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Longitudinal emittance growth due to RF phase errors

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Abstract

The emittance growth from phase error due to the tuning of cavity frequency for a *RHIC* cycle is studied. A tolerable phase angle is given.

Let the emittance of a bunch matached to a bucket be S, which is the area of the bunch in the longitudinal phase space. A bunched beam displaying dipole, quadrupole *etc* motions is not matched to the bucket created by RF systems. A bunched beam (non-interacting particles) is said to match to a bucket when it's free of any forementioned motions. The immediate consequence is that its emittance remains constant. A matched bunch is an idealized concept on which we can measure the effects of undesirable motions.

Let the phase spread in a matched beam be $\Delta\phi$, any subsequently uncorrected errors in the beam phase relative to the RF phase induce dipole motion to the least, eventually lead to filamentation of the beam in the phase space, thus emittance growth. The relative emittance growth $\frac{\delta S}{S}$ to the relative phase error $\frac{\delta\phi}{\Delta\phi}$ relate to each other by (assuming $\Delta\phi$ is not too large.)

$$\frac{\delta S}{S} = 2\frac{\delta\phi}{\Delta\phi} \tag{1}$$

The bunch length of a matched bunch in the transition region shrinks to a minimum, therefore it's the most susceptible to uncorrected phase errors.

Let's consider a cycle of Au^{79+} in *RHIC*. Figure (2) shows the bunch length throughout the cycle; Figure (1) shows the synchrotron frequency throughout the cycle.

Given the cycle as it is, Figure (3) the relative emittance growth for a 5° error; Figure (4) the relative emittance growth for a 0.05° error. At each point on Figure 2, a certain uncorrected phase error is assumed and the emittance growth is recorded. The plots are obtained with assumptions that

- The phase error is not corrected with beam control system for a long time.
- The phase error does not have any accumulative effects.

The synchrotron frequency spread δf_ν in a bunch varies versus the bunch size $\Delta\phi$ as

$$\delta f_\nu = \frac{1}{64}\Delta\phi^2 f_\nu \tag{2}$$

For a given phase error, the larger the synchrotron frequency spread, the faster the filamentation. Figure 5 shows the synchrotron frequency spread in the cycle.

It'd very helpful to relate appreciable emittance growth to the time scale that a pure dipole motion can develop. For a short bunch can survive a lot longer than a long bunch without showing much distortion. This is a difficult question analytically, simulation of particle tracking can partially solve the problem.

The frequency swings about $90KHz$ for the cycle. Suppose a tuner will cover this range in 1000 steps, and each step will introduce a small phase error of a few degrees. Let's ask a question: what is the accumulative phase error at the end of the cycle? The question implies that if we impose a permissible phase error at the end of the cycle, we can set a limit on how much permissible phase error at each tuning step. We approach the question with a simple model. Denote ϕ_i the phase error at step i ($\phi_0 = 0$), $\delta\phi$ is the phase error each step will bring in.

$$\phi_{i+1} = \phi_i \cos(2\pi \frac{f_\nu^i}{f_{step}^i}) - \delta\phi, \tag{3}$$

where f_ν^i and f_{step}^i are the synchrotron frequency and the frequency of tuning steps. It is a summation of a phase error $\delta\phi$ and the previous phase error evolved after a time of $\frac{1}{f_{step}}$.

It's clear that when $f_\nu = k * f_{step}$ (k is an integer.), we have a resonance. That is each step will simply add $\delta\phi$ to the total phase error, it is adding up constructively. When $f_\nu = (k + \frac{1}{2}) * f_{step}$, it is adding up destructively. By inspecting Figure 8, we can see that it will hit on quite a few resonnaces. It is confirmed by plotting ϕ_i at each step, see Figure 9. The worst case happens when the bunch finishes filamentation after each resonnance. In this particular case, the resonances contribute roughly $65\delta\phi$. For one percent of increase in the emittance, the uncorrected phase error is $0.15°$, therefore $\delta\phi \approx 0.0023$. In the presence of a local tuner feedback loop with a gain of 100, that translates into $0.23°$, the permissible phase error at each step. If the beam control system has a gain of 100, that means the permissible phase error will be $23°$ at each step.

