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Tune Spreads and Tune Splitting due to Random Field Errors in RHIC

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AD/RHIC-AP-52

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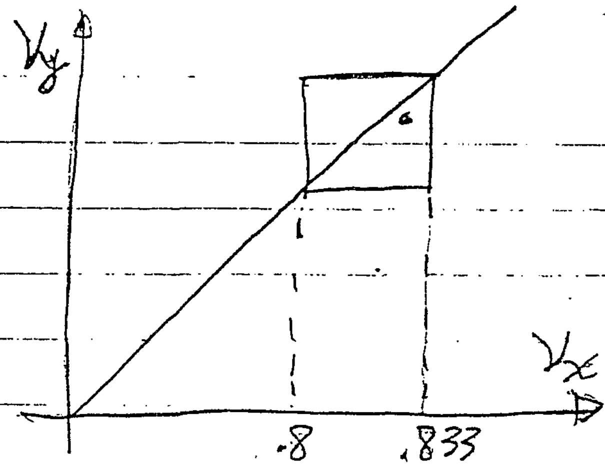
Desired ν-spread

Goal is to stay within box shown — and not cross resonances below 10^{th} order



$\Delta\nu$ (beam-beam) $\simeq 6 \times .004 = .024$

What is $\Delta\nu$ due to random b_K, a_K?

Low Amplitude Non-resonance Result

For small enough $\varepsilon_x; \varepsilon_y, \delta = \Delta p/p$ and not near a resonance

$$\Delta\nu = \Delta\nu(\delta, \varepsilon_x, \varepsilon_y)$$

<u>example</u>, $\Delta\nu$ due to sextupoles

$$\Delta\nu_x = C_x\delta + a\,\epsilon_x + b\,\epsilon_y$$
$$\Delta\nu_y = C_y\delta + c\,\epsilon_x + d\,\epsilon_y$$

To find $\Delta\nu$-spread, compute $\Delta\nu_x$, $\Delta\nu_y$ along the line $\epsilon_x + \epsilon_y = \epsilon_t$, $\epsilon_t = 6.5$ for RHIC at $\gamma = 30$ for several values of δ.

See Fig 1 for sextupole result

For sextupoles, $\Delta\nu \simeq 5 \times 10^{-3}$ for $\beta^* = 3$ lattice.

