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Study on the Results from SYNCH and MAD Programs in Calculating Lattice with Coordinate Rotation

J. Xu

December 1992

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

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Informal Report

Study on the Results from SYNCH and MAD Programs in Calculating Lattice with Coordinate Rotation

Jianming Xu

December 1992

RHIC PROJECT

Brookhaven National Laboratory
Associated Universities, Inc.
Upton, NY 11973

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1. Introduction

In some lattices, it is necessary to introduce coordinate rotation around the beam axis, for example, in such a lattice, where the horizontal deflection and the change in level are mixed. The results from SYNCH and MAD programs in calculating such a lattice have been studied and some discrepancies have been found. The output of SYNCH is shown in Table 1 and outputs from MAD are shown in Table 2 and Table 3. The arrangement of this simple lattice is shown in these outputs. The dispersion functions are discussed first and then the Twiss Parameters (β's and α's).

2. Dispersion Functions

1. Dispersion functions before coordinate rotation at L_{32}.

   The dispersion functions before rotation $\eta_{x0}, \eta'_{x0}, \eta_{y0}, \eta'_{y0}$ from the outputs of MAD and SYNCH are shown as:

   MAD output (Table 2 and 3)

$$\begin{aligned} \eta_{x0} &= 1.146593631, \quad \eta'_{x0} = 0.02084258749; \\ \eta_{y0} &= 0.08803616920, \quad \eta'_{y0} = 0.01253036558. \end{aligned} \tag{1}$$

   SYNCH TRKB (Track Betatron Function) Output (Table 1)

$$\begin{aligned} \eta_{x0} &= 1.146593, \quad \eta'_{x0} = 0.020843; \\ \eta_{y0} &= 0.088036, \quad \eta'_{y0} = 0.012530. \end{aligned} \tag{2}$$

SYNCH MMM (Matrix Multiplication) Output (Table 1, WST)

$$
\begin{aligned}
m_{16} = \eta_{x0} = 1.14670914, \quad \eta'_{x0} = m_{26} = 0.02083611; \\
m_{36} = \eta_{y0} = 0.08803616, \quad m_{46} = \eta'_{y0} = 0.01253036.
\end{aligned}
\tag{3}
$$

From above results, it is clear that without coordinate rotation, the dispersion functions from MAD and SYNCH TRKB output coincide with each other, but η_{0x} and η'_{0x} from SYNCH MMM result differ little from both MAD and SYNCH TRKB outputs.

2. Dispersion functions after a rotation around the beam axis by an angle ξ

The relation between dispersion functions $\eta_x, \eta'_x, \eta_y, \eta'_y$ after rotation and $\eta_{x0}, \eta'_{x0}, \eta_{y0}$ and η'_{y0} before rotation can be expressed as follows

$$\eta_x = \eta_{x0} \cos\xi + \eta_{y0} \sin\xi \tag{4}$$

$$\eta'_x = \eta'_{x0} \cos\xi + \eta'_{y0} \sin\xi \tag{5}$$

$$\eta_y = -\eta_{x0} \sin\xi + \eta_{y0} \cos\xi \tag{6}$$

$$\eta'_y = -\eta'_{x0} \sin\xi + \eta'_{y0} \cos\xi \tag{7}$$

Putting Eq. (1), the MAD output before rotation into Eqs. (4)-(7), we get ($\xi = 0.05$ rad).

MAD by matrix multiplication

$$
\begin{aligned}
\eta_x = 1.149560662, \quad \eta'_x = 0.02144279695; \\
\eta_y = 0.0306203495, \quad \eta'_y = 0.01147301068.
\end{aligned}
\tag{8}
$$

The MAD outputs (from Tables 2 and 3 at R_1) are

$$
\begin{aligned}
\eta_x = 1.149560662, \quad \eta'_x = 0.02144279695; \\
\eta_y = 0.03062034973, \quad \eta'_y = 0.01147301068.
\end{aligned}
\tag{9}
$$

The MAD output (9) coincides with the result from matrix multiplication, Eq. (8).