Acknowlegement

The author would like to thank W. Pirkl for bringing this question to me and holding discussions.

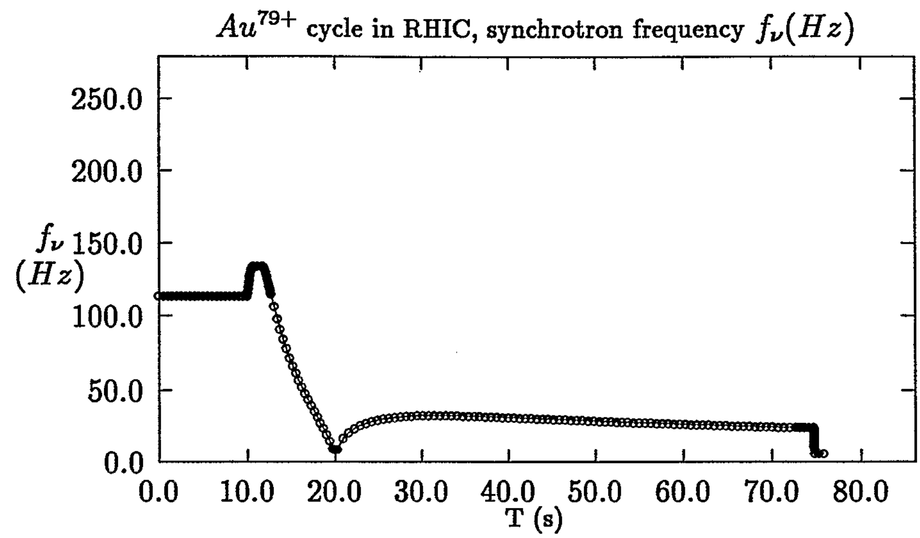

Figure 1: Synchrotron frequency throughout the cycle.

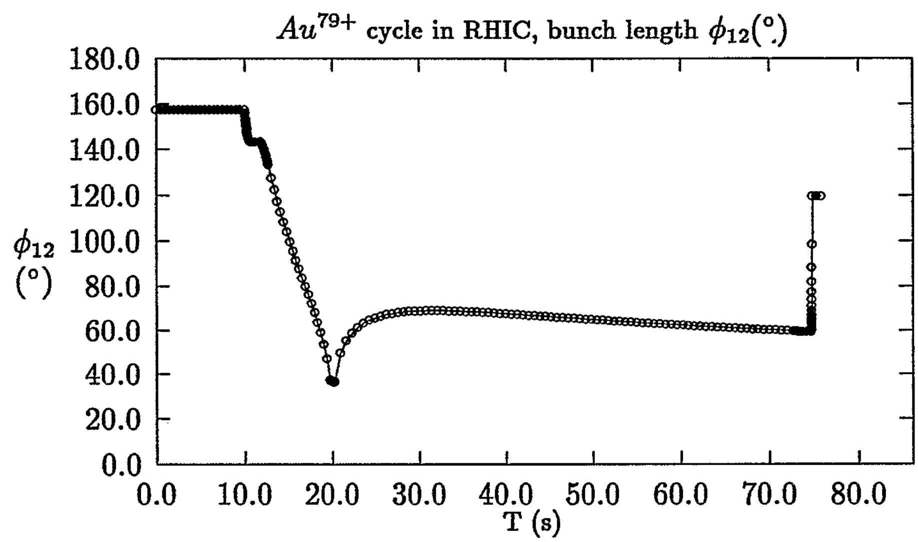

Figure 2: Bunch length throughout the cycle.

