2L] = 122.7354 cm Diameter = 15.2400 cm
Mesh problem length [L] = 61.3677 cm
Full drift-tube gap [2g] = 40.0000 cm
Beta = 0.5729926 Proton energy = 206.567 MeV
Frequency [f] (starting value = 139.000) = 139.958710 MHz
Eo normalization factor (CON(74)=ASCALE) for 1.000 MV/m = 90136.5
Stored energy [U] for mesh problem only = 3101.72217 mJ
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ISEG	zbeg (cm)	rbeg (cm)	zend (cm)	rend (cm)	Emax*epsrel (MV/m)	Power (W)	df/dz (MHz/mm)	df/dr
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1	0.0000	0.0000	0.0000	3.1686	0.0000	0.0000	0.0000	0.0000
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3	20.0000	3.1686	20.0000	2.7241	0.3567	0.0010	0.0000	0.0000
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Wall				Total =	477180.3130			Wall

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Transit time factor [T] = 0.33053
Shunt impedance [Z] mesh problem only, ((Eo*L)**2/P) = 0.78922 Mohm
Shunt impedance per unit length [Z/L] = 1.286 Mohm/m
Effective shunt impedance per unit length [Z/L*T*T] = 0.140 Mohm/m
Magnetic field on outer wall = 23926 A/m
Hmax for wall and stem segments at z= 53.60,r= 2.06 cm = 75036 A/m
Emax for wall and stem segments at z= 24.91,r= 2.00 cm = 156.513 MV/m

Beta	T	Tp	S	Sp	g/L	Z/L
0.57299262	0.33053	0.18483	0.94014	0.06382	0.325904	1.286049

ISEG	zbeg (cm)	rbeg (cm)	zend (cm)	rend (cm)	Emax*epsrel (MV/m)	Power (W)	df/dz (MHz/mm)	df/dr
Wall---Wall								
1	0.0000	0.0000	0.0000	3.1686	0.0000	0.0000	0.0000	0.0000
2	0.0000	3.1686	20.0000	3.1686	0.1433	0.0018	0.0000	0.0000
3	20.0000	3.1686	20.0000	2.7241	0.3567	0.0010	0.0000	0.0000
4	20.0000	2.7241	30.9690	2.7243	105.4048	8765.9063	0.0002	5.8327
5	30.9690	2.7243	30.9690	3.0162	94.6344	1169.9579	0.4215	0.0000
6	30.9690	3.0162	25.0000	3.0162	38.9049	26456.7930	0.0000	-0.1528
7	25.0000	3.0162	25.0000	7.6200	1.3865	12684.8428	-0.1172	0.0000
8	25.0000	7.6200	61.3677	7.6200	30.6048	99416.7813	0.0000	-0.4806
9	61.3677	7.6200	61.3677	4.7333	0.5631	15241.3057	-0.1410	0.0000
10	61.3677	4.7333	54.3150	4.7333	1.5684	47118.2891	0.0000	-0.4334
11	54.3150	4.7333	54.3150	7.4150	24.7467	13812.3125	-0.0652	0.0000
12	54.3150	7.4150	53.7710	7.4150	37.0998	1993.1312	0.0000	0.2884
13	53.7710	7.4150	53.7710	2.0638	24.2711	29881.9375	-0.2080	0.0000
14	53.7710	2.0638	32.0000	2.0638	41.1049	202982.7340	0.0000	-1.3085
15	32.0000	2.0638	31.5000	2.0002	51.4568	2928.4282	0.0159	0.1252
16	31.5000	2.0002	24.8247	2.0002	156.5135	14723.4805	0.0000	7.9571
17	24.8247	2.0002	24.8247	1.8414	153.7094	2.6982	0.4522	0.0000
18	24.8247	1.8414	61.3677	1.8414	65.8864	1.7341	0.0000	0.1138
19	61.3677	1.8414	61.3677	0.0000	0.0000	0.0000	0.0000	0.0000
Wall--------------------------------Total = 477180.3130 -------------Wall								

BEAD.MCD :BEADPULL ANALYSIS 6/22/93

CERAMIC BEAD

$A := \mathrm{READPRN}(cerpull)$

$Measure := A^{<1>}$:Voltage Matrix

$Calibration := 4.09167 \cdot 10^{4} \cdot 2 \cdot \pi \cdot Measure$:Calibration to Frequency

$Sqrtvolt := \overrightarrow{\sqrt{|Calibration|}}$:Square root of each Frequency

