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HYBRID HELICAL SNAKES AND ROTATORS FOR RHIC

E. D. Courant

June 13, 1995

The spin rotators and Siberian snakes presently envisaged for RHIC utilize helical dipole magnets. The snakes and the rotators each consist of four helices, each with a full twist (360°) of the field. Here we investigate an alternate layout, namely combinations of helical and pure bending magnet, and show that this may have advantages.

Requirements.

Each of the two RHIC rings needs two snakes, which should each rotate the spin by 180° about an axis that is in the horizontal plane at 45° from the longitudinal, with the two snakes located in the $Q8-Q9$ straight sections exactly 180° apart. Because of the length of these straight section each snake should be not more than 11 meters long.

For experiments with pure helicity states we need longitudinal polarization at the beam crossing points. Therefore we must rotate the vertical spin in the arcs into a longitudinal spin at the crossing point, and back again. To this end one needs 90° rotators in the long straight section between $Q3$ and $Q4$ on either side of the STAR and PHENIX crossing points in each ring, i.e. four rotators per ring. Since the DO and DX magnets lie between the rotator and the crossing point, and will precess the spin by an angle of $G\gamma\varphi(\varphi = 3.6745$ milliradians = bending angle of DO plus DX; G = 1.7928 = proton anomalous moment), the rotators must rotate the spin from vertical to an angle $G\gamma\varphi$ from longitudinal in the horizontal plane, which varies from 10.9° at injection ($\gamma = 27$) to 101.2° at 250 GeV.

The snakes and rotators should each produce zero net orbit displacement and deflection, and the maximum orbit excursion within each snake or rotator (which will be largest at injection energy) should be as small as possible.

Snakes and rotators accomplishing this can be constructed using combinations of interleaved horizontal, vertical, and/or tilted deflecting magnets, as well as helical deflecting magnets. We now review the methods for calculating the effects of these components.

Conventions

We use a coordinate system with the variables s, x, y(longitudinal, transverse horizontal, transverse vertical), also designated as x_1, x_2, x_3. We use the paraxial approximation, i.e. we assume the motion is primarily along the s direction, and all quantities of higher than first degree in the deviations x and y

from the central reference orbit will be neglected. This enables us to use linear spin rotation matrices, which can be calculated analytically.

We consider two types of magnets: dipoles and helices.

A **dipole** has a constant field $\mathbf{B}$ which may be oriented in the vertical or horizontal direction or tilted in an intermediate direction by an angle α from the vertical; we do not consider solenoidal (longitudinal) fields.

A **helix** is assumed to have fields on the axis

$$B_x = B\cos ks \tag{1a}$$

$$B_y = B\sin ks \tag{1b}$$

where $k = 2\pi/\lambda$, λ being the length of a full twist of the helix. For a right-handed helix k is positive; for a left-handed helix it is negative.

Maxwell's equations require that the fields away from the axis deviate slightly from (1)[1]:

$$B_x = B_0\{[1+\frac{1}{8}k^2(3x^2+y^2)]\cos ks + \frac{1}{4}k^2 xy\sin ks\} \tag{2a}$$

$$B_y = B_0\{[1+\frac{1}{8}k^2(x^2+3y^2)]\sin ks + \frac{1}{4}k^2 xy\cos ks\} \tag{2b}$$

$$B_s = -kB_0(x\sin ks - y\cos ks) \tag{2c}$$

In keeping with our paraxial approximation we ignore the nonlinear parts of (2), so that the transverse fields are still taken to be (1); however the solenoidal field of (2) may be considered.

We neglect all fringe fields, i.e. we calculate as if all fields stepped abruptly from zero to B at the edges.

