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Higher Order Magnet Field Multipoles Aperture Effects, and Tracking Studies

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Review of Tracking Theory (my view)

The instabilities are non-resonant; not associated with the ν-values going to some resonance line, $m\nu_y + n\nu_x = q$

The instabilities are not stochastic - they happen quite fast.

No particular resonance dominates. Classical non-linear theory does not apply. Effect is complicated, and probubly cannot be described by simple analytical results.

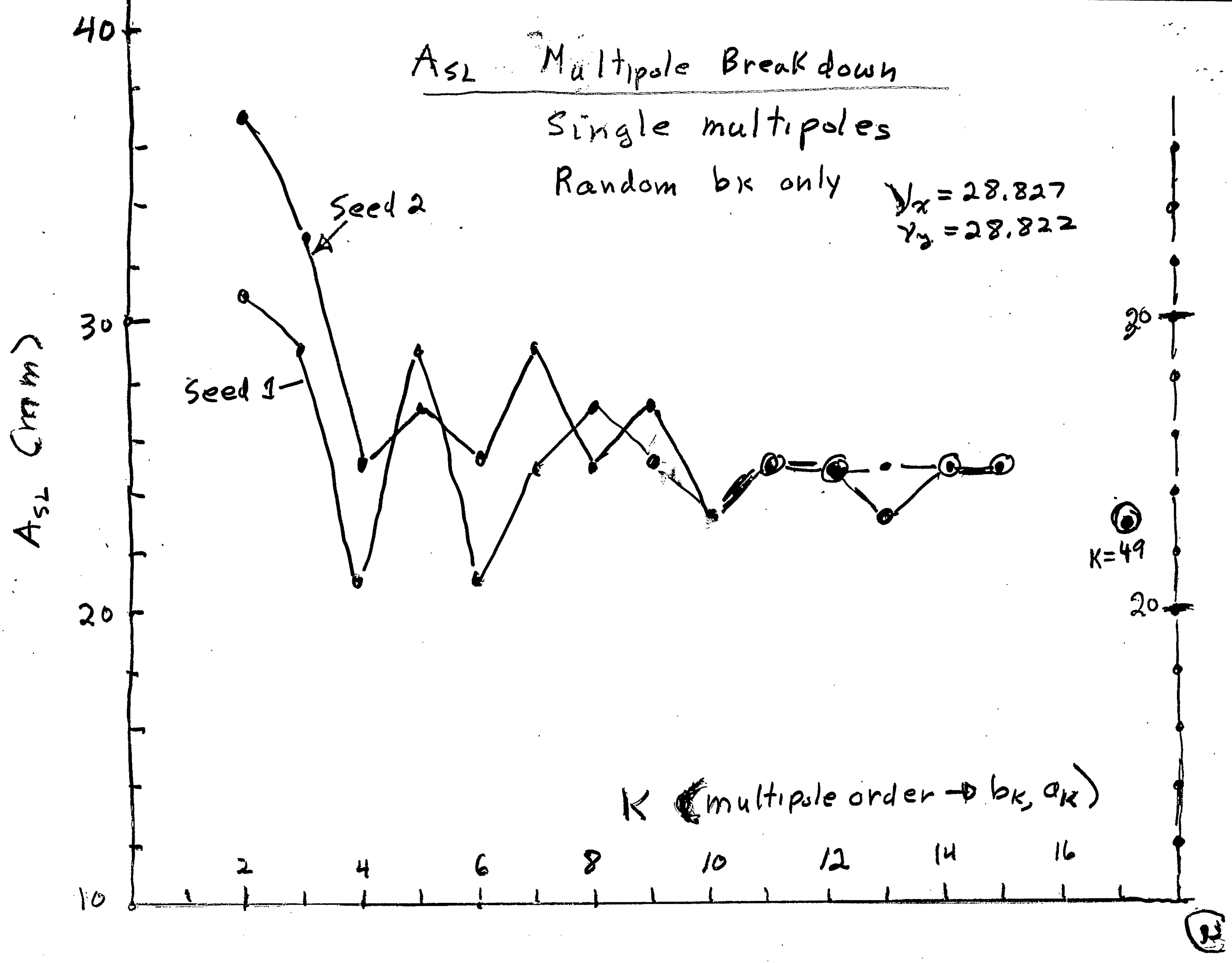
Review of RHIC Results

Random b_K, $b_K \simeq (K+1)\, b_0 / R^K$

$R = 40\ mm$, $b_0 \simeq 1 \times 10^{-4}$

$A_{SL} \simeq 19\ mm$, random b_K only

$\nu_x \simeq \nu_y \simeq 28.824$



ASL Multipole Breakdown
Single multipoles
Random bk only