Using the SYNCH TRKB output before rotation (Eq. (2)) as initial value by means of matrix multiplication Eqs. (4)-(7), we get

SYNCH TRKB by matrix multiplication

$$\eta_x = 1.149560023, \quad \eta_x' = 0.02144319067;$$
$$\eta_y = 0.030620212, \quad \eta_y' = 0.01147262494. \tag{10}$$

But the SYNCH TRKB output after rotation at R_1 is thoroughly different. They are as follows (from Table 1)

$$\eta_x = -1.102947, \quad \eta_x' = -0.020049;$$
$$\eta_y = -0.084685, \quad \eta_y' = -0.012053. \tag{11}$$

The SYNCH TRKB output after coordinate rotation does not fulfill the coordinate rotation relationship, Eq. (4)- (7). It is wrong apparently.

The relation between the SYNCH MMM output before rotation [Eq. (3)] and after rotation (from Table 1, WSRT) fulfill the coordinate rotation relationship [Eqs. (4)-(7)]. The MMM output after rotation (Table 1, WSRT) is

$$\eta_x = 1.14967602, \quad \eta_x' = 0.02143632;$$
$$\eta_y = 0.03061457, \quad \eta_y' = 0.01147333. \tag{12}$$

The data in Eq. (12) differ from those in Eq. (9) (MAD output), because the data before rotation from SYNCH MMM output Eq. (3) differ from both SYNCH TRKB output Eq. (2) and MAD output Eq. (1).

3. Conclusion about dispersion function calculation.

For lattice without coordinate rotation, the MAD TWISS output coincides with SYNCH TRKB output but the SYNCH MMM output differs from them. After coordinate rotation, the MAD TWISS output fulfills the coordinate rotation relationship but SYNCH TRKB output does not fulfill. The SYNCH MMM output after rotation fulfill the coordinate rotation relationship but its data differs from MAD output because its data before rotation differs from both MAD and SYNCH TRKB output. It seems that, the dispersion functions from MAD output are reliable.

3. Twiss Parameters β, α

1. We use the following initial conditions.

$$\begin{aligned} &\beta_{xi} = 40.755237, \quad \alpha_{xi} = 1.992883, \quad \gamma_{xi} = 0.12198635; \\ &\beta_{yi} = 16.997687, \quad \alpha_{yi} = -1.04618, \quad \gamma_{yi} = 0.12322221. \end{aligned} \tag{13}$$

Before rotation, the output of MAD and SYNCH TRKB (at L32) coincides with each other. They are as follows:

MAD output (from Table 2 and Table 3)

$$\begin{aligned} &\beta_{x0} = 11.78884158, \quad \alpha_{x0} = 0.6635168614; \\ &\beta_{y0} = 54.09318824, \quad \alpha_{y0} = -2.370235501. \end{aligned} \tag{14}$$

SYNCH TRKB output (from Table 1)

$$\begin{aligned} &\beta_{x0} = 11.7888, \quad \alpha_{x0} = 0.663517; \\ &\beta_{y0} = 54.0932, \quad \alpha_{y0} = -2.370235. \end{aligned} \tag{15}$$

The initial Twiss parameter Eq. (13) and the final Twiss parameters Eq. (14) or (15) should fulfill the following relation

$$\begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix} = \begin{pmatrix} \mathrm{m}_{11}^2 & -2\mathrm{m}_{11}\mathrm{m}_{12} & \mathrm{m}_{12}^2 \\ -\mathrm{m}_{11}\mathrm{m}_{21} & 1 + 2\mathrm{m}_{12}\mathrm{m}_{21} & -\mathrm{m}_{12}\mathrm{m}_{22} \\ \mathrm{m}_{21}^2 & -2\mathrm{m}_{21}\mathrm{m}_{22} & \mathrm{m}_{22}^2 \end{pmatrix} \begin{pmatrix} \beta_i \\ \alpha_i \\ \gamma_i \end{pmatrix} \tag{16}$$

where m's are the transfer matrix elements. Using the corresponding 4×4 matrix from SYNCH MMM output (Table 1 WST) from Eqs. (13) and (16) we can calculate the β's, α's at L32. The calculated results are shown as following which well coincide with the MAD and SYNCH TRKB output.

$$\begin{aligned} &\beta_{x0} = 11.78884092, \quad \alpha_{x0} = 0.66351697; \\ &\beta_{y0} = 54.09318664, \quad \alpha_{y0} = -2.370235431. \end{aligned} \tag{17}$$

The above result shows that without rotation the β's, α's from MAD and SYNCH TRKB and the 4×4 transfer matrix from SYNCH MMM output are all reliable.