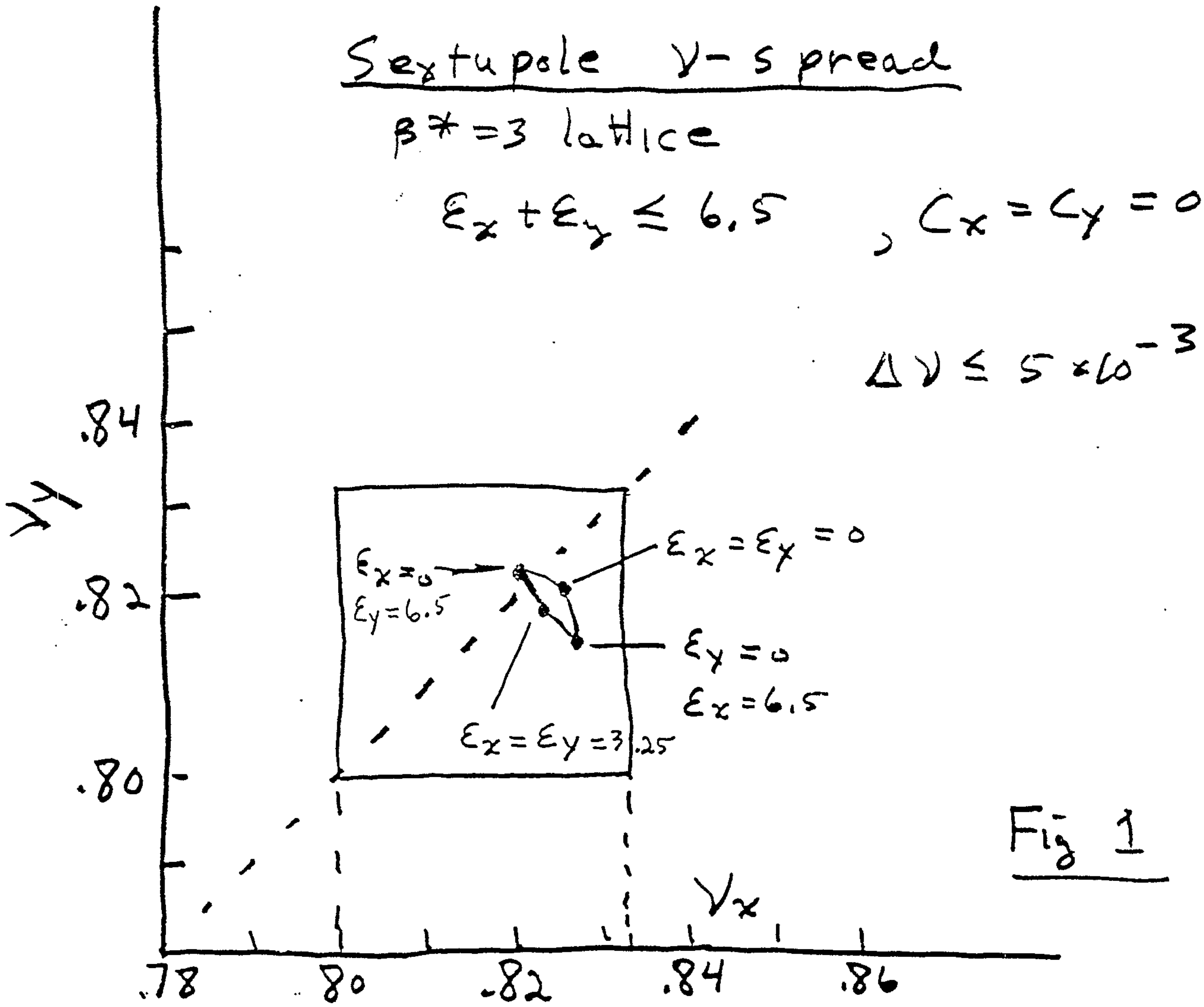

Sextupole ν- spread
β* =3 lattice
εx + εy ≤ 6.5 , Cx = Cy = 0
Δν ≤ 5 ×10-3
εx = εy = 0
εx = 0
εy = 6.5
εy = 0
εx = 6.5
εx = εy = 3.25
νy
νx
.84
.82
.80
.78
80
.82
.84
.86

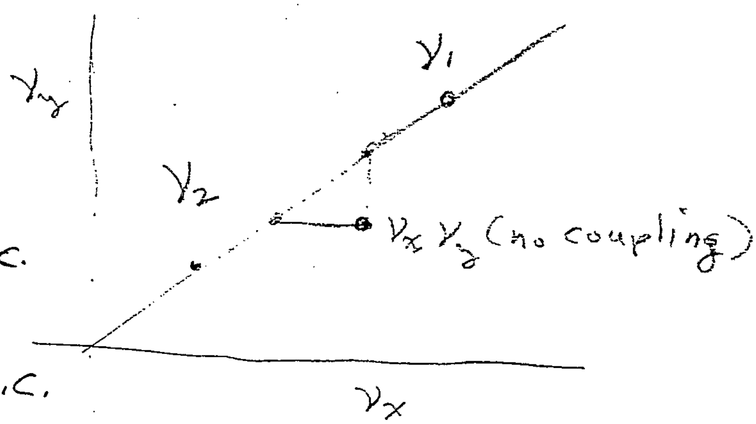
Fig 1

ν - spread near Coupling Resonances

Tracking results point to something going on not indicated by low amplitude non-resonance result.

Linear Coupling Resonance (Review)

Both x and y motion contain two ν-values



$$x \cong A_1 e^{i\nu_1\theta} + A_2 e^{i\nu_2\theta} + c.c.$$

$$y = B_1 e^{i\nu_1\theta} + B_2 e^{i\nu_2\theta} + c.c.$$

Both x and y have the same ν-values ν_1 and ν_2

Splitting $\nu_1 - \nu_2 \simeq 2|\Delta\nu_{11}|$

$\Delta\nu_{11}$ is coupling stop-band

For RHIC, without Magnet Shuffling, $\Delta\nu_{11,max} \simeq .03$ and $\nu_1 - \nu_2 \lesssim .06$

Linear Coupling gives a ν-splitting, but not a ν-spread.

