# The Early Stage Of Ultra-Relativistic Heavy Ion Collisions 

J. P. Blaizot<br>April 1987<br>Collider Accelerator Department<br>Brookhaven National Laboratory

## U.S. Department of Energy <br> USDOE Office of Science (SC)

[^0]
## DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

## AD/RHIC-PH-22

## The Early Stage of Ultra-Relativistic Heavy Ion Collisions

```
            J. P. Blaizot
            SPhT-CEA Saclay
91191 Gif-sur--Yvette Cedex, France
```

A. H. Mueller

Physics Department
Columbia University, NY

April 17, 1987


In order to assess the possibility of producing quark-gluon plaseds in ultra-relativastic heavy ion collisions, it is important to mderstand what happens in the very beginning of such collisions. It is usually assumed that the quanta which are produced in the central rapldity . region quickly reach a state of local thermodynamic equilibrium (1). Such an assumption of thermalization is certainly a convement one, as it leads for example to a simple dynamical model of the collisions based on hydrodynamics [2]. Hoyever, its validity temans to be checked. In particular, one hould like to know how to characterize the quanta which have a chance to thermalize, on which ume scale this thermalization takes place, what is the energy density of the system soon after thermalization, etc... To answer most of these questions rould require solving a kinetic equation, given dpproprtate initial conditions and a detailed knouledge of the actoscopic processes by which the quanta exchange energy, momentum, or pussibly their number. Our goal in this paper will be more modest; He sliall attempt to determine the properties of the quanta which turn Illo free particles during an ultra-relativistic heavy ion collision and which give the dominant contribution to the initial energy density.

Our starting point $2 s$ a parton model. He assume that, in the center of mass frame, where they are fast moving objects, the colliding nuclei may be viened as collections of quasi real particles, the partons, Hith litetime much larger than the collision time. In the spirit of the "sudden approximation" of quanture mechanics, we assume that the nuclear wave functions are esgentially unaltered by the collision; that is, just after the collision, the distribution of partons ill phase space remains identical to what it was just before the collision. Horever, during the collision, some of the partons receive energy and momentum which, we assume, is just enough to put them on their mass shell. They then evolve as free particles, at least over a short period of tilas. Our purpose is to point out some properties of thas system of tree partons at the time it is formed. In particular, we shall find that the dominant contribution to the energy densily comes from partons wath fransverse momenta qrowiny like P1- $A^{1 / 3}$. This is in contrast for example hith other parion models,
such as the dual parton model $|J|$, which use partons with limited transverse momenta. Dine may korry that the lack of partonic saturation in these models will lead to an overestimate of the soft parton contributions in collisions of large nuclei.

Recently, llwa and kajantiol4) presented an attempt to estimate the thermalization time in nuclear collisions. Eventhough we shall not adopt their view on the thermalization problen we would like to mention that our discussion in section 2 expands on thoir treatment of the kinematics of the parton distribution. We shall show in this section that the system of free streaming partons exhibits tro different regimes as a function of time, the first regime being dominated by the longitudinal motion of the fast partons. our arguments concerning the characterization of the partons which get freed during the collisions are developped in section 3. He shall present tho fairly different scenarios which lead to the same $A$ dependence of the partons transverse momenta. The last section of the paper contains numerical estimates of the formation time and the initial energy density.

## 2. THE TREE STREAMIMG REGME.

He assume that,at least for a short period of time, the phase coherence between the partons can be ignored and we describe the system of partons by a classical distribution function $f(p, x)$, where $x$ represents the spaci-time coordinates, and $p$ the monentuma. He shall assume that the distribution is uniform in the transverse direction, i.e. $f(p, x) m\left(p_{r}, p_{a}, z, t\right)$. Longitudinal $(p, z)$ and transverse $\left(p_{r}\right)$ variables play different roles in the present discussion, and in order to expresa easily the consequences of longitudinal lorentz boosts, it is convenient to transform the longitudinal variables. He define a "space-tine rapidity" $ו$ and a proper time $\tau$ :

$$
\begin{equation*}
11=\frac{1}{2} \ln \frac{t+z}{t-z} \quad i=\sqrt{t^{2}-z^{2}} \tag{2,1}
\end{equation*}
$$

in terms of which one has $t=$ ocoshin, $z=1$ sinhy. Siailarly, one introduces a "momentum rapidity" y:

$$
\begin{equation*}
y=\frac{1}{2} \underset{p_{0}-p_{i}}{p_{\theta}+p_{i}} \quad p_{0}=\sqrt{p_{i}^{2}+p_{z}^{2}} \tag{2.2}
\end{equation*}
$$

such that $p_{0}=D_{\text {, }}$ coshy, $p_{2}=p_{\text {s }}$ Einliy.
We shall ignoce in this section the processes which may change locally the number of partons, and also the collisions which change the momenta. Then, the parton distribution obeys the folloring kinetic equation:

$$
\begin{equation*}
0=\frac{\partial f}{\partial t} \frac{P_{z}}{P_{0}} \frac{\partial f}{\partial z}=\cosh (y-\eta) \frac{\partial f}{\partial t}+\sinh (y-\eta) \frac{\partial f}{\partial \partial \eta} \tag{2.3}
\end{equation*}
$$

where we have assumed the partons to be massless. The general solution of this equation is easily seen to be of the form:

$$
\begin{equation*}
f\left(p_{1}, p_{k}, z, t\right)=f\left(p_{1}, H\right) \tag{2.4}
\end{equation*}
$$