2. The 4×4 transfer matrix after rotation from SYNCH MMM output (Table 2, WSRT) fulfill the coordinate rotation relation. The 4×4 matrix after rotation ($\xi = 0.05$) can be got by matrix multiplication, the result is shown as follows:

$$M = \begin{pmatrix} 0.660723045 & 18.98334088 & -0.119785867 & 1.40630563 \\ -0.121925944 & -1.99336127 & -0.006135546 & 0.051179119 \\ -0.03306371 & -0.949958808 & -2.393720579 & 28.10267027 \\ 0.00610138 & 0.099751203 & -0.122608639 & 1.022729245 \end{pmatrix} \tag{18}$$

The SYNCH MMM output is as follows (Table 1, WSRT)

$$M = \begin{pmatrix} 0.66072304 & 18.98334088 & -0.11978587 & 1.40630563 \\ -0.12192595 & -1.99336127 & -0.00613555 & 0.05117912 \\ -0.03306371 & -0.94995881 & -2.39372058 & 28.10267027 \\ 0.006160138 & 0.09975120 & -0.12260864 & 1.02272924 \end{pmatrix} \tag{19}$$

Eq. (19) well coincides with Eq. (18). It shows that the 4×4 matrix from SYNCH MMM output is reliable after rotation also.

3. The projection of the 4-dimensional emittance ellipsoid after rotation on xx' and yy' planes.

From the inverse transformation matrix after rotation (Table 1, IWSR) we can get the 4-dimensional emittance ellipsoid after rotation and then get its projection on xx' and yy' planes. They are expressed as follows:

$$0.120795313x^2 + 2 \times 0.648550256xx' + 11.76053439x'^2 = 1.0113922\epsilon_0 \tag{20}$$

$$0.120966214y^2 - 2 \times 2.336044532yy' + 53.3794012y'^2 = 1.0113922\epsilon_0 \tag{21}$$

where ϵ_0 is the initial x and y emittance. The projections depend upon the ratio of the initial x and y emittance. Here we take the ratio to be one. The corresponding β, α value of these projections are

$$\begin{aligned} \beta_x &= 11.76053439, \quad \alpha_x = 0.648550256; \\ \beta_y &= 53.3794012, \quad \alpha_y = -2.336044532. \end{aligned} \tag{22}$$

4. β, α values after rotation from SYNCH and MAD (at R_1)

SYNCH TRKB output (Table 1)

$$\beta_x = 10.9084, \quad \alpha_x = 0.613963;$$
$$\beta_y = 50.0533, \quad \alpha_y = -2.193219. \tag{23}$$

MAD Twiss output (Table 2)

$$\beta_x = 11.75939402, \quad \alpha_x = 0.6635168614;$$
$$\beta_y = 54.09318824, \quad \alpha_y = -2.370235501. \tag{24}$$

From MAD Twiss couple output (Table 3)

$$\beta_1 = 17.82817229, \quad \alpha_1 = 1.890607154;$$
$$\beta_2 = 120.7788642, \quad \alpha_2 = -5.465936926. \tag{25}$$

There is large difference between the above three sets of Twiss parameters for the same lattice same initial conditions and none of them coincides with the corresponding parameters of the projections of the 4-dimensional emittance ellipsoid on xx' and yy' plane. When the rotation angle ξ is smaller, the discrepancy is smaller but they do not coincide with each other also. For example, if $\xi = 0.0002735$ rad, the results are shown as follows:

Projections on xx' and yy' planes

$$\beta_x = 11.78884012, \quad \alpha_x = 0.663516342;$$
$$\beta_y = 54.0931771, \quad \alpha_y = -2.370234907. \tag{26}$$

SYNCH TRKB output

$$\beta_x = 11.7859, \quad \alpha_x = 0.663354;$$
$$\beta_y = 54.07999, \quad \alpha_y = -2.369653. \tag{27}$$

MAD Twiss output

$$\beta_x = 11.78844069, \quad \alpha_x = 0.6635168117;$$
$$\beta_y = 54.09318420, \quad \alpha_y = -2.370235323. \tag{28}$$

MAD Twiss couple output

$$\beta_1 = 11.79540445, \quad \alpha_1 = 0.668622315;$$
$$\beta_2 = 54.25392611, \quad \alpha_2 = -2.377879573. \tag{29}$$

The discrepancy is clear even though the rotation is as small as 0.0002735 rad. This fact means that, in lattices with transverse coupling, in different programs the β's and α's have different meaning and different value. One has to use these programs carefully in calculating or matching lattices with transverse coupling.