Orbital Motion

The Lorentz force equations, in the paraxial approximation, are

$$x" = \frac{1}{B\rho}(y' B_s - B_y) \tag{3a}$$

[1] J. P. Blewett and R. Chasman, J. App. Phys. **48**, 2692(1977)

$$y" = \frac{1}{B\rho}(B_x - x' B_s) \tag{3b}$$

Dipole, $B_x = B\cos\alpha, B_y = B\sin\alpha$:

$$x = x_0 + (x_0' - \frac{s - s_0}{2\rho}\sin\alpha)(s - s_0) \tag{4a}$$

$$y = y_0 + (y_0' + \frac{s - s_0}{2\rho}\cos\alpha)(s - s_0) \tag{4b}$$

where $\rho \equiv (B\rho)/B, B\rho$ being the magnetic rigidity of the particle.

Helix, fields as in (2):

$$x = x_0 + (x_0' - kr_0\cos ks_0)(s - s_0) + r_0(\sin ks - \sin ks_0) \tag{5a}$$

$$y = y_0 + (y_0' - kr_0\sin ks_0)(s - s_0) - r_0(\cos ks - \cos ks_0) \tag{5b}$$

with

$$r_0 = 1/(k^2\rho) \tag{6}$$

being the radius of a helical orbit in the ideal helical field (1). Note that the solenoidal component of the field, being of higher order, does not enter into the orbit in our approximation.

Spin Motion

The spin vector $\vec{S}$ precesses in a magnetic field, satisfying

$$\frac{d\vec{S}}{ds} = \vec{S} \times \vec{\Omega} \tag{7}$$

where the precession frequency vector $\vec{\Omega}$ is (BMT equation)

$$\vec{\Omega} = [(1 + G\gamma)\vec{B_\perp} + (1 + G)\vec{B_{||}}]/B\rho, \tag{8}$$

$\vec{B_\perp}$ and $\vec{B_{||}}$ being the parts of the field perpendicular and parallel to the particle velocity. The spin motion is conveniently described by the SU2 spinor formalism[2]: We use a two-component spinor ψ; the spin vector $\vec{S}$ is derived from the spinor by

$$\vec{S} = \psi^{\dagger} \vec{\sigma} \psi \tag{9}$$

where $\sigma_1, \sigma_2, \sigma_3$ are the Pauli spin matrices. The precession equation (8) is then equivalent to the spinor equation

$$\frac{d\psi}{ds} = \frac{i}{2}(\vec{\sigma} \cdot \vec{\Omega})\psi \tag{10}$$

which has to be solved for the given field.

Dipole, $B_x = B\cos\alpha, B_y = B\sin\alpha$:

Equation (10) is easily solved, since the coefficients are constant. The solution can be written in matrix form

$$\psi_2 = \mathbf{M}\psi_1 \tag{11}$$

where the SU2 matrix **M** is

$$\mathbf{M} = \exp[\frac{i}{2}\kappa(\sigma_2 \cos\alpha + \sigma_3 \sin\alpha)(s_2 - s_1)]. \tag{12}$$

Helix:

If we take the ideal fields (1) and ignore the solenoidal field (2c) equation (10) becomes

$$\frac{d\psi}{ds} = \frac{i}{2}\kappa(\sigma_2 \cos ks + \sigma_3 \sin ks)\psi \tag{13}$$

where

$$\kappa = \frac{1 + G\gamma}{\rho}. \tag{14}$$

We note that

where

$$\sigma_2 \cos ks + \sigma_3 \sin ks = \sigma_2 e^{iks\sigma_1}. \tag{15}$$

and transform to

$$\varphi = \exp(\frac{i}{2}ks\sigma_1)\psi. \tag{16}$$

[2] B. W. Montague, Physics Reports **113**, 1 – 96(1984)

Then (10) becomes

$$\frac{d\varphi}{ds} = \frac{i}{2}(\kappa\sigma_2 + k\sigma_1)\varphi \tag{17}$$

which is an equation with constant coefficients, and is therefore easily solved:

$$\varphi_2 = \exp[\frac{i}{2}(\kappa\sigma_2 + k\sigma_1)(s_2 - s_1)]\varphi_1 \tag{18}$$

Transforming back to ψ, we find the matrix solution (11) with

$$\mathbf{M} = \exp[-\frac{i}{2}ks_2\sigma_1]\exp[\frac{i}{2}(\kappa\sigma_2 + k\sigma_1)(s_2 - s_1)]\exp[\frac{i}{2}ks_1\sigma_1]. \tag{19}$$

Here ks_1 is the orientation angle between the helical fie ld and the vertical at the entrance, and ks_2 the angle at the end.