νx = 28.827
νy = 28.822
Seed 2
Seed 1
ASL (mm)
40
30
20
10
2
4
6
8
10
12
14
16
K=49
30
20
K (multipole order → bK, aK)

Tracking studies seem to indicate that $A_{SL} \longrightarrow$ constant $\simeq 24$ mm when K gets large

Is this possible?

Is it due to point multipoles being used instead of distributed multipoles?

Point Matipoles versus Distributed Multipoles

Classical N.L. Theory $\rightarrow$ $A_{SL} \xrightarrow[K\to\infty]{}$ constent $\simeq R$ for point b_K

$A_{SL} \xrightarrow[K\to\infty]{} 0$ for distributed b_K

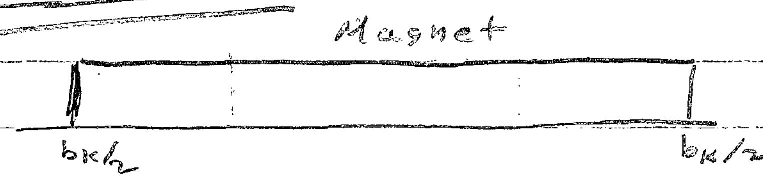
Classical theory Resalt

$$\nu - \nu_{res} \cong \underbrace{\int ds\, e^{i\, p\theta}\, \beta \left(\frac{\beta}{\beta_0}\right)^{(K-1)/2} b_K}_{\text{factor}} \; A_{SL}^{K-1}$$

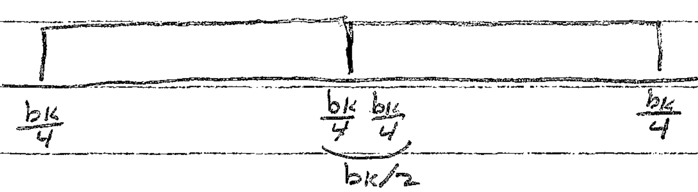
factor $\xrightarrow[K\to\infty]{} 0$ for distributed b_K

