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A Spin Tracking Program for Siberian Snakes

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A FORTRAN program entitled NSNK has been written which will trace the precession of a particle's spin through a lattice with uniformly spaced Siberian snakes. The program calculates 2×2 SU2 matrices for a system whose elements are bending arcs and Siberian snakes (spin rotators), either full (180° rotation) or partial (rotation angle other than 180°).A single depolarizing resonance may be present; the program is unable to cope with a combination of resonances. The number of snakes may be 0, 1, or any even number. A single snake is assumed to rotate the spin around the longitudinal axis; two snakes are assumed to have axes of $\pm 45^\circ$ from the longitudinal (in the horizontal plane); in the case of more than one pair of snakes the precession axes may be either $\pm 45^\circ$ or $\pm 45^\circ/NP$, where NP is the number of snake pairs. The snakes are always assumed to be evenly spaced around the circumference.

The depolarizing resonance is represented by a transverse horizontal field

$$\frac{\epsilon B}{\gamma G} e^{i(\nu\vartheta + p)} \tag{1}$$

where

ϵ = resonance strength

ν = resonance frequency (integer for imperfection resonances, vertical betatron frequency for intrinsic resonances)

γ = particle energy in mass units

$G(= 1.7928$ for protons$)$ = anomalous moment factor

B = normal field in arcs

$p = a$ phase (incrementing by $2\pi\nu$ per turn), specified in units of π.

ϑ = azimuth from starting point (180° from single snake; at position of first snake of first pair otherwise).

Method:

We assume a storage ring with depolarizing field given by (1), with one or more snakes. We represent the spin by 2-component spinors, and traversal of ring elements (arc sections or snakes) by 2×2 SU2 matrices. The matrix for an arc section is obtained from the spinor equation of motion [1]

$$\frac{d\psi}{d\vartheta} = \frac{i}{2} \begin{pmatrix} -\gamma G & \epsilon e^{-i(\nu\vartheta+p)} \\ \epsilon e^{i(\nu\vartheta+p)} & \gamma G \end{pmatrix} \psi \tag{2}$$

which may also be written as

$$\frac{d\psi}{d\vartheta} = \frac{i}{2}(-\gamma G\sigma_3 + \epsilon\sigma_1 e^{i(\nu\vartheta+p)\sigma_3})\psi. \tag{3}$$

If we transform to a precessing coordinate system by

$$\psi = e^{\frac{i}{2}(\nu\vartheta+p)\sigma_3}\varphi \tag{4}$$

we obtain

$$\frac{d\varphi}{d\vartheta} = \frac{i}{2}(-\delta\sigma_3 + \epsilon\sigma_1)\varphi \tag{5}$$

with $\delta = \gamma G - \nu$. The solution, for traversal of an arc from ϑ_1 to ϑ_2, is

$$\varphi(\vartheta_2) = e^{\frac{i}{2}(-\delta\sigma_3+\epsilon\sigma_1)(\vartheta_2-\vartheta_1)}\varphi(\vartheta_1) \tag{6}$$

so that the matrix for the original spinor variable ψ is

$$\mathbf{A} = e^{\frac{i}{2}(\nu\vartheta_2+p)\sigma_3} e^{\frac{i}{2}(-\delta\sigma_3+\epsilon\sigma_1)(\vartheta_2-\vartheta_1)} e^{-\frac{i}{2}(\nu\vartheta_1+p)\sigma_3} \tag{7}$$

The matrix for a (partial) snake, i.e. an element that rotates the spin by an angle α around an axis in the horizontal plane at an angle β from the longitudinal direction, is

$$\mathbf{S} = \exp(-\frac{i}{2}\alpha\sigma_1 e^{i\beta\sigma_3}) = \cos\frac{\alpha}{2} + i(\sigma_1\cos\beta + \sigma_2\sin\beta)\sin\frac{\alpha}{2} \tag{8}$$

[1] E D Courant and R.D Ruth, "The Acceleration of Polarized Protons in Circular Accelerators", BNL Report BNL 51270 (1980)

The program computes the spin transfer matrix for one complete revolution by

multiplying matrices of the forms (7) and (8) as appropriate, starting at $\vartheta = 0$ (or at any other azimuth)

The matrix for the first turn - like any SU2 matrix - can be parameterized in the form

$$\mathbf{M} = \exp(i\pi\nu_s \vec{n} \cdot \vec{\sigma}), \tag{9}$$

meaning that the matrix corresponds to a spin rotation of 2π times ν_s

(the spin tune for one turn) around an axis characterized by the unit vector $\vec{n}$. We may interpret $\vec{n}$ as the "stable spin direction" or "invariant spin vector", when the resonance frequency ν is an integer (imperfection resonance). In the case of ν being the (non-integral) betatron oscillation frequency, the physical significance of the vector $\vec{n}$ is less straightforward. But in any case we begin the tracking run by determining $\vec{n}$ for the first turn, and then setting the initial spin direction to be this $\vec{n}$.

For subsequent turns we increment the particle energy by a constant amount dgg $\equiv \Delta(\gamma G)$. Optionally we may also increment the snake strength, starting from zero and going up to a maximum snake excitation - and then reversing that procedure at the end of the run; however, we may also keep the snake strength constant.The phase of the oscillating depolarizing field increments by $2\pi\nu$ per turn.

For the first turn, and periodically (every N_{print} turns) thereafter we print out the one-turn spin tune ν_s, the components of the stable spin vector n, the component of the actual spin vector, and the inner product of the two - a measure of how closely the actual spin follows the changing stable spin direction.
Input:

The program first prompts for name of output file. If you input a single space, the output file is skipped, and output only appears on screen. Then it asks for "eps, delta,dgg,nures,phizero,Nsnk,Nturn, Nprint, Na, rotang,azimuth" and displays the current values of these parameters. Input from keyboard (comma for keeping current value unchanged): eps=resonance strength, delta= initial distance of γG from the resonance ν; dgg = increment of γG per turn, nures = ν, the resonance frequency (integer for imperfection, betatron tune for intrinsic); phizero = initial phase of oscillations (in units of π); Nsnk = number of snakes (should be 0, 1, or an even number; if Nsnk = a positive even number, snake precession angles are alternately $\pm 90°$/Nsnk; if Nsnk is a negative even number, they are always $\pm 45°$); Nturn = number of turns for the run; Nprint = print spacing (print 0th, 1st and every Nprint-th turn). If the next parameter, Na is not zero, the snakes are gradually turned on from zero to full strength in the first Na turns, and are gradually in the last Na turns; if it is zero, they are always on. rotang is the snake rotation angle in degrees (180 for a full snake). "azimuth" is the azimuth in the ring where the spin is displayed.

The program then goes on to compute for Nturn turns, and prints out (whenever K=1 or a multiple of Nprint):

The number of turns from the beginning, the distance from resonance, the spin tune for one turn, the three components of the stable spin direction (or invariant closed-orbit spin) corresponding to the spin transfer matrix of that turn (or the first turn for $K = 0$); the actual spin of a particle whose initial spin was the stable spin direction of the first turn, and the cosine of the angle between these two spin directions. It then prompts for input parameters for the next case.

The program is unable to cope with more than one resonance at a time; therefore is is expected to give reliable results only if resonances are well separated, i.e. separated by an amount that should be large compared to the width or strength of the resonance.

The program resides on the BNL VAX cluster in directory 2dua3:[courant.pol]nsnk.for (source file) and nsnk.exe (executable). It is also available for PC computers with Fortran compilers.