Snakes and Rotators

Snakes and/or rotators can be constructed by combining dipole and helical magnets. To design a snake or a rotator we have to satisfy the requirements that

(a) the orbit returns to the original values of x, x', y, y', i.e. the net deflection and displacement is zero;

(b) the spin transformation matrix, obtained by multiplying the appropriate matrices of forms (11) and (18) together, produces the desired spin rotation. Any spin transformation, i.e. any SU2 matrix, can be parametrized in the form

a

$$\mathbf{M} = \exp\frac{i}{2}\mu(\vec{n}\cdot\vec{\sigma}) \tag{20}$$

where $\vec{n}$ is a unit vector.

For a snake (180 degree rotator) the parameter μ must equal π or 180°, and the vector $\vec{n}$ (which is the axis of rotation) must lie in the horizontal plane; it is usually desirable for it to make an angle of 45° with the longitudinal direction.

A sure way to ensure that the axis of rotation lie in the horizontal plane is to construct the snake or rotator with reflection symmetry, so that the vertical component of the field is *antisymmetric* about the center, while the horizontal field components are *symmetric.*

This is accomplished by adding to a set of magnets its reflection, consisting of the reflections of each of the elements in reverse order; the reflection matrix for each element is obtained by reversing the order of the factors and changing the sign of the coefficients of σ_3 but not of σ_1 and σ_2. Note that the reflection of a right-handed helix is also right-handed.

To make the spin at the crossing point longitudinal, as is necessary for experiments studying helicity dependence of interactions, one needs *rotators* that change the vertical spin in the arcsto horizontal at the crossing points, followed by the inverse rotator downstream from the crossing point. This may be accomplished by $a90°$ rotation about a horizontal axis (with the same symmetry as the snake described above), but other configurations are also possible. For example, the four-helix rotators described by Ptitsin[3] do not have this reflection symmetry and do not have rotation angles of 90°, but they still rotate a vertical spin into the horizontal plane.

Spreadsheet Calculations

A Lotus-123 spreadsheet program has been written to evaluate the properties of snakes and rotators as functions of their parameters. A combination of horizontal and vertical (or tilted) deflector magnets and/or helix magnets is laid out, the spin matrices multiplied together, and the Lotus procedure "BSOLV" is applied to vary parameters so as to fit the constraints of zero orbit deflection and appropriate spin rotation.

This procedure has been applied to the four-helix snakes and rotators presently envisaged for the RHIC polarized proton project as described by Luccio[4]. We also investigate a "hybrid" configuraton, which consists of a single helix flanked by (horizontally deflecting) dipole magnets.

The procedure used here neglects nonlinear terms in the equations of motion, and also neglects fringing fields; therefore the numerical results for the four-helix configurations are not precisely identical with those given by Luccio, who solves differential equations both for orbital and for spin motion. But the simplified procedure lends itself to easy modification of parameters, and enables one to find parameters that optimize performance subject to given constraints. Once an optimal configuration has been found in this way, one may use the more exact differential equations for fine tuning.

The results, for the four-helix and for the hybrid configurations, are given in Table 1 and 2 and shown in Figures 1 through 4. In each case we

[3] V. Ptitsin, RHIC/*AP*/49(Dec. 1994)
[4] A. Luccio, presentation to RHIC Polarized Proton Review, March 16, 1995

show results both for injection energy ($\gamma = 27$) and maximum storage energy ($\gamma = 268$).

Note that the hybrid snake is significantly shorter than the helical snake, and that the helical magnet for the hybrid is longer than in the 4-helix case; this may make it easier to construct, and certainly reduces the effect of the nonlinear terms in the field (as can be seen from the fact that the nonlinear corrections to the magnetic fields (1) are of the order $(kr)^2 = (2\pi r/\lambda)^2$, where r is the orbit displacement and λ is the helix twist wavelength, i.e. the length of a helical module. The maximum orbit excursion in the hybrid case is 10% more than in the helical case; this is probably not a serious drawback. It may therefore be advantageous to consider choosing the hybrid design rather than the 4-helix design for the snakes.