Where the variables on the l.h.s. are defined in the center mass frame, while on the r.h.s. we have set [5]:

$$
\begin{equation*}
y=p_{Y} t \sinh (y-Y)=p_{z} t-p_{0} z \tag{2.5}
\end{equation*}
$$

The Lorentz invariant may be given the following interpretation: $w / T$ is the longitudinal momentum measured in a frame moving mith rapidity If with respect to the center of mass $f$ rame; on the other hand, $-H / p_{\text {, }}$ is the longitudinal coordinate $z$ in frame moving with rapidity $y$. Note that lor a free parton emanating from the point $z=t=0, H=0$. The parton distribution function may then be calculated in terms of the momentum distribution $f_{0}\left(p_{j}, p_{z}\right)$ at $z=0$ and some initial tine $t_{0}$ :

$$
\begin{equation*}
f\left(p_{1}, H\right)=L_{0}\left(p_{1}, \frac{N}{t_{0}}\right) \quad t_{0}\left(p_{1}, p_{2}\right)=f\left(p_{1}, p_{2}, z=0, t_{0}\right) \tag{2.6}
\end{equation*}
$$

This way of defining the distribution function may look at first somenhat annoying since it introduces an arbitrary time scale $t_{0}$. Homever, we shall see soon that re can take the limit where $\mathrm{l}_{\mathrm{\prime}}$ goes to zero.

A particular choice for the initial distribution function, which is motivated by the $y$-independence of the momentumidistribution,
see eq.(2.12) below, is the follouing:

$$
\begin{equation*}
f_{0}\left(p_{1}, p_{z}\right)=g\left(p_{1}\right) \|\left(p_{z}^{m n x}-\left|p_{1}\right|\right) \tag{2.7}
\end{equation*}
$$

wher $g\left(p_{1}\right)$ is some transverse momentum distribution. The tormula (2.7) assumes that for given $p_{\text {, }}$, and in the plane $z=0$, the density of partons is independent of the longitudinal momentum $p$, up to a maximum value $p_{z}^{m \times x}$. The equation.(2.7), together with (2.6), tinplies:

$$
\begin{equation*}
f\left(p_{T}, p_{z}, z, t\right)=g\left(p_{\tau}\right) 0(c-|H|) \tag{2.8}
\end{equation*}
$$

where we have set $c=t_{0} p_{z}^{m o x}$. In fact, we ahall argue belor that $c$ is a numerical constant of order 1 .

Let us examine some of the implications of the ansatz (2.8). First, it is easy to show, using the formulae (2.5), that:

$$
\begin{equation*}
\theta(c-|H|)=0(\Delta-\eta+\gamma) \theta(\Delta+\eta-\gamma) \tag{2.9}
\end{equation*}
$$

Where we have set:

$$
\begin{equation*}
\Delta=\sinh ^{-1}\left(\frac{c}{7 p_{7}}\right) \tag{2.10}
\end{equation*}
$$

In particular, the initial distribution at $t=0$ has the form: .

$$
\begin{equation*}
f\left(p_{1}, p_{z}, z, t=0\right)=g\left(p_{t}\right) 0\left(\frac{c}{p_{1} \cosh y}-|z|\right) \tag{2,11}
\end{equation*}
$$

This initial distribution coincides with that advocated by liwa and Kajantie. Its structure is easily understood. The partong with rapidity $y$ are.spread over a distance $l / \gamma$, where $l a d c / p$, and ramoshy is the usual Lorentz contraction factor. The diatance 1 may be taken to be of the order of the quantum spreading of the wave function of partons with longitudinal momentum $p_{z}$ io. Since $p_{r}$ is the only energy scale in the problem, it is natural to take $/ / 2 r d z \cdots 1 / \Delta p, n / / p_{1}$, which implies col. Now, the fastest partons occupy a longitudinal size $\cdots 2 c / p_{z}^{m x}$; thus, by a time $\operatorname{mt}_{n}=c / p^{m \prime \prime}$, no such partons remain in the plane $z=0$. This provides an interprutation of the time $t_{0}$ introduced in (2.6) and shows furthermore that, in the very high energy limit, $t_{0}$
is much smaller than any time in the problem.
Another important teature of the distribution (2.8) or (2.11) is illustrated by calculating the number of partons per unit rapidity at $t=0$;

