BNL-102137-2014-TECH

RHIC/AP/26;BNL-102137-2013-IR

# Impedance Parametrization of Simple Geometries


V. Mane


May 1994


Collider Accelerator Department

**Brookhaven National Laboratory**



**U.S. Department of Energy**

USDOE Office of Science (SC)

# DISCLAIMER

# Impedance Parametrization of Simple Geometries

V. Mane and S. Peggs

May 1994

## I. Introduction

RHIC has several pipe diameter transitions of various radii in the IR, in addition to the cold to warm transition. In the low frequency regime, shallow discontinuities in the vacuum chamber, such as transitions, steps and bellows are inductive. The inductance of these objects are given by simple expressions, and are compared with MAFIA results.

## II. Low Frequency Approximation

The objects such as shallow transitions, steps and bellows appear inductive to bunches with bunch length $\sigma > b/2$, where b is the pipe radius. The magnetic flux and hence the inductance of these objects are obtained from the magnetic field due to the beam at the cavity, and the area of the cavity. Given below are the impedance for some of the above mentioned structures [1]. The convention used is that in Chao's text [2].

### *A Pipe Aperture Transition*

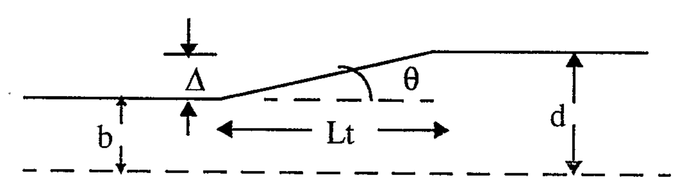


**Figure 1:** Pipe Aperture Transition

$$\frac{Z}{n} = -iZ_0\left(\frac{\beta}{2\pi R}\right)\frac{3}{2}\left(\frac{b\Delta^2}{d^2}\right)\left(\frac{2\theta}{\pi}\right)^{1/2} \qquad \Delta/d \text{ small, } \theta \le \pi/2 \tag{1}$$

where $Z_0 = 377$ ohms is the free space impedance, $\beta = v/c$ is the Lorentz factor, $(2\pi R)$ is the circumference of the accelerator.

*B Bellows*

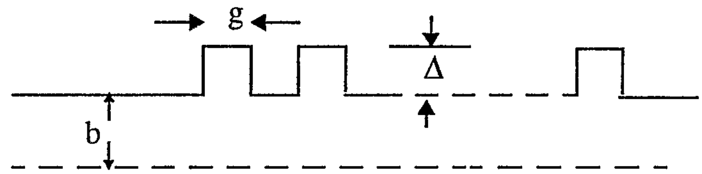


**Figure 2:** Bellows

$$\frac{Z}{n} = -iZ_0\left(\frac{\beta}{2\pi R}\right)gNc\frac{\Delta}{b} \qquad \Delta/b \text{ small} \tag{2}$$

In the above expression, the impedance of a single corrugation is multiplied by Nc, the number of corrugations in the bellows and $Ncg$ is the total corrugation length.

*C Iris*

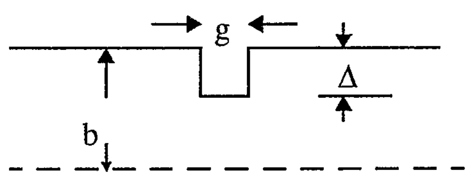


**Figure 3:** Iris

$$\frac{Z}{n} = -iZ_0\left(\frac{\beta}{2\pi R}\right)2\left(\frac{\Delta^2}{b}\right) \qquad \Delta/b \text{ small, } g/b \text{ small} \tag{3}$$

## III. Comparison with MAFIA

Table 1 compares the above results with those from MAFIA [3]. The geometry parameters used are close to RHIC parameters. The calculations are described in details in [4]. The impedance obtained from these expressions are within 20% agreement of the MAFIA results, at frequencies upto 1.5 GHz. In order to satisfy the microwave stability criteria, the total RHIC broadband impedance should be less than 1.5 ohm. The ring broadband impedance is presently estimated to be less than .5 ohm. Therefore, the 20% agreement is good enough to be able to use these expressions to determine the impedance.

At high frequencies the analytical results deviate from the numerical results, as these devices approach resonance. The cutoff frequency, $TM_{01}$ mode, of a pipe with radius b is given by

$$fc = \frac{2.405\,c}{2\pi\ b},$$ c is the speed of light.