In the case of the rotators, on the other hand, the maximum excursion at injection ebergy is significantly larger than in the helical case; moreover the helical magnet has to be made with a short pitch, which may be difficult. The shorter length is hardly an advantage in this case, because the rotators are expected to be placed in the $Q3 - Q4$ straight sections, which have plenty of room (they are $34m$ long). Therefore the *rotators* should probably be made in the 4-helix mode as proposed in Luccio's report.

Table 1: Four Helices

Name	Description	γ	B_{mx} T	Lgth m	BL $T-m$	xmx cm	ymx cm	Axis deg	Rotn deg
Helical snake	4 helices 2.4m each	27.0	3.96	10.56	24.9	1.37	2.96	45	180
		268	4.03	10.56	25.4	0.14	0.30	45	180
Helical rot	4 helices 2.4m each	27.0	2.71	10.56	23.0	2.26	0.94	10.19	90
		268	3.52	10.56	32.5	0.38	0.12	101.2	90

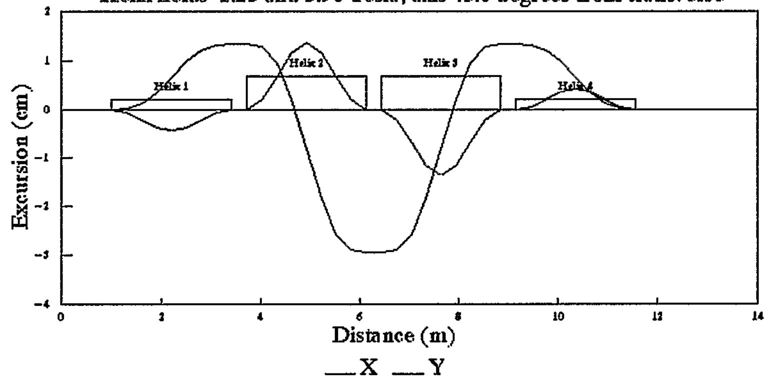
Table 2: Helix and 4 Dipoles

Name	Description	γ	B_{mx} T	Lgth m	BL $T-m$	xmx cm	ymx cm	Axis deg	Rotn deg
Hybrid snake	Full helix & 4 dipoles	27.0	3.87	7.18	22.6	3.34	2.23	45	180
		268	3.95	7.18	23.0	0.34	0.23	45	180
Hybrid rot	Full helix & 4 dipoles	27.0	3.84	7.38	23.4	3.95	1.01	10.19	90
		268	3.91	7.38	23.8	0.13	0.10	101.2	90

Figure 1

Four–helix snake, Energy 24.4 GeV

Helix fields 1.23 and 3.96 Tesla; axis 45.0 degrees from transverse

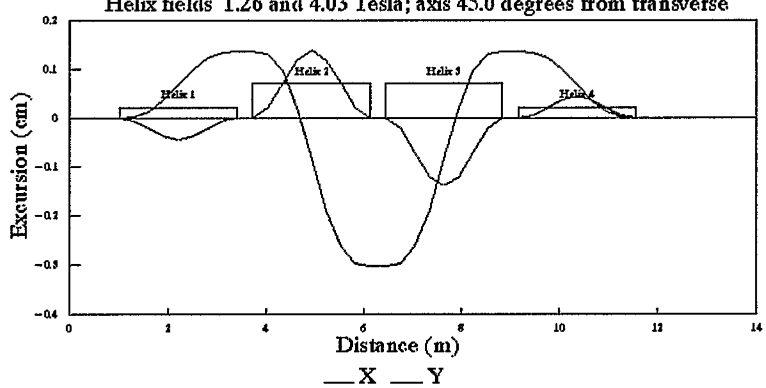


BL(tot)	24.9144
Ltot (m)	10.5600
xmax (cm)	1.3679
ymax (cm)	2.9596

Element	Length (m)	B direction from horiz.	B Field (Tesla)	BL (T–m)
Helix	2.400	–90 to 270	1.232	2.957
Space	0.320			
Helix	2.400	–270 to 90	3.958	9.500
Space	0.320			
Helix	2.400	–90 to 270	3.958	9.500
Space	0.320			
Helix	2.400	–270 to 90	1.232	2.957