$$
\begin{equation*}
\frac{d \|}{d^{2} p_{1} d y}=\pi R^{2} p_{0} \int d z \mathbb{C}\left(p_{1}, p_{z}, z, t=0\right)=2 \pi R^{2} \operatorname{cg}\left(p_{1}\right) \tag{2,12}
\end{equation*}
$$

where $\pi R^{2}$ is the transverse area of the colliding nucled. This distrabution is independent of $y$, as expected from the behaviour of the structure functions at small $x$. In fact, the relation betreen the plase-space distribution function and the structure function is given by:

$$
\begin{equation*}
x G_{A}\left(x, p_{1}^{2}\right)=\int_{0}^{p_{T}^{2}} \frac{d N}{d p_{i}^{\prime 2} d y} d p_{i}^{2}=2 \pi R^{2} c \int_{0}^{p_{T}^{2}} g\left(p_{T}^{\prime}\right) d^{2} p_{1}^{\prime} \tag{2.13}
\end{equation*}
$$

where $G_{A}\left(x, p_{t}^{2}\right)$ is the parton (gluon) density of the nucleus, see section 3 , and $x$ is the fraction of the longitudinal momentum per nucleon carried by the parton.

Let us now evaluate the contributions to the energy deasity and the particle number density at $z=y_{i}=0$, of the partons with transverse monentum $p_{i}$. Using the fact that $d p_{x}=p_{y}$ coshy $d y$, one easily tinds:

$$
\begin{align*}
& \frac{d n(1)}{d^{2} p_{1}}=p_{g} g\left(p, 1 \int_{-\Delta}^{\Delta} d y \operatorname{coshy}=2 g\left(p_{T}\right) \sinh \Delta\right. \\
& \frac{d \in(l)}{d^{2} p_{1}}=p_{Y}^{2} g\left(p_{i}\right) \int_{-\Delta}^{\Delta} d y \cosh ^{2} y=\frac{1}{2} p_{\gamma}^{2} g\left(p_{1}\right)(2 \Delta+\sinh 2 \Delta) \tag{2.14b}
\end{align*}
$$

These furmulae lend themselves to a simple physical interpretation in the two limiting cases of short and long times.

At shot lime, the range of integration is large, Ewn $(1 / 1)$, and partons with all rapidities are found in the plane $z=0$. The lomulae (2.14) give:

$$
\begin{equation*}
\frac{d n}{d^{2} p_{1}} \simeq p_{r} g\left(p_{t}\right) 0^{\Delta} \quad \frac{d t}{d^{2} p_{1}} \simeq \frac{1}{4} p_{t}^{2} g\left(p p_{1}\right) u^{2 \Delta} \tag{2.15}
\end{equation*}
$$

Thus at very short time the dominant contribution to the energy density comes from the partons with a large rapidity. When toit ${ }_{0}$, the energy per particle is of the order of the maximum longitudinal momentum $p_{2}^{m a x} n p_{y} 0^{\Delta} / 2$; at this time, most of the energy is in the longitudinal motion.

For long time, $\Delta$ is small. $\Delta v / 2 t$, and one gets:

$$
\begin{equation*}
\frac{d n}{d^{2} p_{T}}=2 p_{T} g\left(p_{1}\right) \Delta \quad \frac{d s}{d^{2} p_{1}}=2 p_{T}^{2} g\left(p_{T}\right) \Delta \tag{2.16}
\end{equation*}
$$

The quanta fhich remain at $z=0$ after a long time are those which carry Iittle longitudinal momentum. In this regime, the density decreases as $1 / T$, and the energy per particle is simply equal to the transuerso momentum. Note that in both regimes, the particle density, eq. (2.14a), decreases as 1/t (see eq. (2.10)).

The crossing betreen the tho regimes, i.e. the short and long time belaviours, takes place when the rapidity range of the partons which populate the region $z w$ is of order unity, i.e. Aud. In terms of time, this condition is equivalent to $r_{u} p_{\mathrm{t}}{ }^{n} 1$, that is $\eta_{n}$ n $\Delta z$. Thus the time $r_{0}$ is, roughly speaking, the time it takes to the fast partons to leave the collision zone occupied by the slow ones. He shall show in the last section that $r_{0}$ also turns out, to be equal to the "formation time", that is to the time at which the partons get freed because of collisions.

## 3.APPROXMMTE CRITERIA FOR PARTONS SET FREF RURTG THE COLLTSINON.

In this section, He ate going to propose simple criteria for deciding Hbich quanta are freed during a had-on relativistic heavy ion collision. Thest are the partons contailled in the distribution (2.11) which suffer a hard enough interaction to alloh them to convert their momentum into pliysical particles. He shall make estimates of the transverse energy released during the collision in the central unit of
rapidity. He sliall also estimate the energy density at the time the partons are freed from their initial wave function, As we shall see, the partons which dominate the energy density have $p_{r}$ about 1 GeV so that the dynamics is marginally in the weak coupling regime, will3. He might expect perturbation theory to serve as a reasonable guide although нe mould be hard pressed to certify our estimates reliable Hithin a factor of 2