$sumsqrt := \sum \left(Sqrtvolt \cdot .72870 \cdot 10^{-4} \right)$:Sum of the square roots

$sumsqrt = 8.172$

$Radcer := \frac{.1245 \cdot 2.54}{100}$ $Radcer = 0.003$:Radius of the Ceramic Bead (m)

$Fullbeadvol := \frac{4 \cdot \pi \cdot Radcer^{3}}{3}$ $Fullbeadvol = 1.325 \cdot 10^{7}$:Volume of entire Bead (m)

$Radhole := \frac{.04 \cdot 2.54}{100}$ $Volhole := \pi \cdot Radhole^{2} \cdot Radcer$ $Volhole = 1.026 \cdot 10^{-8}$:Volume of hole

$Newbeadvol := Fullbeadvol - Volhole$ $Newbeadvol = 1.222 \cdot 10^{-7}$:Corrected volume of Bead

$length := \frac{50}{1000} \cdot \frac{100}{5000} \cdot 1108 \cdot .072870$ $length = 0.081$:Length of gap

$$Respermeter := \frac{4}{6 \cdot 8.85 \cdot 10^{-12} \cdot \left(2 \cdot \pi \cdot 141.39 \cdot 10^{6}\right)^{2} \cdot Newbeadvol \cdot length} \cdot \frac{11.7}{8.7} \cdot sumsqrt^{2}$$:R/Q in ohms/meter

