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J. Wei

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Collider Accelerator Department  
**Brookhaven National Laboratory**

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# Effect of Longitudinal Variation of Multipoles in QRJ Magnets

Jie Wei

*RHIC Project*

*Brookhaven National Laboratory*

*Upton, New York 11973*

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The recent warm measurements of RHIC QRJ (Q3) quadrupoles indicate that the magnetic multipoles  $a_2$  and  $b_3$  vary significantly lengthwise along the magnet body. In the case of QRJ105, the difference in  $a_2$  between the lead-end half and non-lead-end half is about 10 units, and the difference in  $b_3$  is about 3 units. In the case of QRJ106, the average  $a_2$  is about 6 units. Here, we first estimate the effect of such a longitudinal variation of the multipoles harmonics, and then study the effect of the large average  $a_2$ .

We express the variation of the multipole  $b_n$  along a magnet of length  $L$  by the expression

$$b_n(s) = b_{n0} + b_{n1}(s), \quad \int_0^L b_{n1}(s) ds = 0 \quad (1)$$

where  $b_{n0}$  is the average value of the multipole harmonics, and  $s$  is the distance along the magnet axis. The relative change (kick) in particle action caused by  $b_n$  can be derived to be<sup>1</sup>

$$\frac{\Delta J_x}{J_x} = \left( \frac{10^{-4} G_0}{B_0 \rho R_0^{n-1}} \right) (2J_x)^{\frac{n-1}{2}} \int_0^L 2\beta_x^{\frac{n+1}{2}} \sin \chi_x \cos^n \chi_x b_n(s) ds, \quad (2)$$

where  $R_0 = 0.04$  m is the reference radius,  $G_0$  ( $G_0 = 48$  T/m at storage) is the reference gradient,  $B_0 \rho$  ( $B_0 \rho = 840$  Tm at storage) is the magnetic rigidity, and  $\chi_x$  is the betatron phase. Since the amplitude functions  $\beta_{x,y}$  vary significantly inside the magnet, the effect of variation  $b_{n1}$  is not negligible. From Eq. 2, we define the effective multipole  $\Delta b_{n,eff}$  as

$$\Delta b_{n,eff} = \frac{1}{2} \left\{ \left| \frac{\int_0^L \beta_x^{\frac{n+1}{2}}(s) b_{n1}(s) ds}{\int_0^L \beta_x^{\frac{n+1}{2}}(s) ds} \right| + \left| \frac{\int_0^L \beta_y^{\frac{n+1}{2}}(s) b_{n1}(s) ds}{\int_0^L \beta_y^{\frac{n+1}{2}}(s) ds} \right| \right\}, \quad (3)$$

and similarly the effective  $\Delta a_{n,eff}$ . The effect (e.g. action kick and tune shifts) of the harmonic variation  $b_{n1}$  can be approximated by the effective harmonic  $\Delta b_{n,eff}$  multiplied by the average amplitude function to the appropriate power (Eq. 2).

Figure 1 shows the variation of the amplitude functions  $\beta_{x,y}$  in the Q3 magnet in a typical IR region operating at  $\beta^* = 1$  m. Using a simple model to approximate the variation of

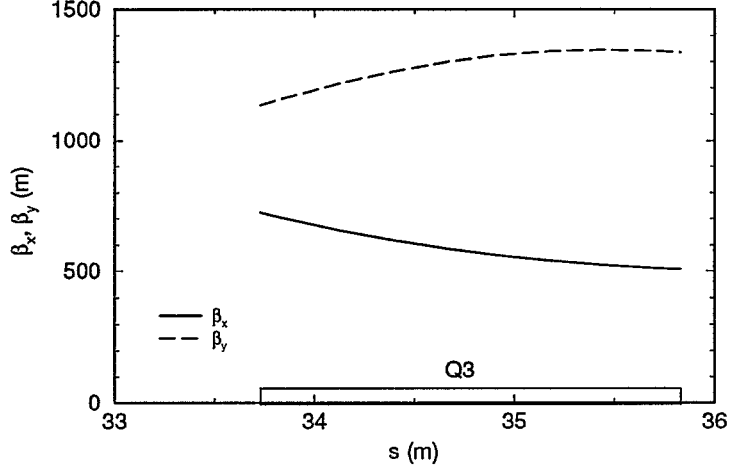


Figure 1: Amplitude function variation in the RHIC Q3 magnet at 6 o'clock outer insertion region during  $\beta^* = 1$  m operation.

the harmonics,

$$b_{n1} = \frac{\Delta b_n}{2} \text{sign} \left( \cos \frac{2\pi\lambda s}{L} \right), \quad (4)$$

where  $\text{sign}(x)$  is equal to 1 if  $x$  is positive, and is equal to  $-1$  if  $x$  is negative, and  $\lambda$  is the frequency of the variation. Using Eq. 3, Table 1 shows the effective values of  $a_{n,eff}$  or

$n$	$\lambda = 1$	$\lambda = 2$	$\lambda = 4$
1	0.055	0.037	0.027
2	0.10	0.075	0.050
3	0.16	0.11	0.080
4	0.26	0.18	0.13
5	0.42	0.28	0.21

Table 1: Effective strengths of the longitudinal multipole variations (i.e., the effective  $a_{n,eff}$  or  $b_{n,eff}$  that correspond to one units of  $\Delta a_n$  or  $\Delta b_n$ , respectively) as functions of the variation cycle  $\lambda$  for various multipole harmonics  $n$ .

$b_{n,eff}$  that correspond to one units of  $a_{n,eff}$  or  $b_{n,eff}$ , respectively, i.e., the relative strength of the multipole variation. Table 1 implies that the effect of the 10 unit difference in  $a_2$  between the lead and the non-lead halves in QRJ magnets is about the same as that of an average  $a_2$  of about 1 unit. Similarly, the 3 unit difference in  $b_3$  can be approximated by an average  $b_3$  of 0.5 units.

If installed in the maximum- $\beta$  location of the IR region where the operating  $\beta^*$  is 1 meter, the magnet with average  $a_2 = 6$  units (similar to QRJ106) produces a significant kick of  $\Delta J_x/J_x \approx 3 \times 10^{-2}$  for a 5- $\sigma$  particle at emittance  $40\pi\text{mm}\cdot\text{mr}$ , and is likely to cause problem. Therefore, measures should be taken to investigate and to correct the large  $a_2$  multipole harmonics. Compared with the  $a_2$ , the effect of longitudinal  $b_3$  variation is less alarming, and is likely be solved by sorting and similar methods.

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## References

1. J. Wei and S. Peggs, RHIC/AP/19 (1993).