Acknowledgments

The author would like to thank Dr. J. Claus and Dr. H. Foelsche for their support and valuable discussions.

Table 1. SYNCH Output

```
         SYNCH RUN W                                                          SYNCH VERSIONIBM 3090
         15-DEC-92  10:23:26

=============================================================================================================
***          SIZE     7    0
***   L01    DRF               //  10.910434
***   L12    DRF               //  14.0517
***   L31    DRF               //  6.11145
***   L32    DRF               //  6.11145
***   KB     =                 //  .086013146
***   TH     =                 //  .0436307780
***   R+     ROTZ              //  90.0
***   R-     ROTZ              //  -90.0
***   WP1    MAG               //  1.8288    0.0       1.8288    -.0125302 $          $
***   DP1    MMM               //  R-    WP1   R+
***   W1D    MAG               //  3.6576    -KB       3.6576    TH        $          $
***   W2F    MAG               //  3.6576    KB        3.6576    TH        $          $
***   R1     ROTZ              //  2.864788976
***   WI     IBET              //  0         40.755237 1.992883  .12198635 .00010772 -9.8343E-6
  *                            //  0         16.997687 -1.04618  .12322221 0.00      0.00

***   BO     SUB       0    0  //
.............................................................................................................
***   WS     BML               //  L01   W1D   L12   W2F   L31   DP1   L32
***   WST    MMM               //  L01   W1D   L12   W2F   L31   DP1   L32
***   WSR    BML               //  WS    R1
***   WSRT   MMM               //  WS    R1
***   TR     TRKB              //  WSR   WI
***          END       0    0  //
.............................................................................................................

***   MAT    SUB       0    0  //
.............................................................................................................
***          WMA       1       //  WST
***          WMA       1       //  WSRT
***          END       0    0  //
.............................................................................................................

***   INM    SUB       0    0  //
.............................................................................................................
***   IWS    INV               //  WST
***          WMA       1       //  IWS
***   IWSR   INV               //  WSRT
***          WMA       1       //  IWSR
***          END       0    0  //
.............................................................................................................

***          CALL              //  BO
```

POS		S	QX	BX	AX	X	DX	QY	BY	AY	Y	DY
0		0.0000	0.000000	40.7552	1.992883	0.000108	-0.000010	0.000000	16.9977	-1.046180	0.000000	0.000000
1	L01	10.9104	0.082917	11.7898	0.661959	0.000000	-0.000010	0.058353	54.4943	-2.390588	0.000000	0.000000
2	W1D	14.5680	0.135874	11.7918	-0.662573	0.081856	0.045950	0.068490	54.5027	2.388542	0.000000	0.000000
3	L12	28.6197	0.229372	54.5080	-2.377362	0.727537	0.045950	0.162557	11.6676	0.659848	0.000000	0.000000
4	W2F	32.2773	0.239503	54.5133	2.376071	0.853115	0.020928	0.216181	11.6386	-0.651090	0.000000	0.000000
5	L31	38.3888	0.263638	30.0242	1.631025	0.981018	0.020928	0.275545	24.1663	-1.398794	0.000000	0.000000
6	DP1	40.2176	0.274386	24.4620	1.410159	1.019215	0.020843	0.286419	29.6916	-1.622532	0.011457	0.012530
7	L32	46.3290	0.332978	11.7888	0.663517	1.146593	0.020843	0.310785	54.0932	-2.370235	0.088036	0.012530
8	R1	46.3290	0.832978	10.9084	0.613963	-1.102947	-0.020049	0.810785	50.0533	-2.193219	-0.084685	-0.012053

*** CALL // MAT

TRANSFER MATRICES R(I,J)