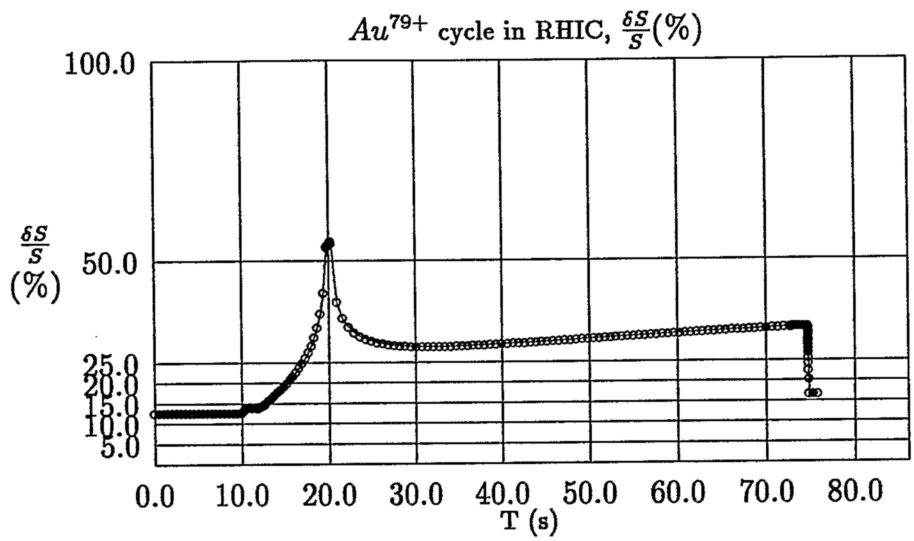

Figure 3: Relative emittance growth due to 5° error.

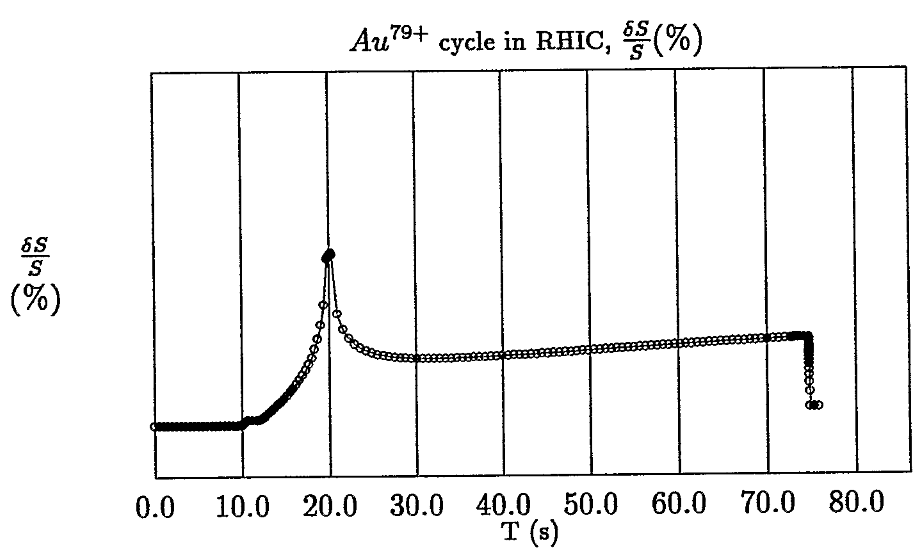

Figure 4: Relative emittance growth due to 0.05° error.

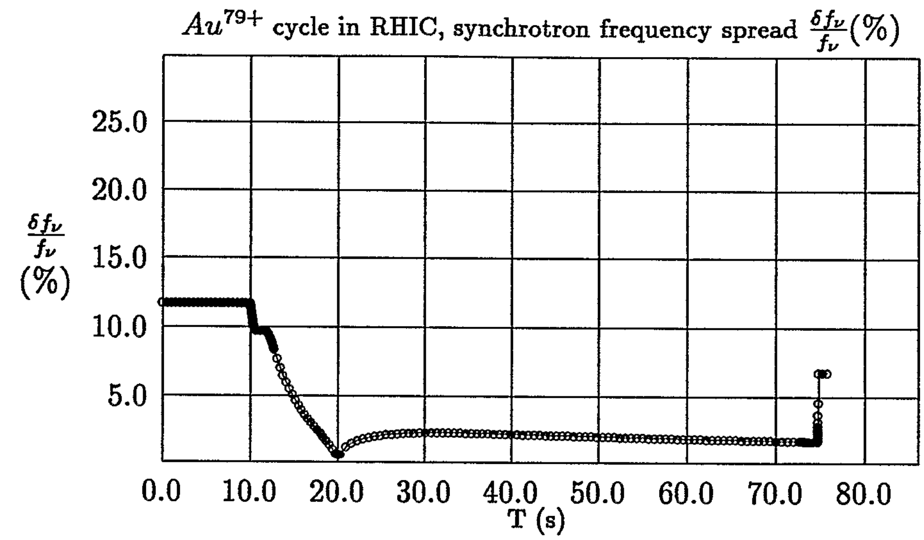

Figure 5: Synchrotron frequency spread throughout the cycle.