To begin, we imagine a head-on heavy ion collision, say in the center of mass system, as described above. He take our partons to be gluons since these are the quanta which dominate the semi-hard collisions with which re are concerned. Then the inclusive gluon "jet" cross section is:

$$
x \frac{d v}{d p_{T}^{2} d x}=2 \int_{0}^{1} \frac{d x_{1}}{x_{1}} x_{1} G_{A}\left(x_{1} \cdot p_{T}^{2}\right) x_{A}\left(x, p_{T}^{2}\right) \frac{d \hat{\sigma}}{d p_{T}^{2}} \theta\left(x_{1} x s-4 p_{T}^{2}\right) \text { (3.1) }
$$

Where $G_{A}\left(x, p_{\top}^{2}\right)$ is the usual gluon density of the nucleus. We shall ignore possible correlation effects in the nucleus which could make $x$ gteater than 1. Also, in our estimates re shall take $G_{A}\left(x, P_{1}^{2}\right)=A G\left(x, P_{T}^{2}\right)$ with $A$ the number of nucleons and $G\left(x, p_{T}^{2}\right)$ the qluon number density of the nucleon, $\hat{v}$ is the gluon-gluon cross section given by [6]:

$$
\begin{equation*}
\frac{d \hat{u}}{d \hat{t}}=\left(\frac{\mathrm{aC}}{\pi}\right)^{2} \frac{\pi^{3}}{2 \hat{s}^{2}}\left(3-\frac{\hat{u} \hat{t}}{\hat{s}^{2}}-\frac{\hat{u} \hat{s}}{\hat{t}^{2}}-\frac{\hat{s} \hat{t}}{\hat{u^{2}}}\right) \tag{3.2}
\end{equation*}
$$

Where the "'" indicates variables directly related to gluon-gluon scattering $\left(\hat{s}=x_{1} \times s, e t c\right)$, and $C_{A} \times 3$. Because the $\frac{d x_{1}}{x_{1}}$ integration in (3.1) 1s logarithmic, the dominant contribution comes from small angle scattering, i.e. the $1 / \mathbf{t}^{2}$ term in (3.2). Thus we approximate:

$$
\begin{equation*}
\frac{d \hat{j}}{d p_{T}^{2}}:\left(\frac{\pi C_{1}}{\pi}\right)^{2} \frac{n^{3}}{2 p_{1}^{1}} \tag{3.3}
\end{equation*}
$$

$$
\begin{equation*}
u\left(p_{t}^{2}, x\right)=p_{t}^{2} \frac{d u}{d p_{t}^{2} d x} \tag{3.4}
\end{equation*}
$$

one finds:

$$
\begin{equation*}
v\left(p_{T}^{2}, x\right)=\left(\frac{k C_{A}}{\pi}\right)^{2} \frac{n^{3}}{p_{T}^{2}} x G_{A}\left(x, p_{i}^{2}\right) \int_{\frac{4 p_{1}^{2}}{1} \frac{d x_{1}}{x_{1}} x_{i} G_{A}\left(x_{1}, p_{1}^{2}\right) ; ~}^{x} \tag{3.5}
\end{equation*}
$$

Noh, at small $x$ the Altarelli-Parisi equation is:

$$
\begin{equation*}
p_{T}^{2} \frac{\partial}{\partial p_{1}^{2}} x G_{A}\left(x, p_{T}^{2}\right)=\frac{u C}{\pi} \int_{x}^{1} \frac{d x_{1}}{x_{1}} x_{1} G_{A}\left(x_{1}, p_{T}^{2}\right) \tag{1.6}
\end{equation*}
$$

so that

$$
\begin{equation*}
\sigma\left(p_{T}^{x}, x\right)=\left(\frac{\alpha C_{A}}{\pi}\right) \frac{\pi^{i}}{p_{r}^{2}} x G_{A}\left(x, p_{T}^{2}\right) p_{T}^{2} \frac{\partial}{\partial p_{T}^{2}} x_{i} G_{A}\left(x_{0} \cdot p_{T}^{2}\right) \tag{3.7}
\end{equation*}
$$

where $x_{0}=4 p_{1}^{2} / x s$. In the central unit of rapidity, we suppose that $x G$ is independent of $x$, see(2.12), so that:

$$
\begin{equation*}
u\left(p_{T}^{2}, x\right) \approx\left(\frac{A}{\pi C_{n}}\right) \frac{\pi^{n}}{2 p_{T}^{2}} p_{T}^{2} \frac{\partial}{\partial p_{r}^{2}}\left(x G_{\Lambda}\left(x, p_{r}^{2}\right)\right)^{2} \tag{1,8}
\end{equation*}
$$

Nor what should wo take for $p_{T}^{2}$ ? That is, which parton transverse momenta are going to dominate the cross section $u$ and hence the produced transverse energy distributions? If $p_{t}^{2}$ is taken very large, $v\left(p_{1}^{2}, x\right) m / p_{1}^{2}$ and the resulting cross section is very sinall. such high transverse momentum qluons are simply virtual fluctuations which are not freed during the collision. According to (3.8), we should choose $p_{1}$ very small to increase 0 . However, (3.8) ceases to be valid when $p_{r}^{2}$ is too small since gluon saturation effects become important. Perharps it is uorth reminding the reader of the physical Idea behind gluon situration $\mid \%$ and why such effects are especially important in large ions.