ELEMENT	X	DX/DS	Y	DY/DS	-DS	DP/P	1
	0.66154981	19.00709480	0.00000000	0.00000000	0.00000000	1.14670914	0.00000000
	-0.12207851	-1.99585557	0.00000000	0.00000000	0.00000000	0.02083611	0.00000000
	0.00000000	0.00000000	-2.39671585	28.13783524	0.00000000	0.08803616	0.00000000
WST	0.00000000	0.00000000	-0.12276206	1.02400899	0.00000000	0.01253036	0.00000000
	-0.15377267	-2.68469968	0.01922422	-0.26242750	1.00000000	-0.03605821	0.00000000
	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000	0.00000000
	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000

LENGTH = 46.32903400 THETA = 0.07473136

TRANSFER MATRICES R(I,J)

ELEMENT	X	DX/DS	Y	DY/DS	-DS	DP/P	1
	0.66072304	18.98334088	-0.11978587	1.40630563	0.00000000	1.14967602	0.00000000
	-0.12192595	-1.99336127	-0.00613555	0.05117912	0.00000000	0.02143632	0.00000000
	-0.03306371	-0.94995881	-2.39372058	28.10267027	0.00000000	0.03061457	0.00000000
WSRT	0.00610138	0.09975120	-0.12260864	1.02272924	0.00000000	0.01147333	0.00000000
	-0.15377267	-2.68469968	0.01922422	-0.26242750	1.00000000	-0.03605821	0.00000000
	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000	0.00000000
	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000

LENGTH = 46.32903400 THETA = 0.07473136

*** CALL // INM

TRANSFER MATRICES R(I,J)

ELEMENT	X	DX/DS	Y	DY/DS	-DS	DP/P	1
	-1.99585557	-19.00709480	0.00000000	0.00000000	0.00000000	2.68469968	0.00000000

	0.12207851	0.66154981	0.00000000	0.00000000	0.00000000	-0.15377267	0.00000000
	0.00000000	0.00000000	1.02400899	-28.13783524	0.00000000	0.26242750	0.00000000
IWS	0.00000000	0.00000000	0.12276206	-2.39671585	0.00000000	0.01922422	0.00000000
	0.02083611	-1.14670914	0.01253036	-0.08803616	1.00000000	0.03605821	0.00000000
	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000	0.00000000
	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000

LENGTH = -46.32903400 THETA = -0.07473136

TRANSFER MATRICES R(I,J)

ELEMENT	X	DX/DS	Y	DY/DS	-DS	DP/P	1
	-1.99336127	-18.98334088	0.09975120	0.94995881	0.00000000	2.68469968	0.00000000
	0.12192595	0.66072304	-0.00610138	-0.03306371	0.00000000	-0.15377267	0.00000000
	0.05117912	-1.40630563	1.02272924	-28.10267027	0.00000000	0.26242750	0.00000000
IWSR	0.00613555	-0.11978587	0.12260864	-2.39372058	0.00000000	0.01922422	0.00000000
	0.02143632	-1.14967602	0.01147333	-0.03061457	1.00000000	0.03605821	0.00000000
	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000	0.00000000
	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000

LENGTH = -46.32903400 THETA = -0.07473136

*** FIN // CORE USE SUMMARY MAXIMUM USED UNUSED

Table 2. MAD TWISS output

```
TITLE, "*UWVGM_A4_BNL.MAD JUL/30/92 AGS RHIC Beam Tr Line U+W+G LINE";
! PARAMETER DEFINITION
!
L01:DRIFT,L=10.910434
W1D:RBEND,L=3.657600,ANGLE=0.043630780,K1=-0.02351628
L12:DRIFT,L=14.05170
W2F:RBEND,L=3.657600,ANGLE=0.043630780,K1=0.02351628
L31:DRIFT,L=6.11145
WP1:RBEND,L=1.82880,ANGLE=0.012530235,TILT
L32:DRIFT,L=6.111450
R1:SROT,ANGLE=0.05
!
WL: LINE=(L01,W1D,L12,W2F,L31,WP1,L32,R1)
TWISS,BETX=40.755237,ALFX=1.992883,BETY=16.997687,&
  ALFY=-1.04618,DX=.00010772,DPX=-9.8343E-6,DY=0.00,&
   DPY=0.00,TAPE
```