Four–helix snake, Energy 250.5 GeV

Helix fields 1.26 and 4.03 Tesla; axis 45.0 degrees from transverse



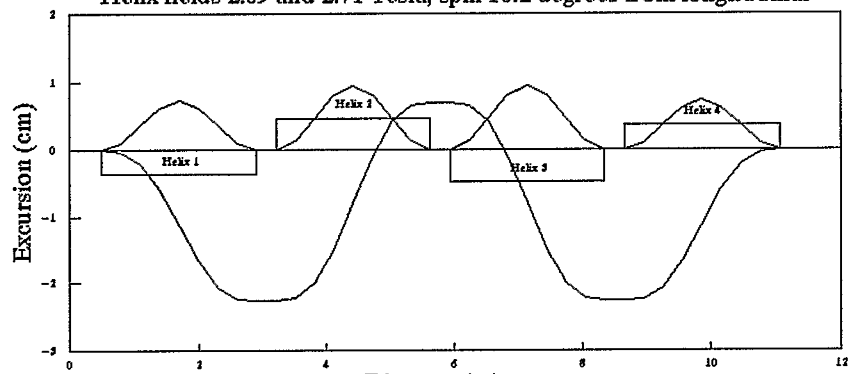
BL(tot)	25.3932
Ltot (m)	10.5600
xmax (cm)	0.1404
ymax (cm)	0.3037

Element	Length (m)	B direction from horiz.	B Field (Tesla)	BL (T–m)
Helix	2.400	–90 to 270	1.256	3.014
Space	0.320			
Helix	2.400	–270 to 90	4.034	9.683
Space	0.320			
Helix	2.400	–90 to 270	4.034	9.683
Space	0.320			
Helix	2.400	–270 to 90	1.256	3.014

Figure 2

Four–helix rotator, energy 24.4 GeV

Helix fields 2.09 and 2.71 Tesla; spin 10.2 degrees from longitudinal



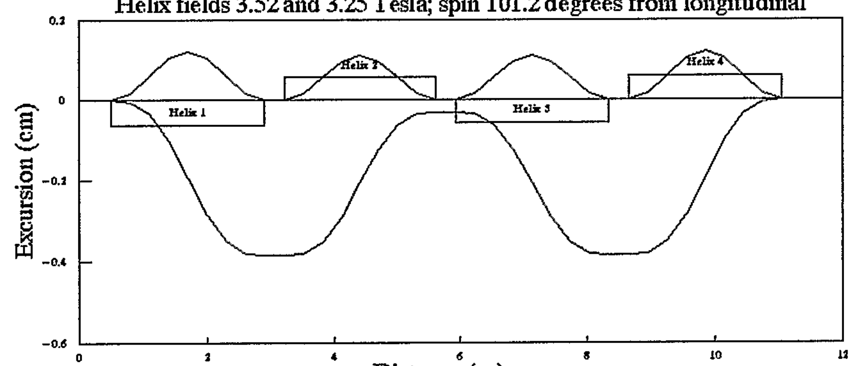
__X __Y

BL(tot)	23.0439
Ltot (m)	10.5600
xmax (cm)	2.2645
ymax (cm)	0.9382

Element	Length (m)	B direction from horiz.	B Field (Tesla)	BL (T–m)
Helix	2.400	–360 to 0	2.086	5.006
Space	0.320			
Helix	2.400	360 to 0	2.715	6.516
Space	0.320			
Helix	2.400	–360 to 0	2.715	6.516
Space	0.320			
Helix	2.400	360 to 0	2.086	5.006