To that end. consider a large nucleus having longitudinal nonentum per nucleon. We suppose that $p$ is much greater than the nucleon mass, m. Then, according to (2.11), the valence quarks in the aucleus, belonging to the individual nucleons, are within a longitudinal region of size proportional to $2 R \mathrm{~m} / \mathrm{p}$ with $R$ the radius of the nucleus. llowever, the glvons and sea quarks, having a particular wlue of $x$, cannot be confined to a longitudinal size smaller than 1/px so that for $x$ s.l/2R $m$ these quanta overlap in longitudinal coordinate space. The transverse size of the gluons is $\left|\Delta x_{T}\right| \sim 1 / p$, so that if $p$, is very large such small quanta will not overlap in the tull three dimensional coordinate space. However, as one considers gluons Hith saaller $p_{\text {, }}$ overlapping configurations become more common. when $X G\left(X, p_{i}^{2}\right)<p_{f}^{2} R^{2}$ different gluons must begin to occupy the same spatial region. Since $x G_{A}\left(x, p_{1}^{2}\right)=A x G\left(x, p_{i}^{2}\right)$ one sees that this dense contiguration is enhanced in largo nuclei with strong interactions expected between the quanta when $P_{r}^{i}, n+A / R^{2}$. (The factor of $x$, to be derived belon, reflects the fact that the overlapping gluons interact (fecombine) with strength a.) Thus, the actual transition from a low dunsily to high density gluonic system accura at $X_{A}\left(x, p_{T}^{2}\right) 2, p_{f}^{2} R^{2} / \alpha$

Nor let's try to make these ideas a little more precise. The usual Altarelli-parisi equation (3.6) is appropriate for a low density system and expiesses the fact that as one looks to smaller transverse sizes, larger $p_{1}$ 's, the gluon density increases because gluon may actually be composed of tho gluons of smaller transverse size. This aspect ut the A-P equation is traditionally called gluon splitting or glunn emmission. However, as the glvon number density becones large, ve may expect the opposite process, gluon recombination, to becone inportant. Gluon recombination, Here tho gluons combine to form a single gluon lorers the number density. This is formally expressed as a higher thist moditication of the usual A-P equations. At small values of $x$ this modified equation lakes the form:


$$
\begin{equation*}
-\left(\frac{n c_{n}}{n}\right)^{2} \frac{n^{2}}{2 p_{1}^{2}} \int_{x}^{1} \frac{d x_{1}}{x_{1}} x_{1}^{2} G_{n}^{(2)}\left(x_{1}, p_{r}^{2}\right) \tag{1.9}
\end{equation*}
$$

where $G_{A}^{(2)}$ is the two gluon distribution of the nucleus. For a spherical nucleus of independent nucleons $\mid 8$ |

$$
\begin{equation*}
\dot{x}^{2} G_{A}^{(2)}\left(x, p_{1}^{2}\right)=\frac{\left(x G_{A}\left(x, p_{1}^{2}\right)\right)^{2}}{8 / 9 \pi R^{2}}=\frac{\Lambda^{2}\left(x G\left(x, p_{1}^{2}\right)\right)^{2}}{8 / 9 \pi R^{2}} \tag{3.10}
\end{equation*}
$$

with $R$ the radfus of the nucleus and $G$ the gluon diatribution of the nucleon.

Clearly, as $p_{r}^{2}$ becomes smaller; recombination becomes more
important. He expect the gluon number density to stabilize when $x G_{A}\left(x, p_{1}^{2}\right) \cdots p_{1}^{2} R^{2} / \mathbb{L}$ Hhich corresponds to

$$
\begin{equation*}
p_{t}^{z} \frac{\partial}{\partial p_{t}^{2}} x G_{A} \simeq x G_{A} \tag{3.11}
\end{equation*}
$$