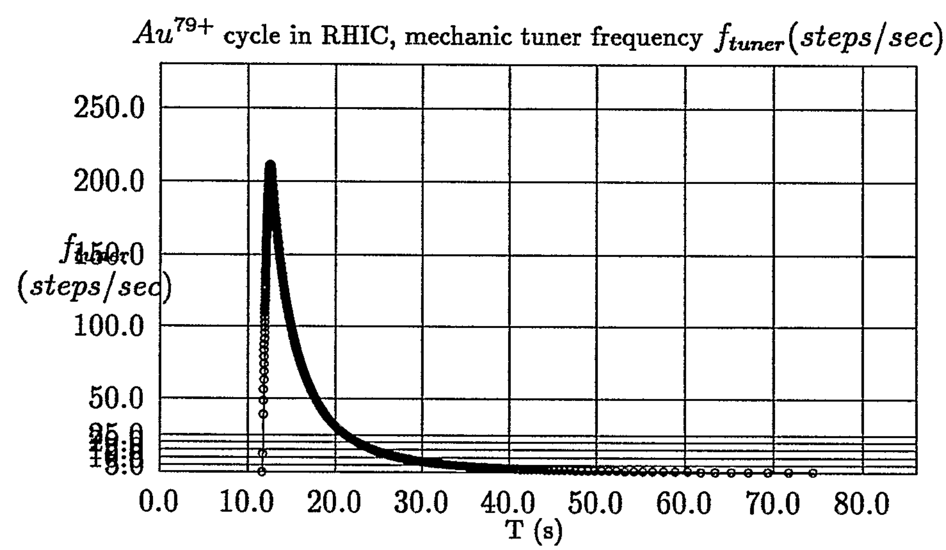

Figure 6: Tuner frequency throughout the cycle.

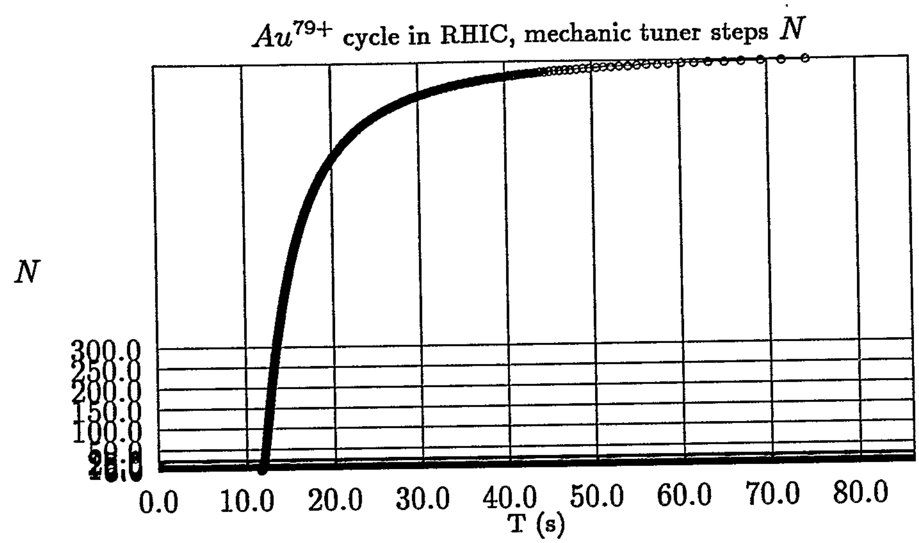

Figure 7: Tuner steps throughout the cycle.