DELTA(P)/P = 0.000000 SYMM = F PAGE 1

ELEMENT SEQUENCE				HORIZONTAL							VERTICAL						
POS. NO.	ELEMENT NAME	OCC. NO.	DIST [M]	BETAX [M]	ALFAX [1]	MUX [2PI]	X(CO) [MM]	PX(CO) [.001]	DX [M]	DPX [1]	BETAY [M]	ALFAY [1]	MUY [2PI]	Y(CO) [MM]	PY(CO) [.001]	DY [M]	DPY [1]
BEGIN	WL	1	0.000	40.755	1.993	0.000	0.000	0.000	0.000	0.000	16.998	-1.046	0.000	0.000	0.000	0.000	0.000
1	L01	1	10.910	11.790	0.662	0.083	0.000	0.000	0.000	0.000	54.494	-2.391	0.058	0.000	0.000	0.000	0.000
2	W1D	1	14.568	11.792	-0.663	0.136	0.000	0.000	0.082	0.046	54.503	2.389	0.068	0.000	0.000	0.000	0.000
3	L12	1	28.620	54.508	-2.377	0.229	0.000	0.000	0.728	0.046	11.668	0.660	0.163	0.000	0.000	0.000	0.000
4	W2F	1	32.277	54.513	2.376	0.240	0.000	0.000	0.853	0.021	11.639	-0.651	0.216	0.000	0.000	0.000	0.000
5	L31	1	38.389	30.024	1.631	0.264	0.000	0.000	0.981	0.021	24.166	-1.399	0.276	0.000	0.000	0.000	0.000
6	WP1	1	40.218	24.462	1.410	0.274	0.000	0.000	1.019	0.021	29.692	-1.623	0.286	0.000	0.000	0.011	0.013
7	L32	1	46.329	11.789	0.664	0.333	0.000	0.000	1.147	0.021	54.093	-2.370	0.311	0.000	0.000	0.088	0.013
8	R1	1	46.329	11.759	0.662	0.333	0.000	0.000	1.150	0.021	53.958	-2.364	0.311	0.000	0.000	0.031	0.011
END	WL	1	46.329	11.759	0.662	0.333	0.000	0.000	1.150	0.021	53.958	-2.364	0.311	0.000	0.000	0.031	0.011

TOTAL LENGTH =	46.329034	MUX =	0.332978	MUY =	0.310785
DELTA(S) =	0.000000 mm	DMUX =	-0.289565	DMUY =	-0.525197
		BETAX(MAX) =	54.513329	BETAY(MAX) =	54.502705
		DX(MAX) =	1.149561	DY(MAX) =	0.088036

WARNING ## TWISS1: TWISS PARAMETERS FOR DELTA(P)/P = 0.00000000 MAY BE WRONG DUE TO COUPLING.

```
        INITIAL          0.000000 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 1.992883000E+00 4.075523700E+01 0.000000000E+00 1.077200000E-04-9.834300000E-06
-1.046180000E+00 1.699768700E+01 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
DRIFT   L01             10.910434 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 6.619589669E-01 1.178975894E+01 8.291732427E-02 4.235189138E-07-9.834300000E-06
-2.390587812E+00 5.449431539E+01 5.835336206E-02 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 1.091043400E+01
RBEND   W1D              3.657600 4.363078000E-02-2.351628000E-02 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
-6.625729332E-01 1.179179474E+01 1.358738998E-01 8.185562552E-02 4.595041089E-02
 2.388542037E+00 5.450270480E+01 6.849037676E-02 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 1.456803400E+01
DRIFT   L12             14.051700 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
-2.377361680E+00 5.450804395E+01 2.293720334E-01 7.275370142E-01 4.595041089E-02
 6.598476797E-01 1.166764703E+01 1.625567451E-01 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 2.861973400E+01
RBEND   W2F              3.657600 4.363078000E-02 2.351628000E-02 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 2.376071221E+00 5.451332857E+01 2.395034915E-01 8.531150845E-01 2.092845051E-02
-6.510899602E-01 1.163857385E+01 2.161809221E-01 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 3.227733400E+01
DRIFT   L31              6.111450 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 1.631024961E+00 3.002416061E+01 2.636382729E-01 9.810182634E-01 2.092845051E-02
-1.398793640E+00 2.416633498E+01 2.755453954E-01 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 3.838878400E+01
RBEND   WP1              1.828800 1.253023500E-02 0.000000000E+00 0.000000000E+00
 1.570796327E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 1.410158863E+00 2.446200698E+01 2.743864553E-01 1.019215199E+00 2.084258701E-02
-1.622531821E+00 2.969159039E+01 2.864194321E-01 1.145749847E-02 1.253040058E-02
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 4.021758400E+01
DRIFT   L32              6.111450 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 6.635168641E-01 1.178884146E+01 3.329783392E-01 1.146593628E+00 2.084258701E-02
-2.370235501E+00 5.409318824E+01 3.107848662E-01 8.803641511E-02 1.253040058E-02
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 4.632903400E+01
SROT    R1               0.000000 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 5.000000000E-02 0.000000000E+00 0.000000000E+00 0.000000000E+00
 6.618594538E-01 1.175939391E+01 3.329783392E-01 1.149560671E+00 2.144279822E-02
-2.364314848E+00 5.395806793E+01 3.107848662E-01 3.062059551E-02 1.147304566E-02
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 4.632903400E+01
 0.000000000E+00 0.000000000E+00 4.632903400E+01
-6.663191159E-01 3.329783392E-01-2.895654389E-01 5.451332857E+01 1.149560671E+00
-6.854956584E-01 3.107848662E-01-5.251966506E-01 5.450270480E+01 8.803641511E-02
```