Distributed Multipoles in Tracking

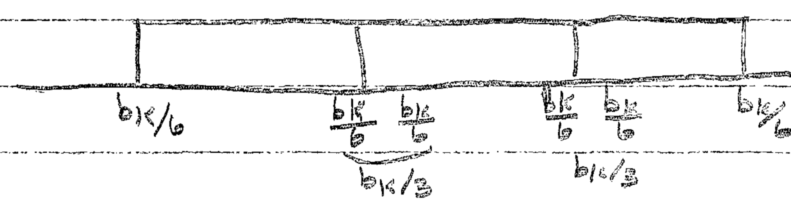
point b_K



2 intervals

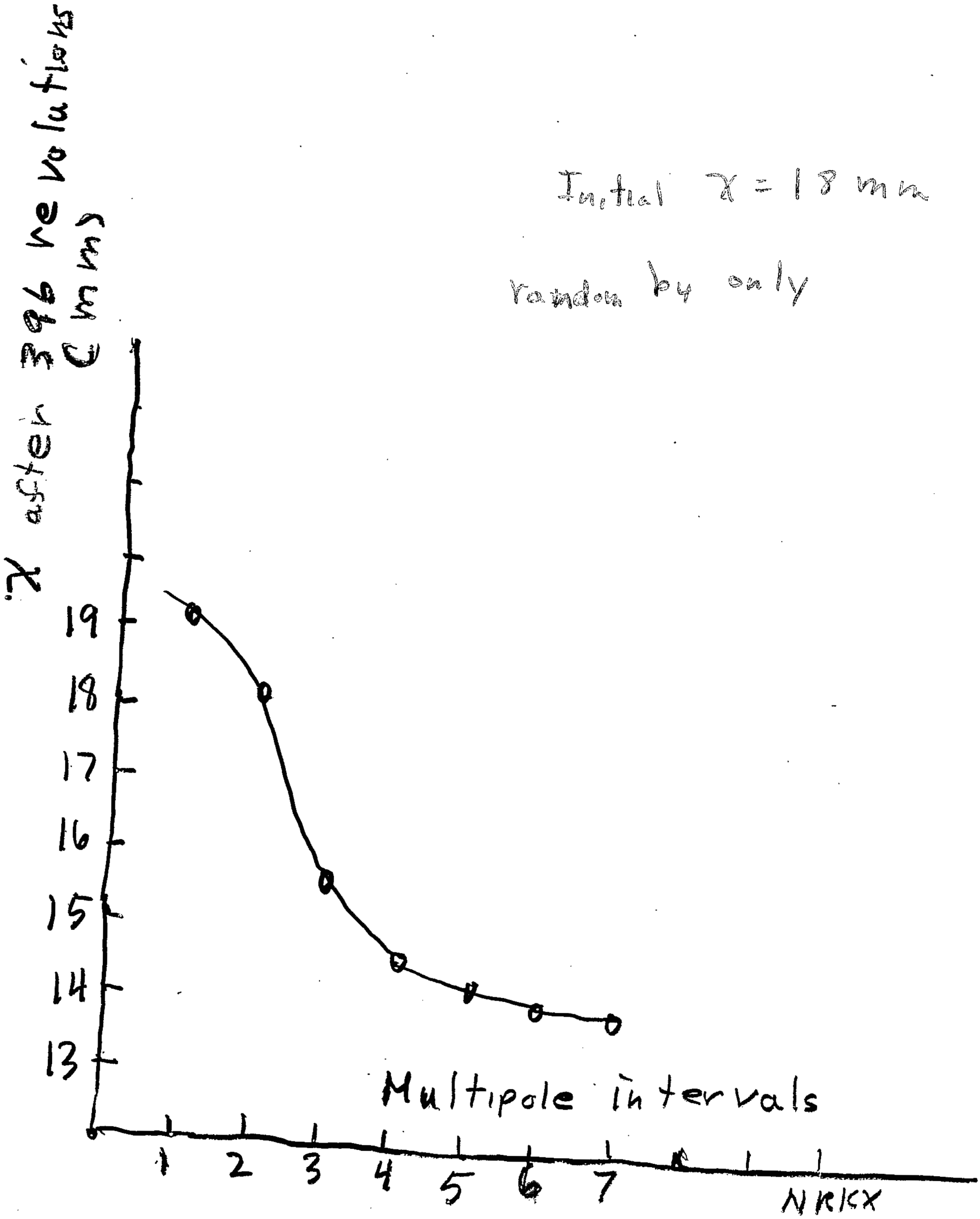


3 intervals



Equivalent to using 2^{nd} order Runge-Kutta.

Convergence as Multipole Intervals are Increased



Distributed Multipole Results

Increasing the number of intervals to describe the multipoles, changes results significantly. However, A_{SL} (the stability limit) is not changed. Occasionally A_{SL} is increased by 2 mm for some runs.

Point multipoles appear to give essentially the correct result for A_{SL}.

