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Accelerator Physics Coordinate Conventions

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Accelerator Physics coordinate conventions

Now that application and utility software and databases are beginning to proliferate in earnest, it is time to establish unique nominal coordinate conventions inside the RHIC Accelerator Physics (RAP) group. It is fortunate that unique internal conventions are (appear to be) possible, so that we can hand off to our users data that conform closely to these internal standards. The surveyors, for example, are not so fortunate - they have to juggle between the 5 or 6 coordinate frames that exist on the Brookhaven site. The three main user groups considered here are the survey group, the engineering drawing group, and control room console operators.

This note has been prepared in consultation with Mike Hemmer and Sue Norton, who represent the surveyors and the engineering drawers, respectively. Both of these groups expect survey information from RAP in the "RHIC survey frame", which is defined in detail below. A minor detail is that Mike expects the unit of length to be meters, while Sue expects international inches[1].

Since beams in the Blue and Yellow rings rotate clockwise and counterclockwise, the possibility of confusion exists. For example, what is the direction of positive dispersion? What happens at the split in the transfer line, as it approaches RHIC? It is up to RAP to act as the software conscience of the console in resolving these issues, and to best streamline the control room high level paradigm. They are addressed in the "beam coordinates" section, below.

The RHIC survey frame

Beams cross at six locations in RHIC, forming a regular hexagon that lies in the plane that contains both Blue and Yellow rings. In surveyors jargon[2], the center of this hexagon is called the Machine Center Point, or MCP. The location of the MCP in the Euclidian RHIC survey frame is given by coordinates (N,E,W), defined to be exactly

$$\begin{pmatrix} N \\ E \\ W \end{pmatrix}_{MCP} \equiv \begin{pmatrix} 32284.517011 \\ 30230.237553 \\ 0.000000 \end{pmatrix} \qquad 1$$

where the units are meters. The N-axis points approximately from the 6 o'clock crossing point to the 12 o,clock ("north"), while the E-axis points approximately east. Both N and E axes lie in the RHIC plane. The W axis points vertically upwards. Therefore, (N,E,W) forms a left handed coordinate system.

In RAP software - such as "dbsf", "survey", and "fiduce" - MAD conventions are used for the "global reference system" that corresponds to the RHIC survey frame[3,4]. In this convention, (X,Y,Z) form a right handed system in which Y is the vertical axis. That is, there is an implicit correspondence given by

$$\begin{pmatrix} N \\ E \\ W \end{pmatrix} \longleftrightarrow \begin{pmatrix} X \\ Z \\ Y \end{pmatrix} \qquad 2$$

In fact, a single point in the beam line has 7 global survey coordinates in RAP software - 4 displacements and 3 angles. The fourth displacement, s, measures the arc distance that has been traversed along the design trajectory from a fixed reference point.

Three angles describe the orientation of the "local reference system" (x,y,z) that travels with the design orbit, relative to the "global reference system" (X,Y,Z). The angles θ, ϕ, and ψ are defined as in the MAD program[4]. The horizontal azimuth θ is measured relative to the "E" axis. The other two angles ϕ and ψ are respectively the vertical pitch angle ($\phi > 0$ upward) and the roll angle ($\psi > 0$ having the vertical tilt to the right side for an observer looking along the direction of motion). The rotations are applied in the order

$$R_y(\theta)\, R_x(\phi)\, R_z(\psi) = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad 3$$

Note that, since Blue and Yellow lie in a plane, $\phi = \psi = 0$ everywhere in RHIC. By contrast, all three angles may be non zero in the transfer line.

