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Decoherence of Betatron Oscillations in RHIC

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RHIC PROJECT

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Introduction

Measurements of tune and coupling are commonly made by giving the beam a small transverse displacement from the reference orbit and measuring the oscillation amplitudes and phases at the beam position monitors (BPM's). In a lattice with non-zero chromaticity, the momentum spread of the beam causes a spread in the betatron frequency. This tune spread results in filamentation in transverse phase space which damps the coherent signal. This note presents a calculation to estimate the minimum rate of decoherence we can expect in RHIC. The calculation deals only with tune spread from chromaticity and momentum spread and assumes chromaticity is linear with momentum deviation.

Calculation

Consider a beam bunch with a Gaussian momentum distribution,

$$n(p)=\frac{N}{\sqrt{2\pi}\sigma_p}\exp-\left(\frac{(p-p_0)^2}{2\sigma_p^2}\right)$$

where N is the number of particles, p_0 is the momentum of the reference particle and σ_p is the rms width of the momentum distribution. This bunch is displaced from the reference orbit such that it has a betatron amplitude of X=1. The actual amplitude around the ring varies as $\beta^{1/2}$. Also on consecutive passes past a given point on the ring the bunch steps around the origin in phase space in steps of 2π times the fractional tune. At an amplitude maximum in the lattice the displaced beam can be represented in phase space like this.

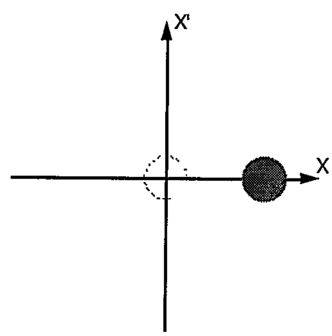

Figure1. A beam bunch is initially displaced to a contour in transverse phase space with a radius along the X axis equal to 1. This is the distribution at an amplitude maximum.

As this bunch circulates in the ring, it rotates about the center of phase space at the betatron frequency. There is a spread in the rotation rate proportional to the momentum spread times the chromaticity. After many rotations in phase space the distribution spreads azimuthally, so at an amplitude maximum the distribution is this.

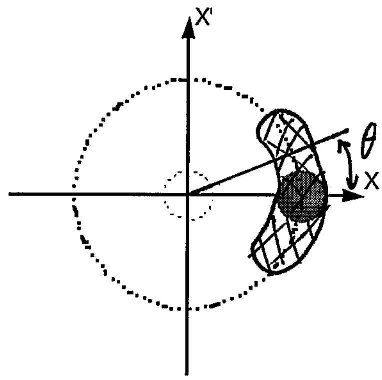

Figure 2. The phase-space distribution after the reference particle has traveled an integral number of betatron wavelengths.

with a Gaussian distribution in the coordinate θ of,

$$n(\theta) = \frac{N}{\sqrt{2\pi}\sigma_\theta} \exp-\left(\frac{\theta^2}{2\sigma_\theta^2}\right)$$

where

$$\sigma_\theta = 2\pi m \xi \left(\sigma_p \big/ p_0 \right)$$

Here ξ is the chromaticity and m is the number of turns since initial displacement. As in fig. 1, this is the representation of the bunch at an amplitude maximum. Over time the distribution continues to spread azimuthally in transverse phase space until the particles populate an entire annular region with unit radius. In this limit the particles are distributed uniformly about the origin in transverse phase space and the dipole signal disappears.

The BPM's measure amplitude of the mean displacement in X. For the beam shown in fig. 2 this amplitude is,

$$\langle X \rangle = \frac{\int n(\theta) x(\theta) d\theta}{\int n(\theta) d\theta}$$

Since x(θ) = cos θ the measured signal is,

$$\langle X \rangle = \frac{1}{\sqrt{2\pi}\sigma_\theta} \int \cos\theta \exp\left(\frac{-\theta^2}{2\sigma_\theta^2}\right) d\theta$$

$$\langle X \rangle = \exp\left(\frac{-\sigma_\theta^2}{2}\right)$$

After m turns the mean displacement is,

$$\langle X(m)\rangle = \exp\left[-\frac{1}{2}\left(2\pi m\xi\frac{\sigma_p}{p}\right)^2\right]$$

Application to RHIC

The most recent estimates for the momentum spreads in RHIC are from RHIC/AP/145. This gives targets of,

Beam	Injection (σ_p/p x 10^4)	Store(begin) (σ_p/p x 10^4)	Store (end) (σ_p/p x 10^4)
Gold	4.7-7.4	4.2-6.4	
Protons	5.1	3.4	5.7

The design chromaticity at injection is ξ = 3 and at storage ξ = -2. The decoherence time does not depend on the sign of the chromaticity. Plotted here are betatron amplitudes as functions of number of turns for these two chromaticities and various values of σ_p/p. These curves are normalized to a unit initial displacement. The actual amplitudes depend on the size of the initial displacement and the value of the β function at the location of the measurement.