A_{SL} for High order bk

$$A_{SL} \xrightarrow[K \to \infty]{} 24 \text{ mm}$$

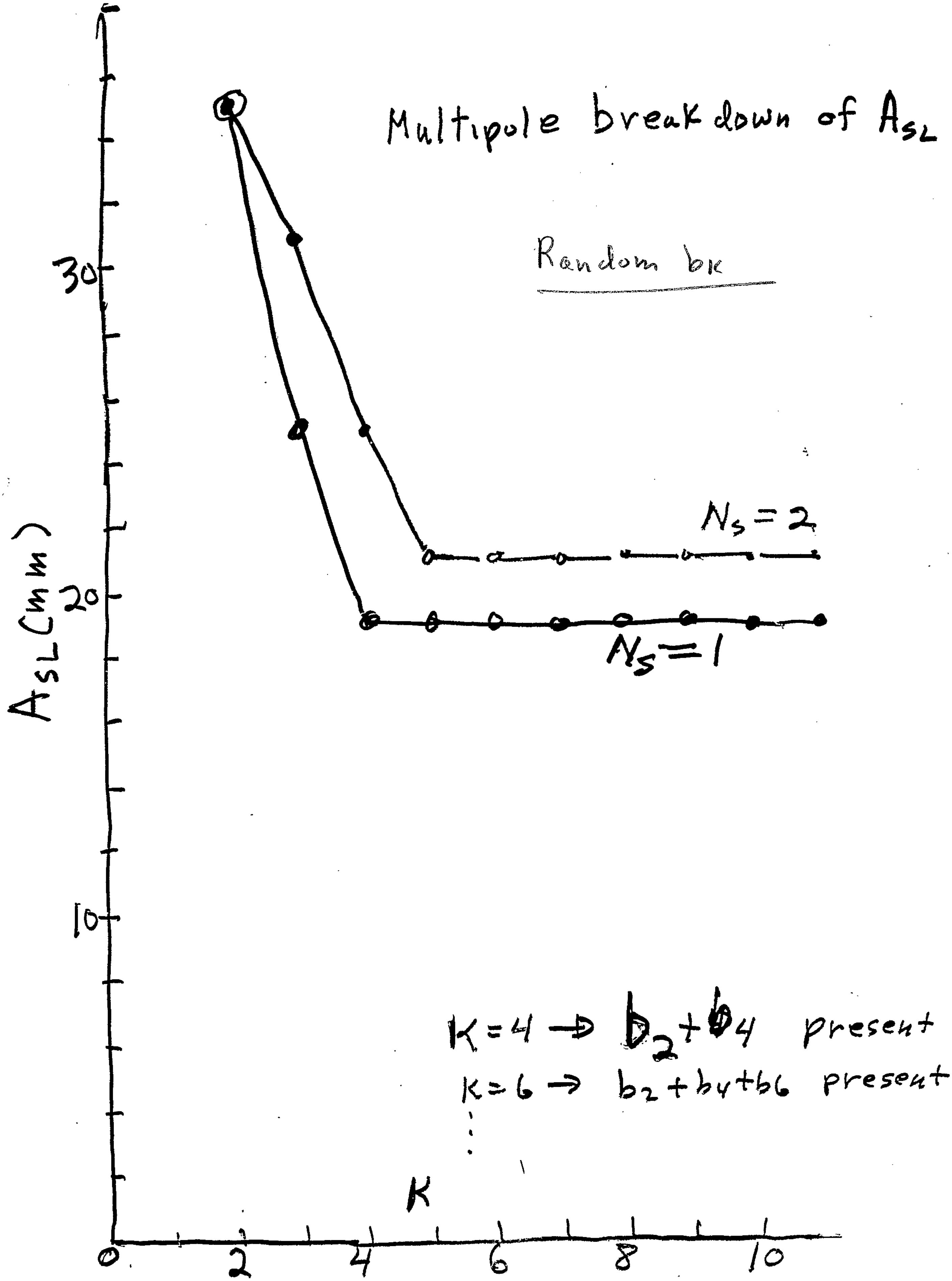
for bk random case

What are the consequences?

How many bk needed to determine A_{SL}?

I dont need up to $K = 50$
I just need up to $K \cong 6$

(see next slide)


Multipole breakdown of A_{SL}
Random bK
$N_s = 2$
$N_s = 1$
A_{SL} (mm)
30
20
10
0
2
4
6
8
10
K
K=4 → $b_2 + b_4$ present
K=6 → $b_2 + b_4 + b_6$ present
⋮

Higher b_k produce a wall at about 24 mm; For $x \leq 24$ mm, they have little effect. Thus if lower b_k produce $A_{SL} \simeq 19$ mm, the higher b_k do not affect A_{SL}.