Coordinates of some key locations

A straight line from 6 o'clock to 12 o'clock collision points lies exactly 1.999898 degrees to the east of the N-axis, as shown in Figure 1 of reference 2. That is,

$$E_6 < E_{MCP} \qquad 4$$

In revision 0.3 of the RHIC92 lattice[5], the distance from the 6 o'clock crossing to the 12 o'clock crossing is exactly 1181.163278 meters. Note that this number is not entirely constrained from revision to revision of the lattice, and varies by 38 microns from the nominal value of 1181.163316 meters reported elsewhere[2]. Putting all this together, the design location of the 6 o'clock crossing point in the RHIC survey frame is

$$\begin{pmatrix} N \\ E \\ W \\ \theta \\ \phi \\ \psi \end{pmatrix}_6 = \begin{pmatrix} X \\ Z \\ Y \\ \theta \\ \phi \\ \psi \end{pmatrix}_6 = \begin{pmatrix} 31694.295102 \\ 30209.627602 \\ 0.000000 \\ 3.106687849 \\ 0.000000000 \\ 0.000000000 \end{pmatrix} \qquad 5$$

where the displacements are in meters and the angles are in radians. The angle theta is exactly $180 - 1.999898 = 178.000102$ degrees.

The location of the MCP is absolutely fixed to the RHIC survey frame, by definition, so the design values in equation 5 are unambiguous: they are not sensitive to the presence of unavoidable surveying errors. In order to know the location of the beginning of the transfer line from the AGS to RHIC, it is necessary to know where the AGS is in the RHIC survey frame, a measurement that, by contrast, is susceptible to measurement error. At the time of writing, the best estimate for the location of the AGS design trajectory for extracted beam at the center of the extraction septum is

$$\begin{pmatrix} N \\ E \\ W \\ \theta \\ \phi \\ \psi \end{pmatrix}_{septum} = \begin{pmatrix} X \\ Z \\ Y \\ \theta \\ \phi \\ \psi \end{pmatrix}_{septum} = \begin{pmatrix} 31199.604935 \\ 30286.568504 \\ 1.711363 \\ 2.101037618 \\ 0.000150293 \\ -0.000078145 \end{pmatrix} \qquad 6$$

where the units are meters and radians. Note that the AGS is 1.711363 meters above the RHIC plane. In fact, the RHIC and AGS planes are tilted relative to each other by an angle of about 0.2 milliradians. This angle is negligible, in the sense that it is smaller than typical closed orbit angles in the AGS. Continuing on a short distance downstream from the AGS septum, the coordinates of the beginning of the U-line are

$$\begin{pmatrix} N \\ E \\ W \\ \theta \\ \phi \\ \psi \end{pmatrix}_{U\ begin} = \begin{pmatrix} X \\ Z \\ Y \\ \theta \\ \phi \\ \psi \end{pmatrix}_{U\ begin} = \begin{pmatrix} 31208.166445 \\ 30281.536417 \\ 1.712859 \\ 2.099848558 \\ 0.000150000 \\ 0.000078000 \end{pmatrix} \qquad 7$$

Note that the vertical height has changed slightly, due to the non-coplanarity of the AGS and RHIC.

The Blue transfer line and the Blue collider ring merge at the easterly end of quadrupole 8 of the 6 o'clock interaction region. Propagating the appropriate beamlines to this point using the "rhic92r0p3" and "ags_to_rhic" lattices, the coordinates of the Blue ring injection point become

$$\begin{pmatrix} s \\ \\ N \\ E \\ W \\ \theta \\ \phi \\ \psi \end{pmatrix}_{B\ inject} = \begin{pmatrix} s \\ \\ N \\ E \\ W \\ \theta \\ \phi \\ \psi \end{pmatrix}_{B\ inject} = \begin{pmatrix} 126.408198 \\ \\ 31699.644901 \\ 0.000000 \\ 30083.357683 \\ 3.062337945 \\ 0.000000000 \\ 0.000000000 \end{pmatrix} \qquad 8$$

Similarly, the coordinates of the Yellow ring injection point are

$$\begin{pmatrix} s \\ \\ N \\ E \\ W \\ \theta \\ \phi \\ \psi \end{pmatrix}_{Y\ inject} = \begin{pmatrix} s \\ \\ N \\ E \\ W \\ \theta \\ \phi \\ \psi \end{pmatrix}_{Y\ inject} = \begin{pmatrix} 3707.436802 \\ \\ 31690.824174 \\ 0.000000 \\ 30335.963129 \\ 0.009445100 \\ 0.000000000 \\ 0.000000000 \end{pmatrix} \qquad 9$$

where the units, as usual, are meters and radians.