$Respermeter = 868.682$

$Shunt := Respermeter \cdot length$

$Shunt = 70.137$:R/Q in ohms

APPENDIX 4: MATHCAD Calculations for Ceramic Bead

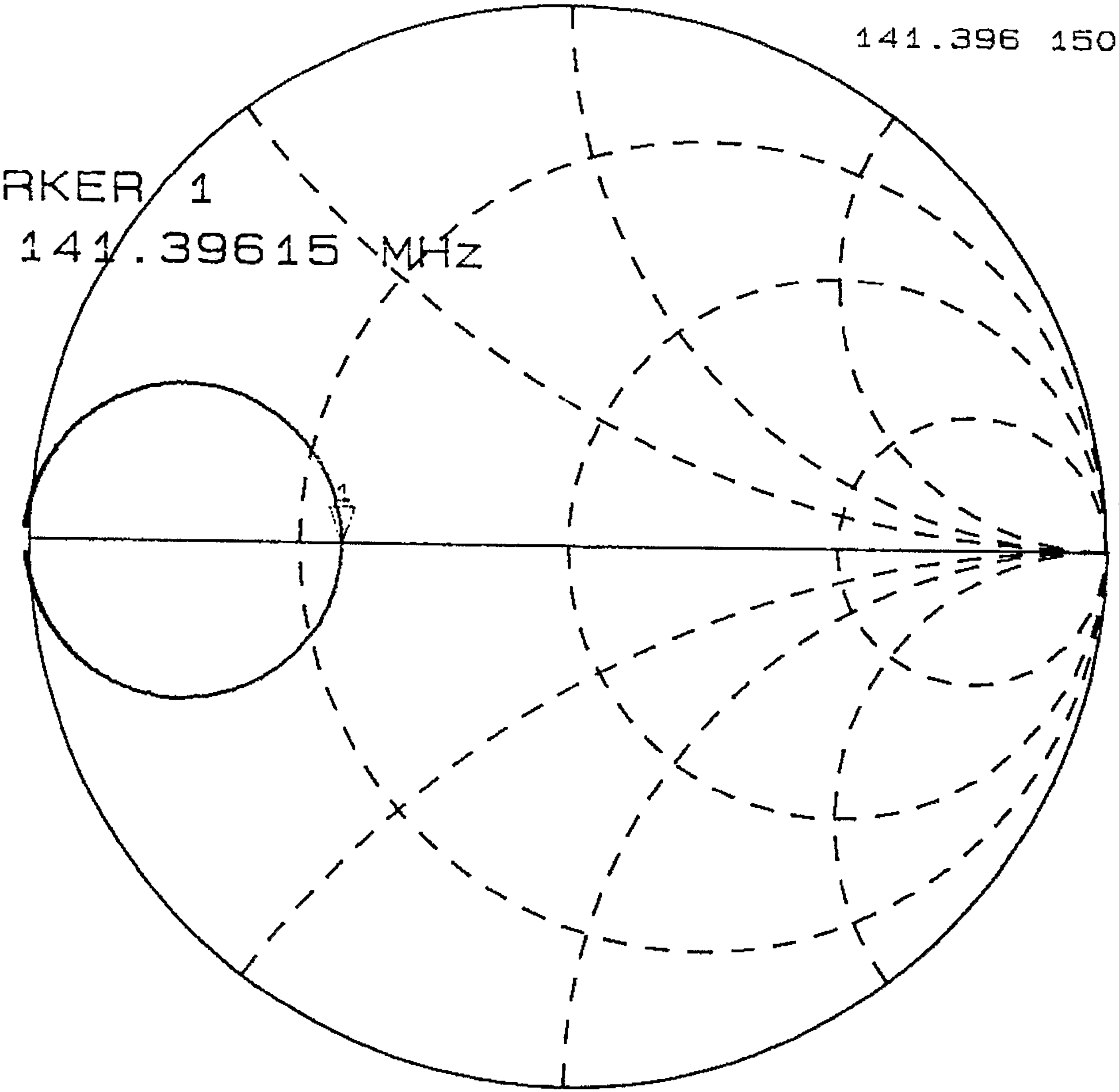
APPENDIX 5:Smith Chart

CH1 S_{11} 1 U FS 1: 20.37 Ω 0.0088 Ω 9.8925 pH

141.396 150 MHz

Cor