The largest $p_{T}^{2}$ at which (3.11) holds should determine the $p_{1}^{2}$ to be usod in (3.8) to give the freed transverse energy in the collision. Honever. (3.11) is a little hard to use with (3.9) because the $x$-integration in that equation is not limited to very small $x$. It is easier instead to use the weaker equation

$$
\begin{equation*}
x \frac{\partial}{\partial x} p_{1}^{2} \frac{\partial}{\partial p_{1}^{2}} x G_{A}\left(x, p_{t}^{2}\right)=0 \tag{3.12}
\end{equation*}
$$

as a criterion for the saturation region. Eq.(3.12) follous from the expectation that $X_{A}\left(x, p_{r}^{2}\right)$ become independent of $x$ in the saturation region. Eqs.(3.12) and (3.9) give

$$
\begin{equation*}
\left(\frac{\alpha C_{A}}{\pi}\right) \frac{n^{3}}{2 p_{t}^{2}} x^{4} G_{A}^{(2)}\left(x, p_{F}^{2}\right)=x G_{A}\left(x, p_{t}^{2}\right) \tag{3.13}
\end{equation*}
$$

or
$\because$

$$
\begin{equation*}
\operatorname{AxG}\left(x, r_{1}^{2}\right)=\frac{16}{9 \pi} \frac{p_{1}^{2} K^{2}}{\sqrt{2} C_{A}} \tag{3.14}
\end{equation*}
$$

Thas qives $p_{1}^{2}$ as

$$
\begin{equation*}
p_{1}^{2}=\frac{9 \pi}{16} \alpha C_{A} \frac{A \times G}{R^{2}} \tag{3.15}
\end{equation*}
$$

Differentiation of (3.14) with respect to $p_{1}^{2}$ and using (3.8) ylelds:

$$
\begin{equation*}
v\left(p_{1}^{2}, x\right)=\frac{16}{9} \pi R^{2} A x G \tag{3.16}
\end{equation*}
$$

Eq. ( 3.16 ) qives the number of produced gluons per unit rapidity in a head-on collision of two spherical nuclei as:

$$
\begin{equation*}
\frac{d N}{d y}=\frac{\sigma}{\frac{\sigma}{9}-\pi R^{2}}=2 \times \times 6 \tag{3.17}
\end{equation*}
$$

and the tranavarso enargy as:

$$
\begin{equation*}
\frac{d E_{1}}{d y}=2 p_{q} A \times G \tag{3.18}
\end{equation*}
$$

The tactor $8 / 9$ in eq. (3.17) comes from an averaging over impact pasameter. Eqs.(3.17) and (3.18) are rather remarkable in that our determination of $p_{T}^{2}$ by the equality of emission and recombination leads to a produced number of gluons per unit rapidity exactly trice the number in the wave function. This factor of 2 is the factor explicitely exlibited in (3.1). In a frame where the measured gluon han emall rapidity we interprot this factor of two as corresponding to the gluon coming from aither the tro difierent colliding nuclei. In any case, it is remarkable that the recombination calculation carried out in Rof.s and the calculation done here uning (3.3) exactly componate leaving the aimple expresoiona (3.17) and (3.18).

In order to obtain from eqs.(3.17-18) number and enerqy densities, we assume that the newly freed partons obey the same free streaming kinematics as described in the previous section. Thus partons with transverse momentum $p_{1}$ occupy a volume $V=2 c \pi R^{2} / p_{T}$, see 12.11). He shall turthermore assume, and will justify in the next section. that all the partons within a rapidity range $\Delta+\boldsymbol{l}$ conttibute to the densities in the plane $z=0$. He obtain thus a number density:

$$
\begin{equation*}
\left.n=\frac{2 \Delta}{V} \frac{d N}{d y}=\frac{3}{2} \int^{\alpha-\hat{A}} \frac{\alpha\left(\frac{A x G}{\pi}\right.}{R^{2}}\right)^{\rho / 2} \tag{3.19}
\end{equation*}
$$

and an enorgy density:

$$
\begin{equation*}
=z \frac{2 \Delta}{V} \frac{d \Sigma_{1}}{d y}=\frac{9}{8} u C_{A}\left(\frac{A x G}{R^{z}}\right)^{x} \tag{3.20}
\end{equation*}
$$

While the explicit numbers which may be extracted trom (3.19-20) should only be taken as rough estimates (ase section 4), wis bulieve the and $A$ dependences exhibited in (3.15)-(3.20) are correct predictions of $Q C D$, at least for large enough $A$.

Before we go and discuss the time at which this energy is freed, He would like to make a somewhat different estimate of which gluons are freed in a head-on ultra-relativistic heavy ion collision. The mechandsm which we are about to discuss is subleading, by a power of $x$, in the amount of energy freed and we are not able to qive a systematic account of this order a correction. Nevertheless, because of its intuitive appeal and because the resulting estimates aro not too much smaller than those contained in (3.19-20), we should like to outline this simple mean free path argument. He emphasize that this is not an alternate version of our previous estimates but a discussion of a separate physical mechanism.