Non-Linear Coupling Resonance ν-shifts

ν-splitting depends on ϵ_x, ϵ_y

Fig 2 shows Tracking Result for ν_y versus x_0, initial x-betatron oscillation, when $y_0 = 0$ or $\delta = .005$.

ν_y splitting increases with x_0
At largest x_0, $x_0 = 18$ mm, $\epsilon_x = 6.5$,
ν-splitting is $\Delta\nu = .034$

This ν-splitting is a ν-spread; all intermediate ν-values reached at lower values of x_0.

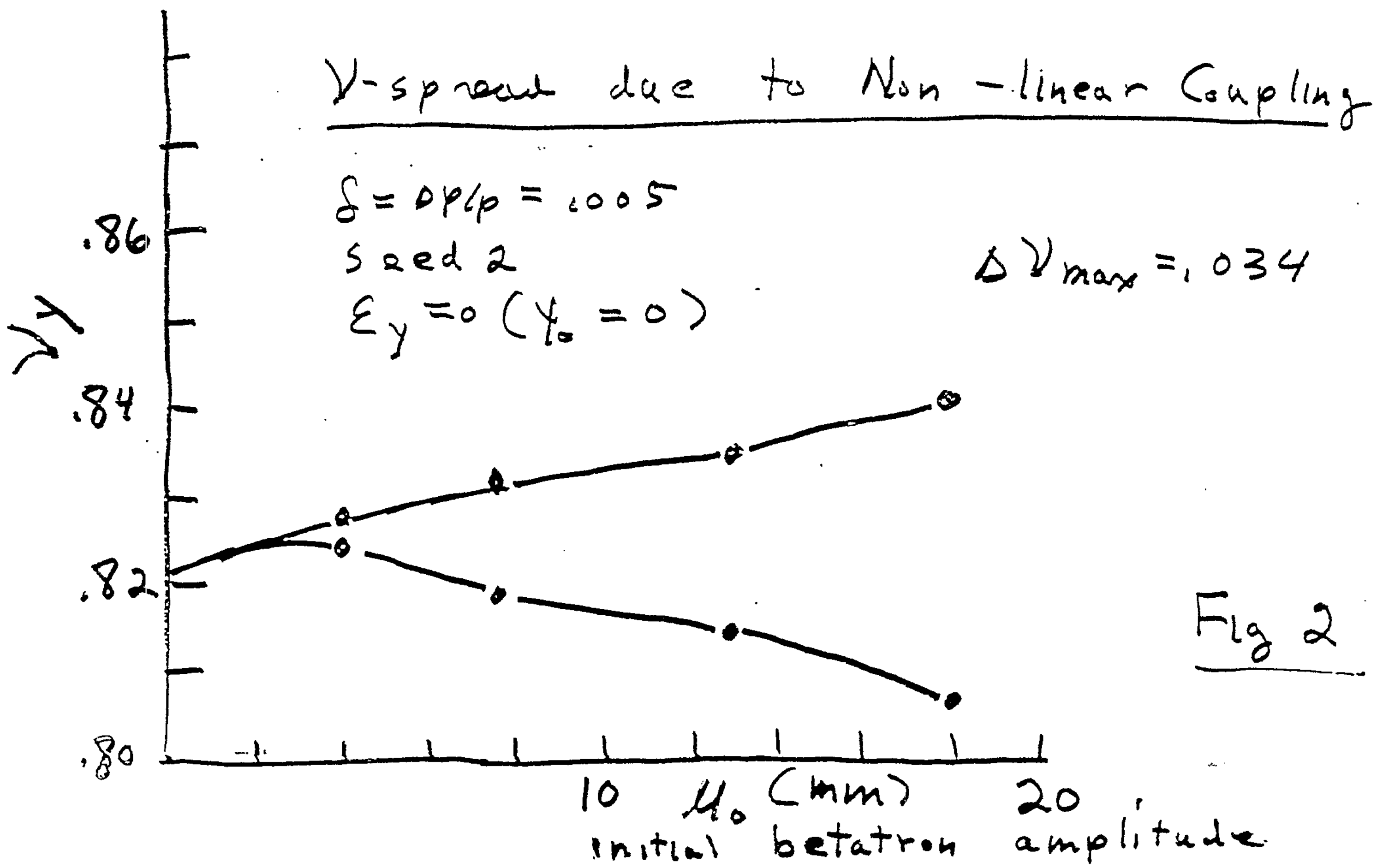

ν-spread due to Non-linear Coupling
δ = Δp/p = .005
Seed 2
$\mathcal{E}_y$ = 0 (Y_0 = 0)
Δν$_{max}$ = .034
ν
.86
.84
.82
.80
10
20
U_0 (mm)
initial betatron amplitude