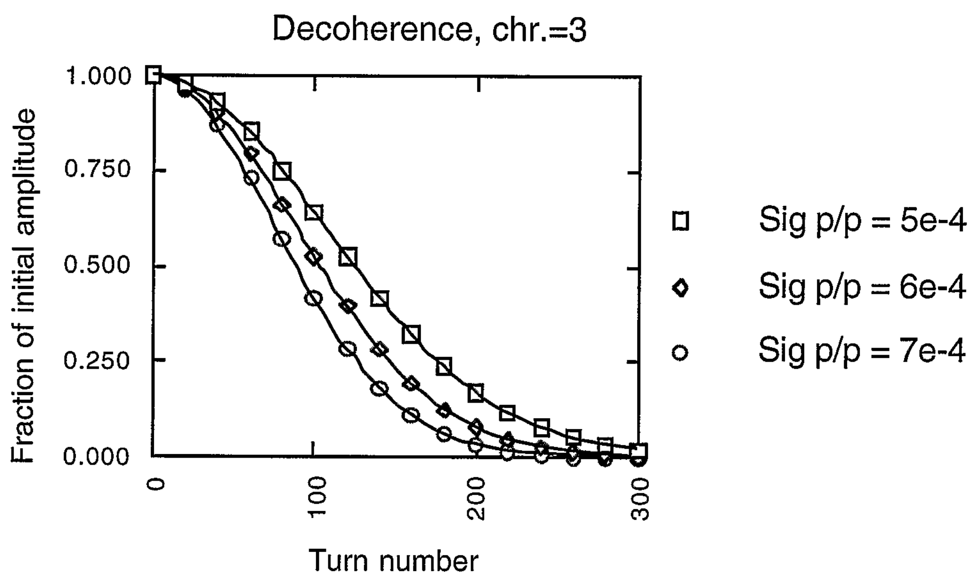

Figure 3. Decoherence of betatron oscillation for chromaticity=3 (injection) plotted for three values of σ_p/p.

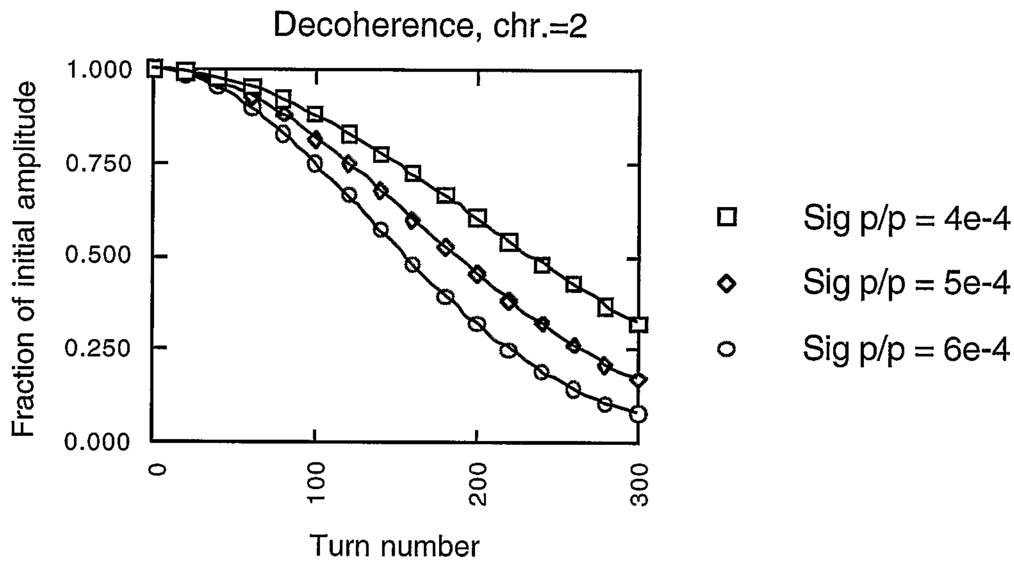

Figure 4. Decoherence of betatron oscillation for chromaticity=2 (storage) plotted for three values of σ_p/p.

Conclusion

For beams at injection, the signal from any transverse kick will decay to about 50% in 100 turns and will be completely gone in 200 turns. This sets a maximum record length if tune and coupling measurements are to be made by measuring ringing from a single kick. Also if significant transverse oscillations exist from injection errors they will have to be damped in the first hundred turns to limit emittance growth. The decoherence time at full energy is about twice that at injection.