The transition has a resonance at 1.9 GHz, the cutoff frequency of the larger pipe (radius d). The iris has a resonance at 3.3 GHz, the cutoff frequency of the pipe with radius b. The bellows have a broad resonance at high frequencies given by

$$f_r = \frac{1.69}{2\pi}\left(\frac{c}{\Delta}\right)\left(\frac{\Delta}{b}\right)^{0.43}$$

**Table 1:** Comparing analytical and numerical results

| Object Name | Geometry Parameters | Frequency [MHz] | \|Z\| analytical [ohm] | \|Z\| MAFIA [ohm] |
|---|---|---|---|---|
| Pipe Transition | b = 3.5 cm<br>d = 6 cm<br>Lt = 10 cm | 1000 | 4.5 | 4.4 |
| | | 1500 | 6.8 | 8.7 |
| Bellows | b = 3.5 cm<br>Δ = 1 cm<br>g = .25 cm<br>Nc* = 30 | 1000 | 27 | 23.8 |
| | | 1500 | 40 | 38 |
| Iris | b = 3.5 cm<br>Δ = 1.5 cm<br>g = 1 cm | 1000 | 16.1 | 14.8 |
| | | 1500 | 24.2 | 21.5 |

* Number of corrugations in the bellows

## IV. Z/n Plots

To reduce the aperture transition impedance, the transitions will be made smoothly. A 5:1 transition ratio (ratio of transition length to depth Lt/Δ) will be used in RHIC, whereever possible. Eq (1) was used to determine the impedance, with the inner radius b varying from 3 cm to 7 cm, and the outer radius d varying from 4 cm to 11 cm. Figure 4 gives a plot of Z/n contours from 100 μohm to 700 μohm, for the above mentioned transition radii and 5:1 transition ratio. This plot will help determine the impedance of various transitions in the IR. Equations (2) and (3) were used to obtain Z/n contours for the bellows [Figure 5] and for the iris [Figure 6], respectively.

## V. Conclusion

The impedance of bellows, transition and iris calculated from MAFIA are in 20% agreement with analytical results, upto a frequency of 1.5 GHz.

All transitions in RHIC will be made with a 5:1 transition ratio, if possible.

The impedance of the transitions, bellows and iris can be obtained from the Z/n contours [Figures 4, 5, 6].

## VI. References


[1] K. Bane, SLAC-PUB-4618, May 1988

[2] A. W. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, John Wiley & Sons.