Fig 2

ν-Spread Study for 10 Sets of Random Field Errors

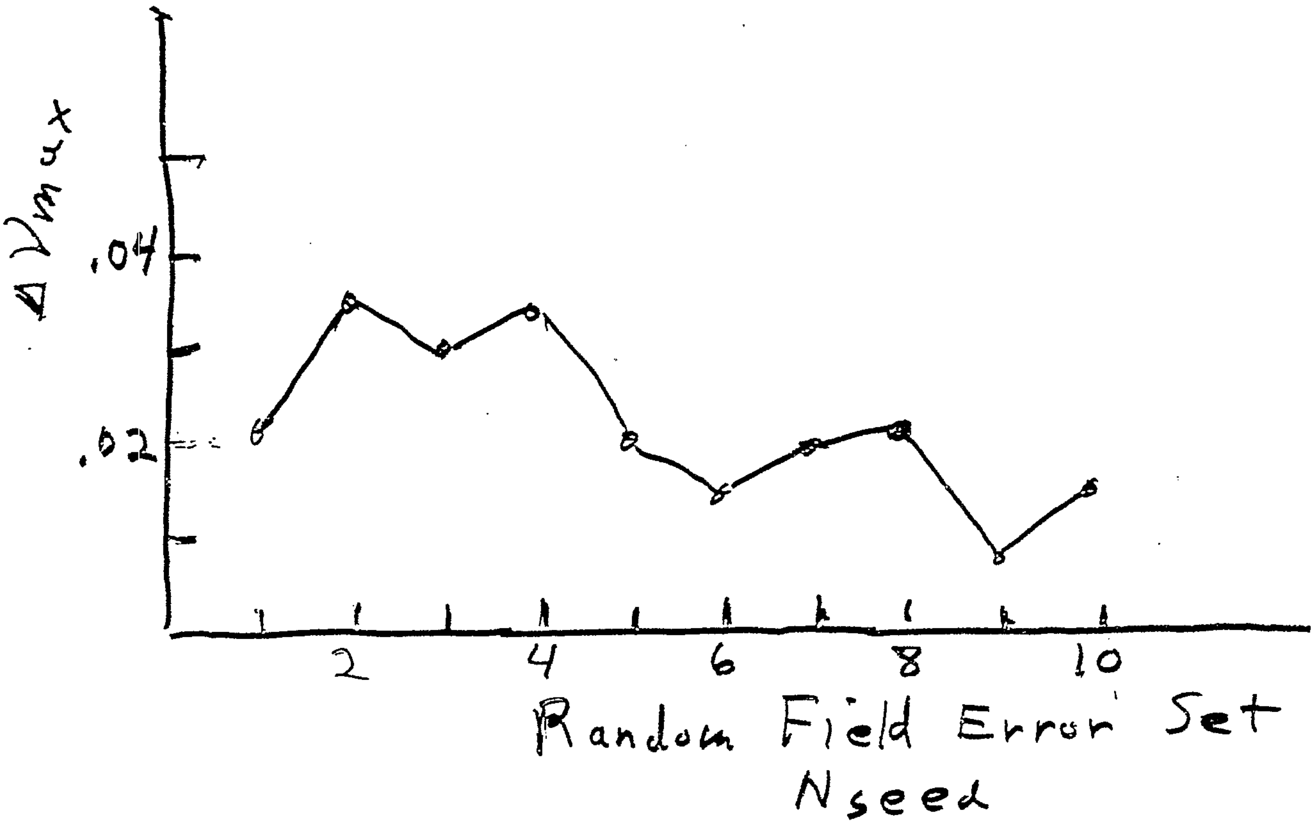
$\epsilon_x + \epsilon_y = 6.5$

		$\epsilon_x = \epsilon_y = 3.25$	$\epsilon_x = 0, \epsilon_y = 6.5$	$\epsilon_y = 0, \epsilon_x = 6.5$	$\Delta\nu_{max}$
Seed 3					
$\delta = .005$	ν_x	.828 .817 .823 .832	.827 .818 .823 .831	.830.	
	ν_y	.822 .818 .828 .833	.822 .819 .828 .831	.830 .817	.029
$\delta = -.005$	ν_x	.818	.823 .833	.839	
	ν_y	.818	.822 .833	.810 .839	
Seed 1					
$\delta = .005$	ν_x	.818	.827 .821 .815	.834	
	ν_y	.819	.821 .815 .827	.834 .814	.020
$\delta = -.005$	ν_x	.821 .825	.828 .820	.818	
	ν_y	.823 .818 .820	.820	.824 .819	
Seed 2					
$\delta = .005$	ν_x	.830	.828 .825	.841	
	ν_y	.820	.825 .822 .829	.841 .807	.034
$\delta = -.005$	ν_x	.827	.820 .830	.831	
	ν_y	.821	.820	.831 .816	
Seed 4					
$\delta = .005$	ν_x			.834	
	ν_y			.816 .834	.033
$\delta = -.005$	ν_x			.842	
	ν_y			.809 .842	
Seed 5					
$\delta = .005$	ν_x			.833 .837	
	ν_y			.817 .833	.019
$\delta = -.005$	ν_x			.834	
	ν_y			.834 .815	
Seed 6					
$\delta = .005$	ν_x			.829	
	ν_y			.829 .819	.024
$\delta = -.005$	ν_x			.835	
	ν_y			.811 .835	

	$\varepsilon_x = \varepsilon_y = 3.25$	$\varepsilon_x = 0$, $\varepsilon_y = 6.5$	$\varepsilon_y = 0$, $\varepsilon_x = 6.5$	$\Delta \gamma_{max}$
Seed 7				
$\delta = .005$ γ_x			.831	
γ_y			.817 .831	.019
$\delta = -.005$ γ_x			.835	
γ_y			.816 .835	
seed 8				
$\delta = .005$ γ_x			.826	
γ_y			.826 .822	.022
$\delta = -.005$ γ_x			.837	