Higher b_k Limit A_{SL} to $A_{SL} \leq 24$ mm

Even if I correct many of the lower b_k, say for $k \lesssim 10$, I cant do better than $A_{SL} \sim 24$ mm

Systematic b_K

<u>Possible problem.</u> I expect the higher sytematic b_K to be larger than the higher random b_K

$$b_K \simeq \frac{b_0}{R^K} \times 10^{-4}$$

$b_0 \cong (K+1) \times 1 \times 10^{-4}$ random b_K

$b_0 \simeq 300 \times 10^{-4}$ not unlikely for systematic b_K

$b_0 \sim 1$ is possible for systematic b_K

Systematic b_K, RHIC Results

b_K, systematic used (from RHIC Proposal)

Dipole

R = 40mm

K	b_K'	$b_K * R^K / 10^{-4}$
2	17	44
4	-5.9	-38
6	1.6	17
8	-.4	-10
10	.1	7
12	-.1	-10
14	-.1	-72
16	-.13	-239
18	.080	378
20	.0030	36

Quads

K	b_K'	$b_K * R^K / 10^{-4}$
5	—	0
9	—	.005
13	.2	94
17	-.12	-361
21	.0075	-145
25	.015	1914

↑ x(k+1) for random b_K

Results for $\left\{\begin{array}{l}\text{Dipoles } K = 14 \rightarrow 20 \\ \text{Quads}\end{array}\right\}$ from H. Hahn Tech Note

Note, $A_{SL} \sim b_K^{1/K}$ or large changes in b_K produce small changes in A_{SL} for large K

FNAL b_K, systematic

measured results

K	b_K ($in^{-K}/10^{-4}$)	$b_K * R^K/10^{-4}$ $R = 38$ mm
2	.99	2.2
3	−.27	− .91
4	−.76	−3.8
5	−.05	− .38
6	6.69	76.
7	.02	.34
8	−15.69	−404
9	.01	.38
10	5.25	302
12	−1.1	−142
14	.12	35.

Aperture Results (Tracking Results)

A_{SL} including b_{K}, systematic

For b_{K}, sys ($K=1 \to 20$) + b_{K}, ran ($K=1 \to 10$)