[3] T. Weiland, Particle Accelerators 15 (1984), pp. 245-292

[4] V. Mane, BNL 48375, May 1993

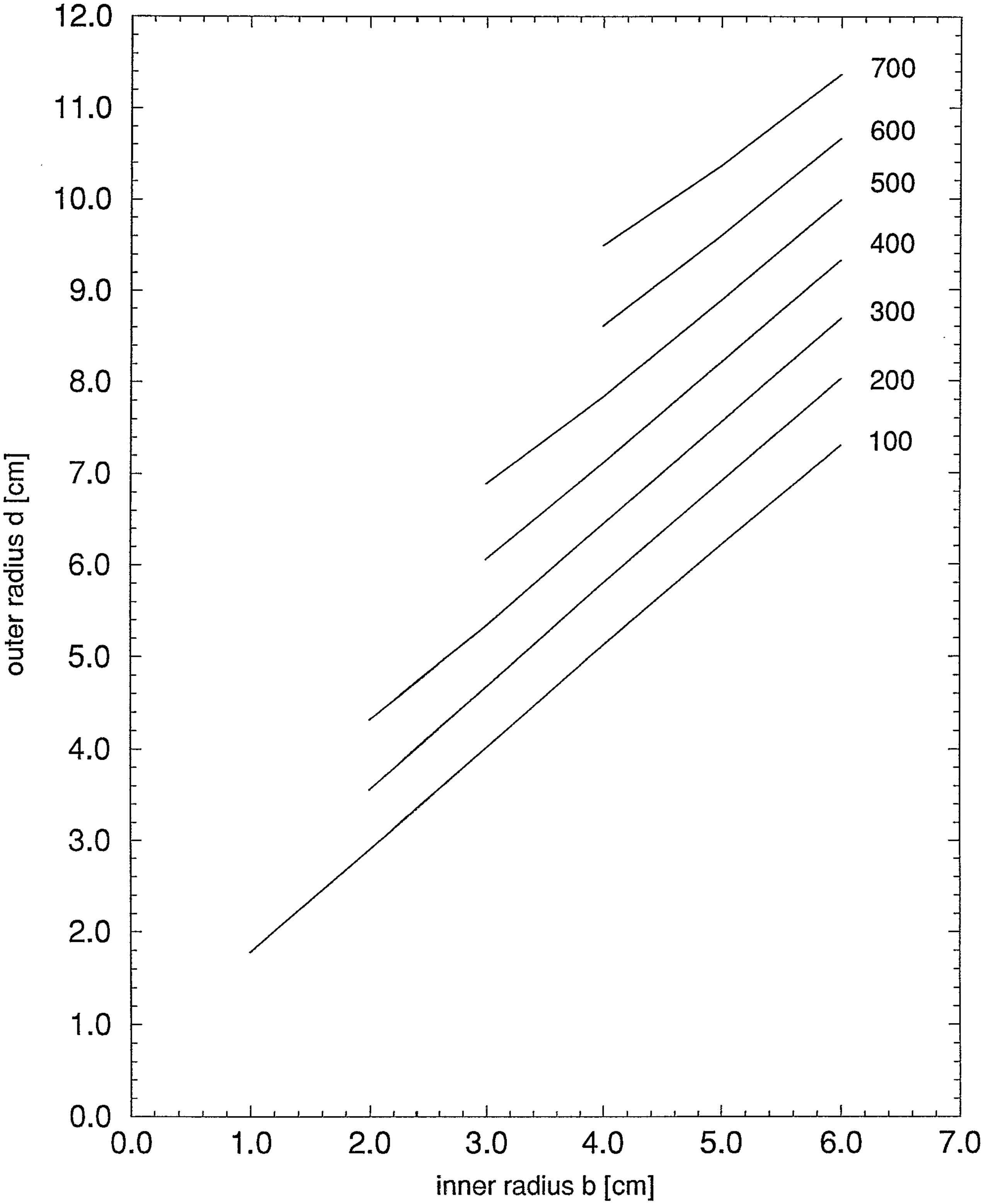


Fig 4: Pipe Transition Z/n Contours [μohm] 5:1 Transition Ratio