Consider a gluon, say in the right moving nucleus, fust before the collision. He suppose that this gluon can have rido angle scatterings Hith those gluons lett moving wilh respect lo it and occupying the one unit of rapidity bordering it. (Beyond this one unit of rapidity, the scatterings are predominantly small angle and are exactly those scatterings covered by (3.8).) Let us focuss on the mean free path $\lambda$ of our right moving gluon. He have:

$$
\begin{equation*}
\lambda=\frac{1}{n_{،} u}: \frac{1}{\frac{A \times G}{1 R^{2} \Delta 7}} \tag{13.21}
\end{equation*}
$$

where "is the cross section for wite ande qluon-qluon scattering and $\Delta z$ us the longitudinal width occupied by the aluons through which our
raght moving gluon passes. This gluon should be ereed if it has one Hade angle scattering as it passes through the distance $\Delta z$. Thus $\lambda \leqslant \Delta z$ is the criterion for converting the gluon from virtual to real. This requires

$$
\begin{equation*}
\frac{A x G}{\pi R^{2}} \dot{u} \geqslant 1 \tag{3.22}
\end{equation*}
$$

We eslimale $\hat{o}$ by taking

$$
\begin{equation*}
\hat{u}=\left.|\Delta \hat{t}| \frac{d \hat{u}}{d \hat{t}}\right|_{\hat{A}+1 \times x i-2 \hat{u}} \tag{3.23}
\end{equation*}
$$

HIth $|\Delta \ddot{t}| \approx \stackrel{n}{s} / 2$. Then

$$
\begin{equation*}
\bar{u}=\frac{27 \pi}{16 \ddot{s}}\left(u C_{A}\right)^{2} \tag{3.24}
\end{equation*}
$$

Substituting (3.24) into (3.22) and taking $t=-p_{t}^{2}$ one finds

$$
\begin{equation*}
P_{T}^{*} \leqslant\left(\operatorname{KC}_{A}\right)^{2} \frac{27}{32} \frac{\Lambda x G}{R^{2}} \tag{3.25}
\end{equation*}
$$

in contrast to our previous result (3.15).
The $p_{1}^{2}$ qiven by $(3,25)$ is higher order in a compared to that given by (3.15), however in practice there is only a factor of tro difterence between the two estimates. Since both gluons involved in the scattering described by (3.24) remain in about the same unit of rapidity we arrive at a transverse energy per unit rapidity

$$
\begin{equation*}
\frac{d E_{1}}{d y}: 2 \mu_{1} A \times G \tag{3.26}
\end{equation*}
$$

and, using the sant volume and rapidity tange as before, an energy density

$$
\begin{equation*}
E \therefore \frac{2 p_{T}^{2} A \times G}{\pi R^{2}}=\frac{27}{16 \pi}\left(\alpha C_{A}\right)^{2}\left(\frac{\Lambda x G}{R^{2}}\right)^{2} \tag{3.27}
\end{equation*}
$$

about a factor tho less than that obtained in (3.20).

## 4. DISCUSSION:

He turn nor to the question of the formation time $\mathbf{1}_{0}$, i.e. the time at which the partons are treed. In tact, it is oasy to see that this time $1 s$ of order $1 / p_{1}$ in either of our dynamical estimates given in the previous section. This follows from the fact that a gluon which gets treed in the central unit of rapidity must have undergone a scattering of momentum transfer $p_{\text {, }}$ with partons in the neighbouring rapidity slices. Thus $f_{0}$ is the time during which the partons in the contral rapidity unit ovarlap with those partons with which they may interact. Referring to the discussion at the end of section 2 we see that $i_{0}$ is also the time bordering between the long and short time behaviours in the free streaming, regine. Viening the collision in a alightly more general frame, suppose the unit of rapidity which we are considering is centered about $y=y_{0}$. Then using (2.5) and (2.8) we dee that at time the partons in this rapidity unit are located within

$$
\begin{equation*}
z_{ \pm}=t \tanh y_{0}+\frac{t}{\cosh ^{2} y_{0}}+\frac{1}{p_{1} \cosh y_{0}} \tag{4.1}
\end{equation*}
$$

Where the first $\pm$ in (4.1) comes from the unit rapidity spread and the second $t$ comes from the original specad, dua to the uncertainty relation, at $t=0$. At $t m, \cosh y_{0} / p_{r}$ we see that

$$
\begin{equation*}
z_{t}=\frac{1}{p_{1}} \sinh y_{a}+\frac{1}{p_{r} \cosh y_{a}} \tag{4.2}
\end{equation*}
$$

With the spread in $\Delta z$ due to the differences in rapidity being comparable to the original uncertainty spread. Partons with rapidity greater than $y_{0}+1 / 2$ or less than $\gamma_{0}-1 / 2$ have separated from those centered about $y_{0}$ at the time to $\cosh \gamma_{0} / \mu_{1}$. Thus nur physical picture holds together. By the tiate the collissons nucessary to free the
partons occupying a unit of rapidity have occured, these partons have physically separated, in the longitudinal direction, from the partons corrsponding to difierent rapidity intervals.