Del

MARKER 1
141.39615 MHz



f_0 = 141.39615 MHz

f_2 = 141.43415 MHz

f_1 = 141.35835 MHz

Q = 1865.3846

CENTER 141.375 000 MHz SPAN 2.000 000 MHz

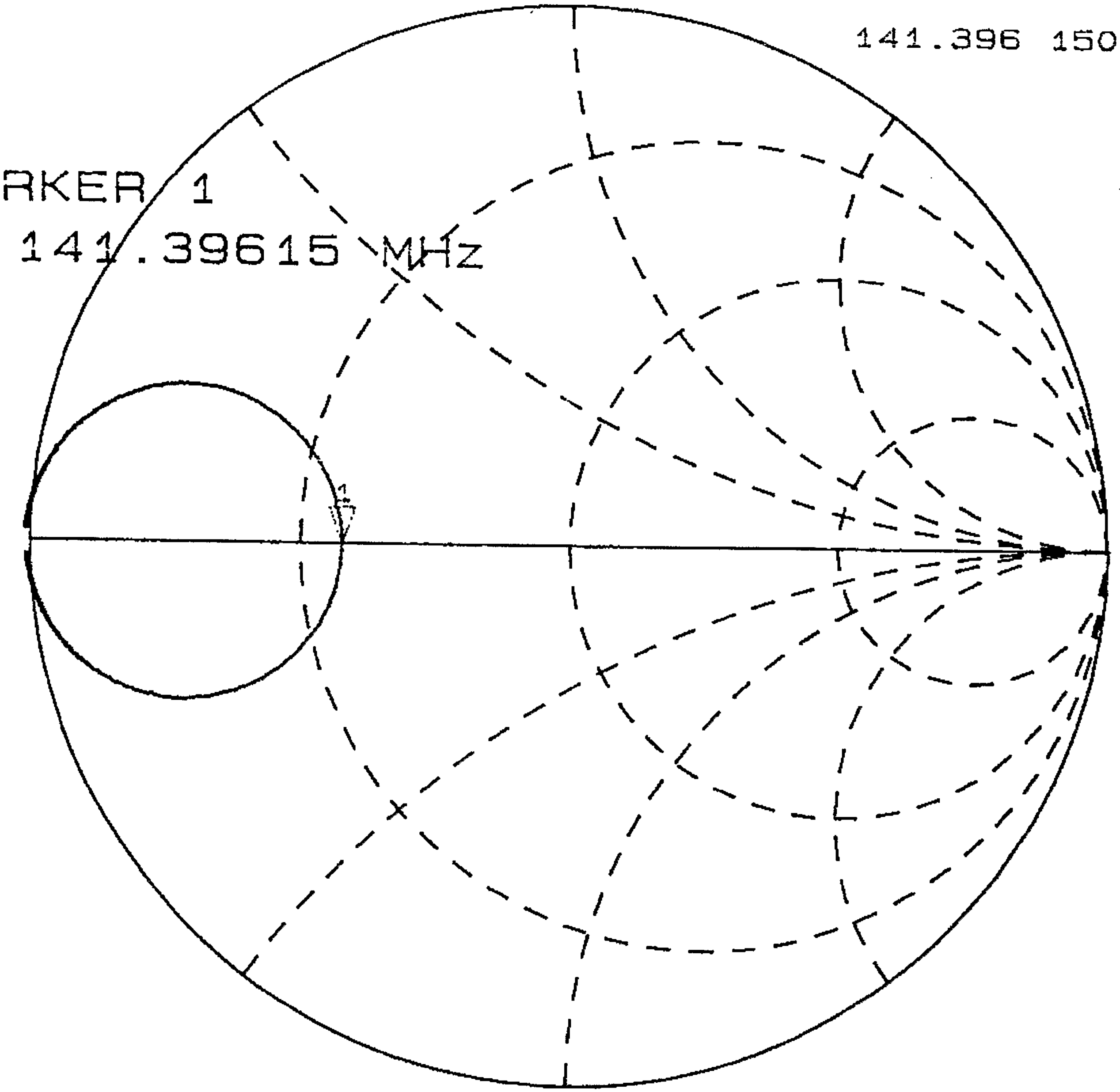
APPENDIX 5:Smith Chart

CH1 S_{11} 1 U FS 1: 20.37 Ω 0.0088 Ω 9.8925 pH

141.396 150 MHz