$$A_{SL} = 15 \text{ mm}$$

For b_{K}, sys ($K=1 \to 20$) ; no b_{K}, ran

$$A_{SL} = 19 \text{ mm}$$

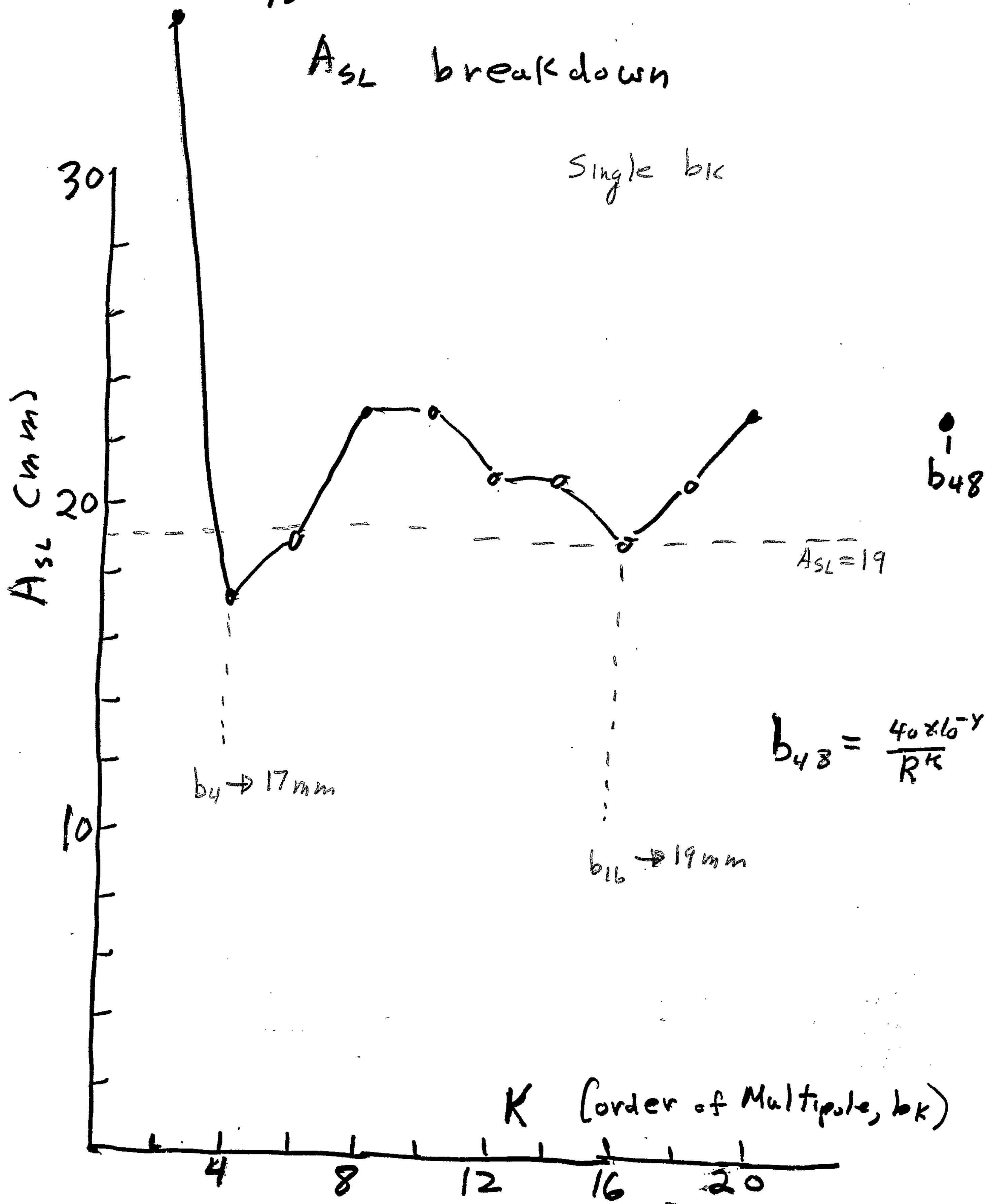
same as b_{K}, random case

b_{K}, systematic, multipole breakdown

(see next page)

Systematic b_K

A_{SL} breakdown



For $b_{K,sys}$ ($K=10 \rightarrow 20$) + $b_{K,ran}$ ($K=1 \rightarrow 10$)

$$A_{SL} = 17 \text{ mm}$$

Even if all lower $b_{K,sys}$ for $K=1 \rightarrow 9$ are eliminated, one gets $A_{SL} = 17$ mm

There is the possibility, and some indication, that proper choice of lower b_K ($b_K \neq 0$) can increase A_{SL}

Non-point $b_{K,sys}$ tests

Distributed $b_{K,sys}$ produce only small changes in the results.

FNAL Aperture Results

$b_{K,sys}$ all alone $\rightarrow$ $A_{SL} = 21\,mm$ ($\beta_x = 100$)

addition of $b_{K,ran}$ reduces A_{SL} to
$A_{SL} = 19\,mm$ ($\beta_x = 100$; $y \cong 0$ results)

Tracking results of Gelfand and Willeke

Willeke says Tevatron aperture largely due to $b_{K,sys}$. I think both, $b_{K,sys}$ and $b_{K,ran}$, are important

Note, Tevatron A_{SL} for $x \simeq y$ may be $A_{SL} \simeq 15\,mm$.

Conclusions for RHIC Magnets

1) $A_{SL} \simeq 17$ mm may result from Systematic b_K.

2) Possibility of choosing lower b_K to improve A_{SL}. $b_K = 0$ is not necessarily optimum solution.

3) Watch out for very large higher b_K (b_{16} etc. . .)