In order to get numerical estimates, let's take $n^{1 / n}{ }^{n}$, $R=1.2 A^{1 / 3} \mathrm{fn}, \mathrm{xG}=3$ and $\mathbb{x}=1 / 3$, i.e. ${ }^{\prime} C_{A}=1$ (the value $x G=3$ is reasonable, even traditional, but at this time it is not a well determined quantity, experimentally). Then (3.15) gives $p_{\mathrm{r}} \approx 0.94 \mathrm{GeV}$, i.e. ${ }^{1} 00.21 \mathrm{~m} / \mathrm{c}$, and one finds:

$$
\begin{gather*}
\frac{d M}{d y} \approx 1300  \tag{4.3a}\\
\frac{d E_{r}}{d y} \cdot \approx 1.2 \mathrm{TeV} \tag{4.3b}
\end{gather*}
$$

n $\approx 37 / \mathrm{fm}^{3}$
© $\because 35 \mathrm{GeV} / \mathrm{Fm}^{\prime \prime}$ (4.3d)

Our second estimate qives $p_{r} \approx 0.65 \mathrm{GeV}$, eq.(3.25), and hence $y_{0} n 0.3 \mathrm{~mm}$, $n: 25 / \mathrm{fm}^{3}$, and en: $17 \mathrm{GeV} / \mathrm{fm}^{3}$. These large numbers reflect the large value of the optimum $p$, in larqe nuclei. They correspond for Aal, that is lor proton-proton or proton-nucleus collisions, to $p_{T}$ njeohev and an $1 \mathrm{GeV} / \mathrm{fm}^{3}$ which look like reasonable values. The A dependence that we have found for the energy density is quite similar to that obtained in other models $|9|$, but for quite different physical reasons. in our approach a non trivial $A$ dependence is contained in the parton transverse momenta.

Finally, we have said nothing about the thermalization of the partons set free during a heavy ion collision. In fact we have little to say on this subject. Nevertheless, there is one amusing calculation which can be done at this stage to perhaps get an indication of how far trom equilibriun our initial distribution is. For a free boson gas in equilibrium one has

$$
\frac{0^{3 / 4}}{n}=\frac{\pi^{7 / 2}}{2(3) 30^{3.4}}: 1.7
$$

while from 13.19$)$ one has $: 1: 1 / n=1.3$ and from (3.24), (3.25) and $(3.26)$ one has $s^{3 / 4} / n: 1.1$. Mwa and kajantie 141 tre a refined version of this type of comparison to argue about the thermalization time.

However, eventhough the moments of the distribution may not be too tat from those of an equilibrium distribution, as the numbers we just gave seem to indicate, it is clear that the thermalization time must depend on a colliaion rate and can't be determined fom kinematical consideration alone. In fact, we would like to arque difterently and assume that the distribution function of the newly formed partons is much like the free streaming distribution (2.8) in the long time regime where the space-time rapidity $\eta$ and the momentum space rapidity $y$ are strongly correlated, i.e. Tly. The Hay such a distribution approaches equilibrium has been studled by Baym $|9|$, using a simple collision time approximation. He finds that after a time twall, with 11 the collision time, the energy density is uithin 208 of its local equilibrium value. As an order of magnitude, he may take our mean free
 (3.25). (Note that 0 becomes infinite in the limit of vanishing coupling strength, as it should.) With the number given above, one thus finds $0 \mathrm{~m}, 3 \mathrm{fm}$, and a thermalization time $1_{1} 2.211 \cdots 0.6 \mathrm{fm}$. Let ins emphasize again that this is meant to serve only as a rough estimate and not as a substitute to a decent treatment of the thermalization problem. In any case, energy densities such as those given above are sufficiently high that heavy fon collisions involve new and interesting aspects of $Q C D$ independently or not a true equilibrium is reached.

## referehices.

(1) Quark Matterig4, Proc. of the $4^{\text {in }}$ Int. Conf. on Ultra-relativistic Nucleus-nucleus Collisions, K.kajantie, ed., Lucture Hoten in Physica 221 (springer, 1985). Quark Matter'日6, proc. of the 5': Int. Cont, on Ultra-Relativiatic Nucleus-Nucleus Collisions, H.Gyulassy of al, eds.. Nucl. Phys. $\AA$ (in press).
(21 J.D.Bjorken, Phys.Rev.D2](1983)140.
(3) A.Capella, C.Pajares and A.V.Romallo, Nucl.Phys. $\mathrm{R}_{241}(1984) 75$.
[4] Ilud and K.Kajantie, Phys.Rev.Lett. 56 (1986) 696.
[5] A.Hialas and H.Czyz, Phys.Rev.D30(1984)2371.
161 B.Combridge, J.kripfgans and J.Ranft, Phys,Lett.10(1977)234; J.F.Owens. E.Raya and M.Gluck, Phys.Rev.D18(1978)1501.
[7| L.V.Gribov, E.M. Levin and M.G.Ryskin, Phys.Reports 100 (1983) 1. (8) A. ll. Hueller and J.Qili, nucl. Phys. B 268 (1968) 427.
(9) J.P.Blaizol, Proc, of the XXVI'n Cracon School of Theoretical Physics, Zakopane, Poland (1986); to appear in Acta Physica Polinica. (10) G.Baym, Phys.Lett. 138 (1984)18


[^0]:    Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No.DE-AC02-76CH00016 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