Cor
Del

MARKER 1
141.39615 MHz



CENTER 141.375 000 MHz SPAN 2.000 000 MHz

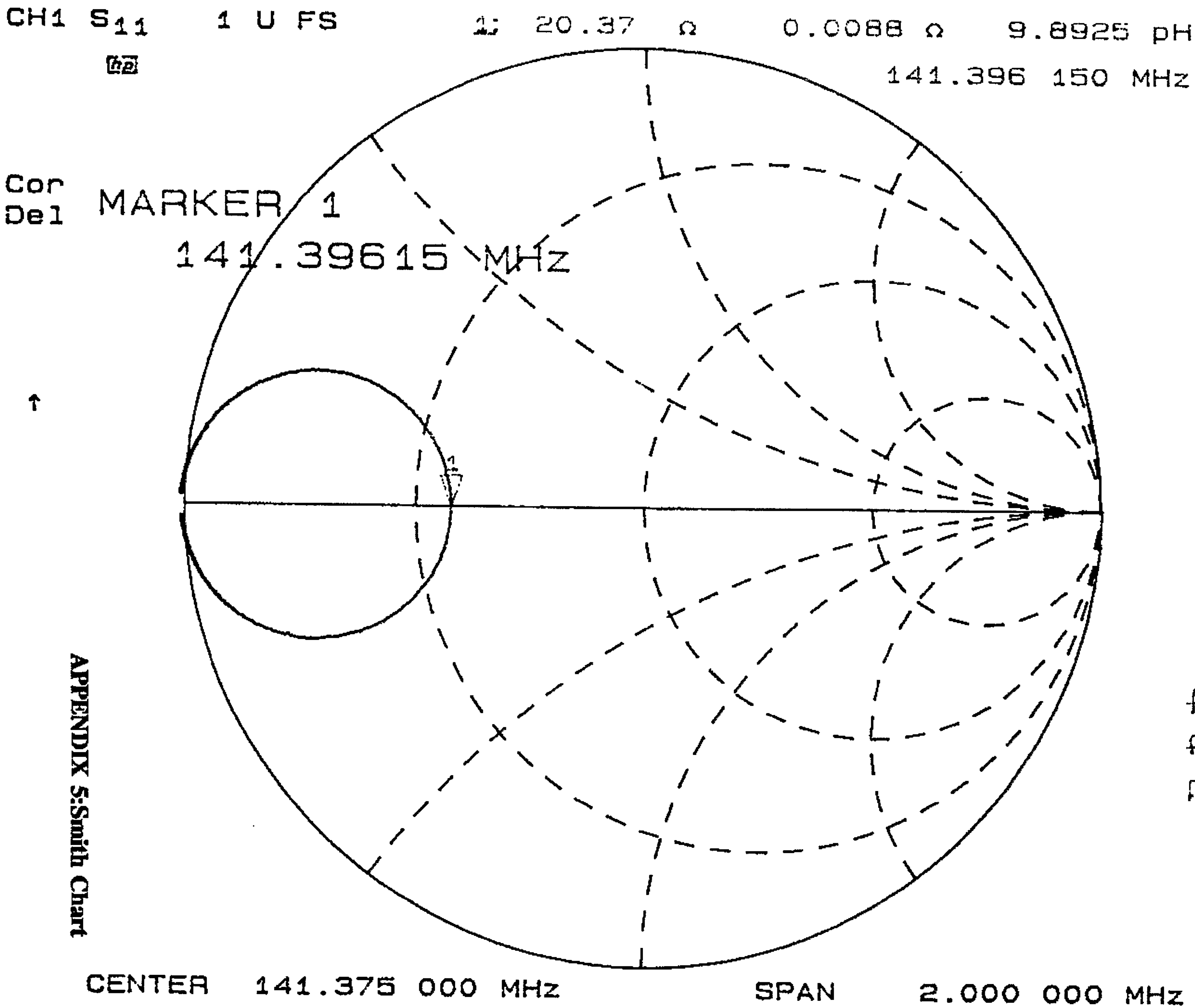
f_0 = 141.39615 MHz

f_2 = 141.43415 MHz

f_1 = 141.35835 MHz

Q = 1865.3846

APPENDIX 5:Smith Chart

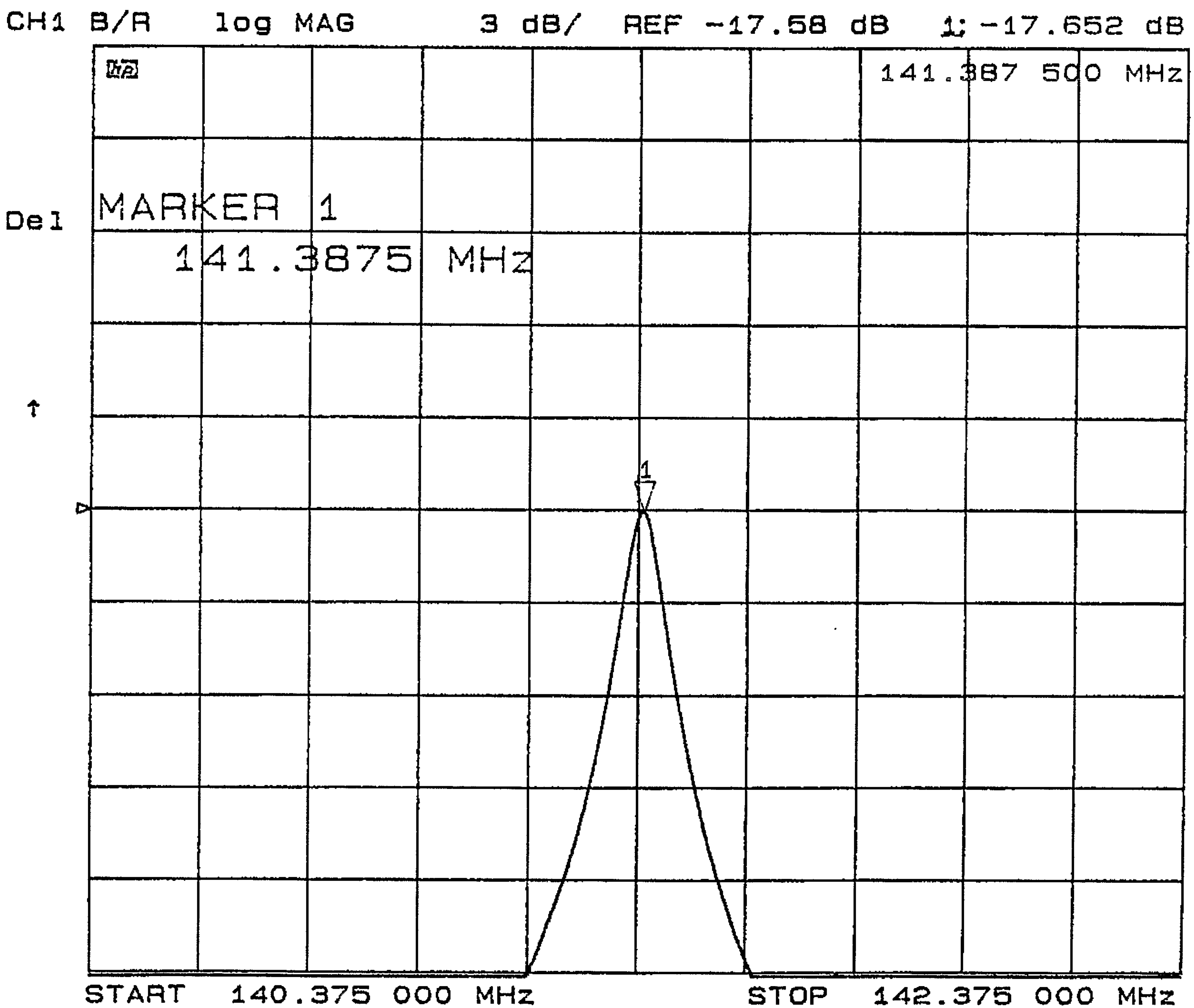