Table 3. MAD TWISS Couple Output

```
! PARAMETER DEFINITION
!
L01:DRIFT,L=10.910434
W1D:RBEND,L=3.657600,ANGLE=0.043630780,K1=-0.02351628
L12:DRIFT,L=14.05170
W2F:RBEND,L=3.657600,ANGLE=0.043630780,K1=0.02351628
L31:DRIFT,L=6.11145
WP1:RBEND,L=1.82880,ANGLE=0.012530235,TILT
L32:DRIFT,L=6.111450
R1:SROT,ANGLE=0.05
!
WL: LINE=(L01,W1D,L12,W2F,L31,WP1,L32,R1)
TWISS,BETX=40.755237,ALFX=1.992883,BETY=16.997687,&
  ALFY=-1.04618,DX=.00010772,DPX=-9.8343E-6,DY=0.00,&
   DPY=0.00,TAPE,couple
```

DELTA(P)/P = 0.000000 SYMM = F PAGE 1

ELEMENT SEQUENCE				MODES			COUPLING			ORBIT		DISPERSION	
POS. NO.	ELEMENT NAME	OCC. NO.	DIST [M]	BETA1 [M] / BETA2 [M]	ALFA1 [1] / ALFA2 [1]	MU1 [2PI] / MU2 [2PI]	R(1,1) [1] / R(2,1) [1/M]	R(1,2) [M] / R(2,2) [1]	COSPHI [1] / SINPHI [1]	X(CO) [MM] / Y(CO) [MM]	PX(CO) [.001] / PY(CO) [.001]	DX [M] / DY [M]	DPX [1] / DPY [1]
BEGIN	WL	1	0.000	40.755	1.993	0.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000
				16.998	-1.046	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000
1	L01	1	10.910	11.790	0.662	0.083	1.000	0.000	1.000	0.000	0.000	0.000	0.000
				54.494	-2.391	0.058	0.000	1.000	1.000	0.000	0.000	0.000	0.000
2	W1D	1	14.568	11.792	-0.663	0.136	0.668	0.771	1.000	0.000	0.000	0.082	0.046
				54.503	2.389	0.068	-0.172	1.298	1.000	0.000	0.000	0.000	0.000
3	L12	1	28.620	54.508	-2.377	0.229	-1.748	43.571	1.000	0.000	0.000	0.728	0.046
				11.668	0.660	0.163	-0.172	3.714	1.000	0.000	0.000	0.000	0.000
4	W2F	1	32.277	54.513	2.376	0.240	3.015	64.260	1.000	0.000	0.000	0.853	0.021
				11.639	-0.651	0.216	0.370	8.207	1.000	0.000	0.000	0.000	0.000
5	L31	1	38.389	30.024	1.631	0.264	5.274	82.185	1.000	0.000	0.000	0.981	0.021
				24.166	-1.399	0.276	0.370	5.948	1.000	0.000	0.000	0.000	0.000
6	WP1	1	40.218	24.462	1.410	0.274	5.957	82.176	1.000	0.000	0.000	1.019	0.021
				29.692	-1.623	0.286	0.370	5.272	1.000	0.000	0.000	0.011	0.013
7	L32	1	46.329	11.789	0.664	0.333	8.219	64.169	1.000	0.000	0.000	1.147	0.021
				54.093	-2.370	0.311	0.370	3.011	1.000	0.000	0.000	0.088	0.013
8	R1	1	46.329	17.828	1.891	1.279	12.355	96.797	0.663	0.000	0.000	1.150	0.021
				120.779	-5.466	0.320	0.558	4.499	1.249	0.000	0.000	0.031	0.011
END	WL	1	46.329	17.828	1.891	1.279	12.355	96.797	0.663	0.000	0.000	1.150	0.021
				120.779	-5.466	0.320	0.558	4.499	1.249	0.000	0.000	0.031	0.011