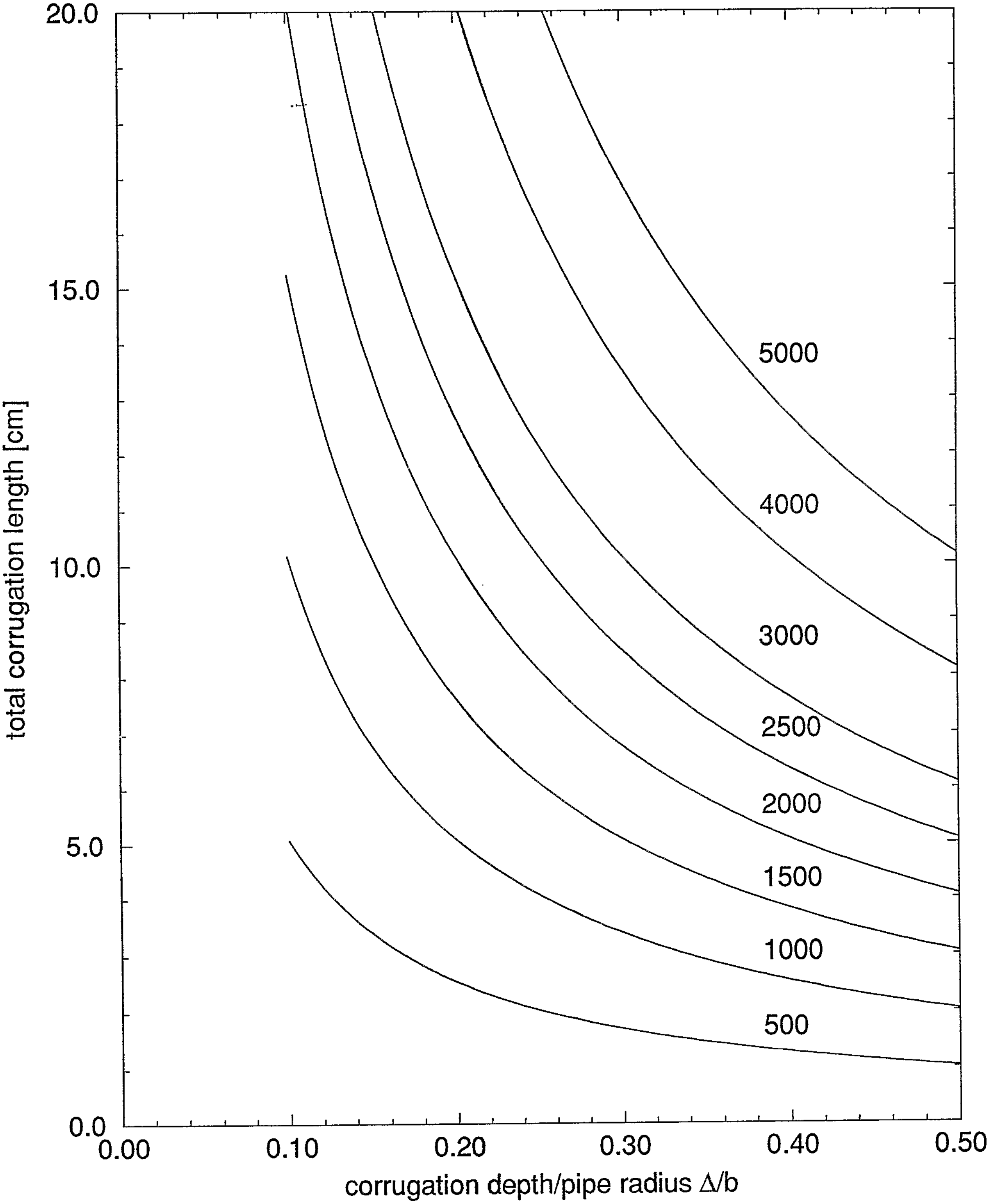


Fig 5: Bellows Z/n Contours [μohm]

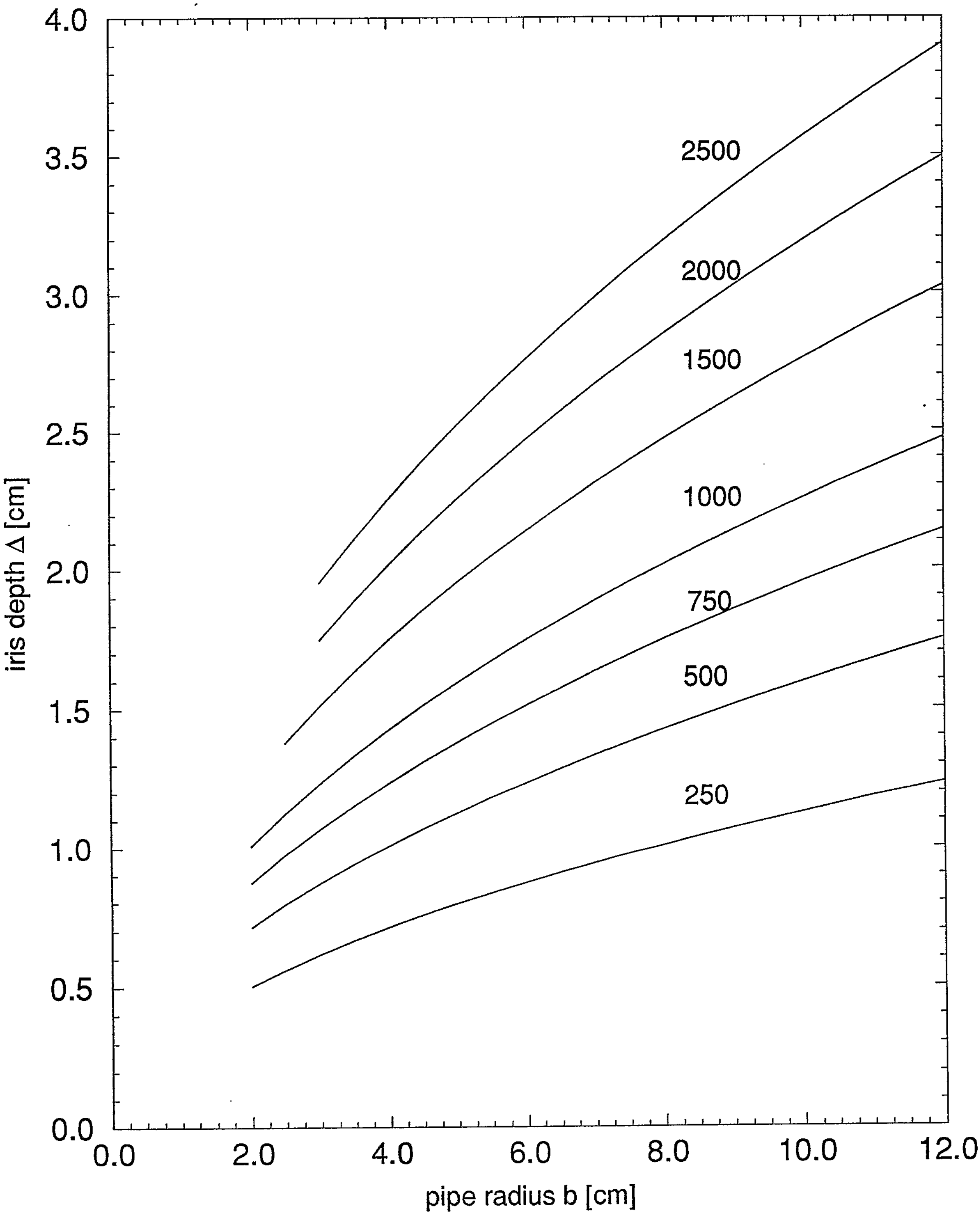


Fig 6: Iris Z/n Contours [μohm]