Four–helix rotator, energy 250.5 GeV

Helix fields 3.52 and 3.25 Tesla; spin 101.2 degrees from longitudinal



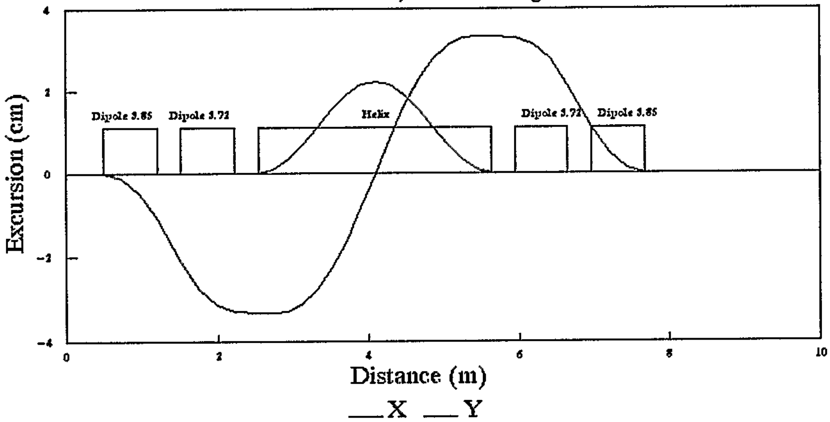
__X __Y

BL(tot)	32.4637
Ltot (m)	10.5600
xmax (cm)	0.3844
ymax (cm)	0.1224

Element	Length (m)	B direction from horiz.	B Field (Tesla)	BL (T–m)
Helix	2.400	–360 to 0	3.517	8.440
Space	0.320			
Helix	2.400	360 to 0	3.246	7.792
Space	0.320			
Helix	2.400	–360 to 0	3.246	7.792
Space	0.320			
Helix	2.400	360 to 0	3.517	8.440

Figure 3

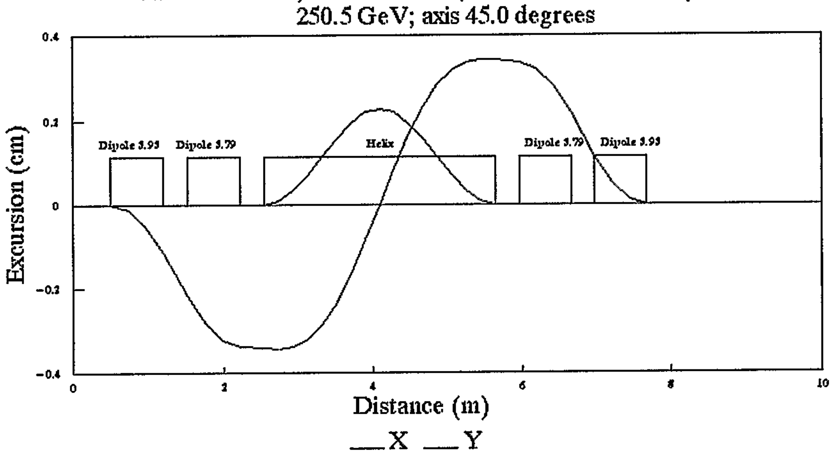
RHIC Snake, 3.87 T helix; 3.72 & 3.85 T dipoles
24.4 GeV; axis 45.0 degrees



BL(tot)	22.6036
Ltot (m)	7.1800
xmax (cm)	3.3436
ymax (cm)	2.2330

Element	Length (m)	B direction from horiz.	B Field (Tesla)
Dipole	0.700	90.000	3.853
Space	0.320		
Dipole	0.700	−90.000	3.717
Space	0.320		
Helix	3.100	0 to 360	3.873
Space	0.320		
Dipole	0.700	90.000	3.717
Space	0.320		
Dipole	0.700	−90.000	3.853