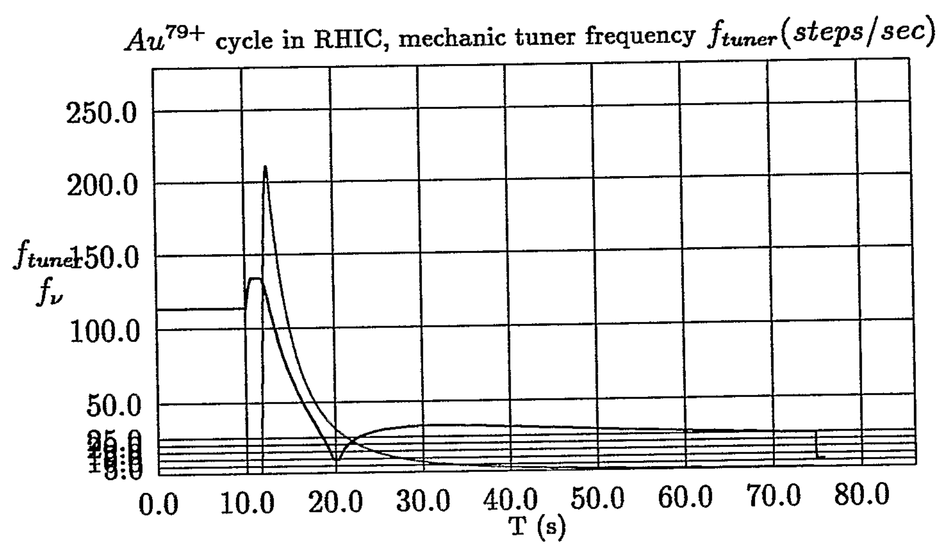

Figure 8: Overlay of f_{ν} with f_{tuner}.

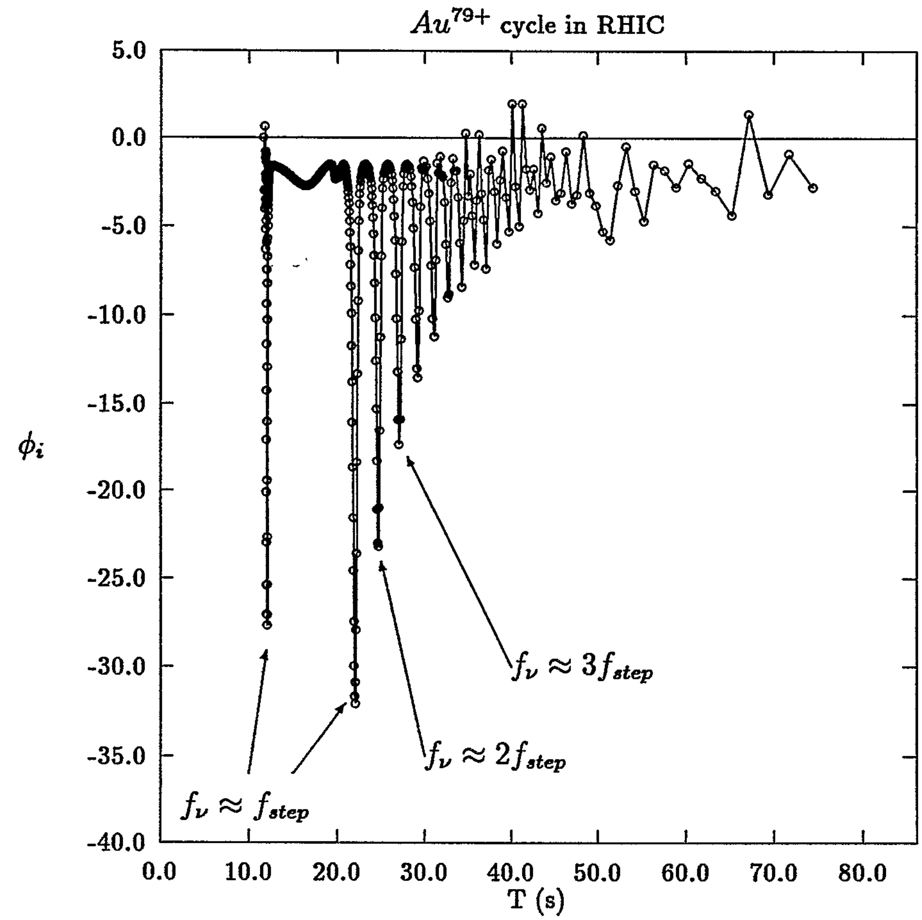

Figure 9: ϕ_i evolution.