The local beam coordinate system

The MAD default convention, followed throughout RAP software whenever possible, is that the x and y axes of the local (x,y,z) coordinate system point leftward and upward in the horizontal and vertical planes, respectively, as the design orbit moves forward. A positive dipole angle means that the design orbit veers to the right - if the beamline is closed, it propagates in a clockwise direction, for an observer looking down on the ring. This convention is natural for the Blue ring, in which an ion moves clockwise, in the same direction as the design orbit. Difficulties occur in the Yellow ring if the design orbit propagates with the particles in a counter clockwise direction. For example, the arc dipole bend angle becomes negative, the x-axis points toward the center of the ring, and the natural horizontal dispersion is negative. These difficulties are overcome by carefully and deliberately distinguishing between the propagation of the design orbit and the propagation of ions.

In both Blue and Yellow rings, therefore, the convention is to propagate the design orbit in a clockwise direction. This is the default way that beamlines are stored in the lattice databases[5,6]. From this it follows that the x-axis always points radially outward from the center of a ring, that the arc dipole bending angles are positive, and that the horizontal dispersion is naturally positive. (This picture of harmony is only slightly marred by the convention that a positive kick of $\Delta x'$ in a horizontal dipole corrector kicks the beam outwards towards positive x - a dipole corrector with a positive horizontal angle has a vertical B-field in the opposite direction to that of an arc dipole.)

In the transfer line, the design orbit is propagated from the AGS towards RHIC. The local x-axis points left (usually approximately west), the y-axis initially points up, and a positive dipole angles turn the design orbit to the right. Thus, in the final quarter circle arc into the Blue ring, the dipole angles are negative and the natural horizontal dispersion is negative, while the arc into the Yellow ring has positive dipole angles and positive natural dispersion. The local coordinate system of the Blue transfer arc merges gracefully with that of the Blue ring[7].

The local Yellow transfer system unfortunately does not merge seamlessly with the local Yellow ring system. The s-axis, x-axis, and the z-axis directions flip by exactly 180 degrees at this interface, because the design orbit propagation vectors point head to head. The y-axis is invariant. That is, the locations and slopes of the same trajectory point in the two local reference systems are related by

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ z' \end{pmatrix}_{YT} = \begin{pmatrix} -x \\ +x' \\ +y \\ -y' \\ -z \\ +z' \end{pmatrix}_{YR} \qquad 10$$

where subscripts YT and YR refer to the yellow transfer line or ring, and where a prime denotes differentiation with respect to the s coordinate. The conditions for optical matching at this point are therefore

$$\begin{pmatrix} \beta_x \\ \alpha_x \\ \beta_y \\ \alpha_y \\ \eta_x \\ \eta'_x \\ \eta_y \\ \eta'_y \end{pmatrix}_{YT} = \begin{pmatrix} \beta_x \\ -\alpha_x \\ \beta_y \\ -\alpha_y \\ -\eta_x \\ \eta'_x \\ \eta_y \\ -\eta'_y \end{pmatrix}_{YR} \qquad 11$$

where η is the dispersion.

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References

1 One international inch is exactly equal to 2.54 centimeters, unlike the US survey inch.

2 M. Goldman, "The RHIC Reference Geometry", AD/RHIC/RD-43, August 1992.

3 LAMBDA collaboration, "LAMBDA manual", RHIC/AP/13, October 93..

4 H. Grote and F. C. Iselin, "The MAD Program", Version 8.1, CERN/SL/90-13(AP).

5 Saltmarsh et al, "Lattice Information Service for the RHIC project", RHIC/AP/6, August 93.

6 The user may still follow (say) the Yellow ring in a counter clockwise direction, by inserting a minus sign in front of the beamline name, or by reordering the sequence in which lists are ordered and reported. Note, however, that investigations of long term tracking behavior do NOT have to propagate particles in the actual direction they go, since it is trivial to show that the equations of motion are invariant under time reversal (assuming the fields are static).

7 It is not self evident that the roll angle ψ is zero at the end of the Blue transfer arc, since the transfer line does not lie in a plane. However, the roll angle is in fact zero, and thus the coordinate systems do merge seamlessly, due to design choices in the way that the transfer line dipoles are rolled.