TOTAL LENGTH = 46.329034 MU1 = 1.278829 MU2 = 0.320316
DELTA(S) = 0.000000 mm
BETAX(MAX) = 54.513329 BETAY(MAX) = 120.778865
DX(MAX) = 1.149561 DY(MAX) = 0.088036

```
        INITIAL          0.000000 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 1.992883000E+00 4.075523700E+01 0.000000000E+00 1.077200000E-04-9.834300000E-06
-1.046180000E+00 1.699768700E+01 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
DRIFT   L01             10.910434 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 6.619589669E-01 1.178975894E+01 8.291732427E-02 4.235189138E-07-9.834300000E-06
-2.390587812E+00 5.449431539E+01 5.835336206E-02 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 1.091043400E+01
RBEND   W1D              3.657600 4.363078000E-02-2.351628000E-02 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
-6.625729332E-01 1.179179474E+01 1.358738998E-01 8.185562552E-02 4.595041089E-02
 2.388542037E+00 5.450270480E+01 6.849037676E-02 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 1.456803400E+01
DRIFT   L12             14.051700 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
-2.377361680E+00 5.450804395E+01 2.293720334E-01 7.275370142E-01 4.595041089E-02
 6.598476797E-01 1.166764703E+01 1.625567451E-01 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 2.861973400E+01
RBEND   W2F              3.657600 4.363078000E-02 2.351628000E-02 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 2.376071221E+00 5.451332857E+01 2.395034915E-01 8.531150845E-01 2.092845051E-02
-6.510899602E-01 1.163857385E+01 2.161809221E-01 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 3.227733400E+01
DRIFT   L31              6.111450 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 1.631024961E+00 3.002416061E+01 2.636382729E-01 9.810182634E-01 2.092845051E-02
-1.398793640E+00 2.416633498E+01 2.755453954E-01 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 3.838878400E+01
RBEND   WP1              1.828800 1.253023500E-02 0.000000000E+00 0.000000000E+00
 1.570796327E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 1.410158863E+00 2.446200698E+01 2.743864553E-01 1.019215199E+00 2.084258701E-02
-1.622531821E+00 2.969159039E+01 2.864194321E-01 1.145749847E-02 1.253040058E-02
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 4.021758400E+01
DRIFT   L32              6.111450 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00
 6.635168641E-01 1.178884146E+01 3.329783392E-01 1.146593628E+00 2.084258701E-02
-2.370235501E+00 5.409318824E+01 3.107848662E-01 8.803641511E-02 1.253040058E-02
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 4.632903400E+01
SROT    R1               0.000000 0.000000000E+00 0.000000000E+00 0.000000000E+00
 0.000000000E+00 5.000000000E-02 0.000000000E+00 0.000000000E+00 0.000000000E+00
 1.890607166E+00 1.782817215E+01 1.278828965E+00 1.149560671E+00 2.144279822E-02
-5.465936986E+00 1.207788647E+02 3.203164257E-01 3.062059551E-02 1.147304566E-02
 0.000000000E+00 0.000000000E+00 0.000000000E+00 0.000000000E+00 4.632903400E+01
 0.000000000E+00 0.000000000E+00 4.632903400E+01
-6.663191159E-01 1.278828965E+00 0.000000000E+00 5.451332857E+01 1.149560671E+00
-6.854956584E-01 3.203164257E-01 0.000000000E+00 1.207788647E+02 8.803641511E-02
```