RHIC Snake, 3.95 T helix; 3.79 & 3.93 T dipoles
250.5 GeV; axis 45.0 degrees

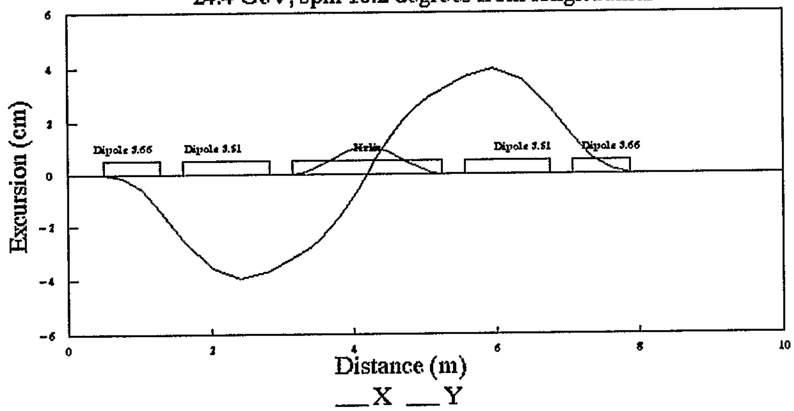


BL(tot)	23.0383
Ltot (m)	7.1800
xmax (cm)	0.3431
ymax (cm)	0.2291

Element	Length (m)	B direction from horiz.	B Field (Tesla)
Dipole	0.700	90.000	3.927
Space	0.320		
Dipole	0.700	−90.000	3.788
Space	0.320		
Helix	3.100	0 to 360	3.948
Space	0.320		
Dipole	0.700	90.000	3.788
Space	0.320		
Dipole	0.700	−90.000	3.927

Figure 4

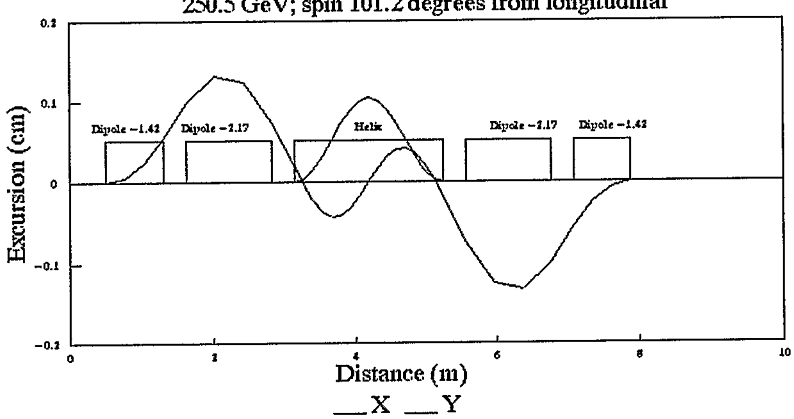
Rotator, 3.84 T Helix; 3.51 & 3.66 T dipoles
24.4 GeV; spin 10.2 degrees from longitudinal



BL(tot)	23.3961
Ltot (m)	7.3800
xmax (cm)	3.9463
ymax (cm)	1.0147

Element	Length (m)	B direction from horiz.	B Field (Tesla)
Dipole	0.800	90.000	3.664
Space	0.320		
Dipole	1.200	−90.000	3.510
Space	0.320		
Helix	2.100	0 to 360	3.835
Space	0.320		
Dipole	1.200	90.000	3.510
Space	0.320		
Dipole	0.800	−90.000	3.664

Rotator, 3.91 T Helix; −2.17 & −1.42 T dipoles
250.5 GeV; spin 101.2 degrees from longitudinal



BL(tot)	23.8460
Ltot (m)	7.3800
xmax (cm)	0.1315
ymax (cm)	0.1041

Element	Length (m)	B direction from horiz.	B Field (Tesla)
Dipole	0.800	90.000	−1.425
Space	0.320		
Dipole	1.200	−90.000	−2.167
Space	0.320		
Helix	2.100	0 to 360	3.909
Space	0.320		
Dipole	1.200	90.000	−2.167
Space	0.320		
Dipole	0.800	−90.000	−1.425