f_0 = 141.39615 MHz

f_2 = 141.43415 MHz

f_1 = 141.35835 MHz

Q = 1865.3846

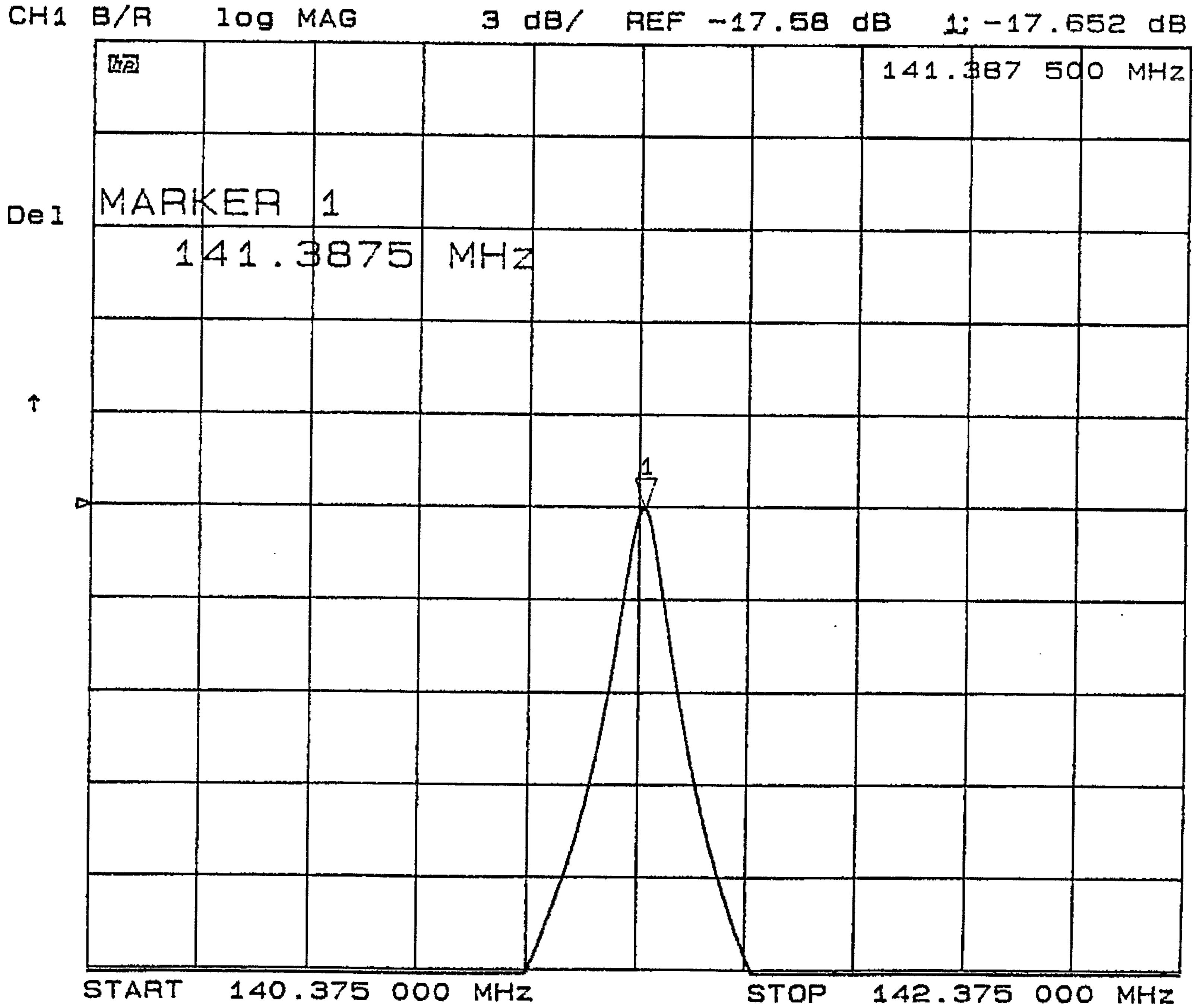


$f_0 = 141.3875$ MHz

$f_2 = 141.4226$ MHz

$f_1 = 141.349$ MHz

$Q_L = 1917.0848$



$f_0 = 141.3875$ MHz

$f_2 = 141.4226$ MHz

$f_1 = 141.349$ MHz

$Q_L = 1917.0848$