γ_y			.814 .836	
Seed 9				
$\delta = .005$ γ_x			.828	
γ_y			.821 .825 .828	.011
$\delta = -.005$ γ_x			.826	
γ_y			.826 .820 .831	
seed 10				
$\delta = .005$ γ_x			.829	
γ_y			.819 .830	.015
$\delta = -.005$ γ_x			.832	
γ_y			.817 .832	

ν-splitting Versus N_{seed}

$\varepsilon_y = 0$, $\mu_0 = 18$ mm



Scenario for ν-spread

at the start

$\Delta\nu$ beam-beam ~ .024

$\Delta\nu$ (betatron osc.) ~ small

after many hours

Beam grows due to intrabeam scattering

$\Delta\nu$ beam-beam ~ .008

$\Delta\nu$ (betatron osc.) ~ .02

Also, thes $\Delta\nu$ are to some extent not additive

$\Delta\nu_{beam-beam}$ → large at small betatron osc.
Small at large betatron osc

$\Delta\nu$ (betatron osc.) → Small at small betatron osc.
Large at large betatron osc.

Multipole Breakdown – Tune Splitting

Initial $\varepsilon_y = 0$, $\varepsilon_x = 6.5$, $u = 18mm$

Kmax		Seed 3	Seed 8
no b_K, a_K	ν_x	.829	.829
	ν_y	.819	.819
2	ν_x	.827	.842
	ν_y	.818, .827	.807 .841
3	ν_x	.825	.846
	ν_y	.817, .825	.846 .801
4	ν_x	.839	.841
	ν_y	.801, .839	.841 .806
5	ν_x	.838	.840
	ν_y	.803, .838	.840 .807
6	ν_x	.837	.839
	ν_y	.837, .804	.839 .807
7	ν_x	.836	.839
	ν_y	.836, .804	.839 .807
8	ν_x	.836	.840
	ν_y	.836, .805	.840 .807
9	ν_x	.837	.840
	ν_y	.837, .805	.840 .806
10	ν_x	.836	.840
	ν_y	.836, .806	.840 .807
$\Delta\nu_{max}$		.030	.033
δ		-.005	-.005

Which multipoles b_K, a_K are responsible for the ν-splitting? ~~In above,~~ Kmax is the highest multipole present.

Answer not clear.