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AN ANALYSIS OF THE RHIC FRONT END INJECTION

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Section I - Introduction

The desired goal of the RHIC project is to produce ^{197}Au + ^{197}Au collisions, in the collider, at sufficient energy and intensity, to both produce and detect a quark-gluon plasma.[1]

At the present time, the initial source of ions is injected into a Tandem. Although long term, future developments, have addressed the need to replace this ion source and Tandem,[2] all the calculations discussed here will be based on current or very near future Tandem specifications.

The very definition of "current" is derived and can be very confusing. Let us consider in detail the "number of particles" N^{AGS} that finally arrive at the AGS using the existing or near future ion source characteristics.

In Figure 1 the four preinjector arrangements to be discussed and compared in this manuscript are shown. Arrangement A corresponds to the established preinjector for heavy ions.

We will consider the four separate arrangements; A, B, C and D for getting N^{AGS}. All four will modify the definition of space charge limit at the booster. These four arrangements will highlight the current scheme and indicate advantages and disadvantages of an intermediate Linac. In addition, a simplified scheme will be considered to highlight the importance of stripping foil efficiencies.

The essence of the injection problem is this; the vacuum requirements of the AGS demand that fully stripped ions be injected, and yet we have a low intensity source injected into a Tandem. Fully stripped ions in the AGS are desirable, for they can only be depleted from the AGS beam by electron pickup. The probability for this to occur decreases approximately linearly as the kinetic energy of the ion increases.[3]

In this manuscript all of our calculations will be for ^{12}C ions or ^{197}Au ions, that is "best case" or "worst case" calculations. Often, for matters of clarity, we will simply drop the "best case" analysis. At the end of this manuscript we will also relate our results to the future possibilities of using Uranium beams.

In general the number of particles reaching the AGS is given by,

$$N_{AGS} = \underbrace{(N_T S_T T_T)}_{\text{Tandem}} \times \underbrace{(S_F T_L)}_{\text{Linac}} \times \underbrace{(S_B T_B)}_{\text{Booster}}$$

We will take $T_T = .75$ for all ions, $T_L = .5$ for all ions, $T_B = 1$

where N_T number of particles at source
S_i stripping foil efficiency for most probable charge state
T_i Transmission probability through device.

Now S_T (^{12}C) = .39 (charge state 5)
S_T (^{197}Au) = .19 (charge state 13)

The values T_T S_T have been tabulated;[1] T_L is conservatively taken as .5 throughout this manuscript, but values as large as .8 are considered realistic.[4]

Not only do we want to get fully stripped ions into the AGS, but it is also necessary to respect the space charge limits of intermediate devices. The four possible options, A, B, C and D will be analyzed in this respect.

At the present time currents of 80 μA or 200 μA, for ^{197}Au or ^{12}C respectively, are considered reasonable.[4] In the near future (end of '88) currents for 300 μA for ^{197}Au are expected with improvements of source.[4] Although several factors determine the final number of particles in the collider, the current of the source and the pulse length injected into the Tandem are critical. We note, limits of the present Tandem have not been probed with high current and long pulse length operation.[4] At what pulse length and current the Tandem becomes inefficient, and the foil lifetimes under such adverse conditions, are not known in quantitative experimental detail.

Let us calculate the number of particles produced by the Tandem as a function of pulse length.

$$N^T = I^S \times P^L/e$$

where I^S is current of source
P^L is pulse length into Tandem
e is electron charge.

Table 1 - Number of Particles Emitted by Tandem.

Label	I^S	P^L	N^T	$N^T T_T T_S^T$ (^{12}C)	$N^T T_T T_S^T$ (^{197}Au)
1.	80 μA	80 μs	4.0 x 10^{10}	1.17 x 10^{10}	.569 x 10^{10}
2.	80 μA	110 μs	5.5 x 10^{10}	1.606 x 10^{10}	.780 x 10^{10}
3.	80 μA	200 μs	10.0 x 10^{10}	2.92 x 10^{10}	1.423 x 10^{10}
4.	80 μA	500 μs	25.0 x 10^{10}	7.30 x 10^{10}	3.56 x 10^{10}
5.	110 μA	80 μs	5.5 x 10^{10}	1.606 x 10^{10}	.780 x 10^{10}
6.	110 μA	110 μs	7.55 x 10^{10}	2.209 x 10^{10}	1.071 x 10^{10}
7.	110 μA	200 μs	13.73 x 10^{10}	4.02 x 10^{10}	1.96 x 10^{10}
8.	110 μA	500 μs	34.32 x 10^{10}	10.00 x 10^{10}	4.893 x 10^{10}
9.	200 μA	80 μs	10.0 x 10^{10}	2.92 x 10^{10}	1.423 x 10^{10}
10.	200 μA	110 μs	13.73 x 10^{10}	4.02 x 10^{10}	1.960 x 10^{10}
11.	200 μA	200 μs	25.0 x 10^{10}	7.30 x 10^{10}	3.558 x 10^{10}
12.	200 μA	500 μs	62.4 x 10^{10}	18.26 x 10^{10}	8.89 x 10^{10}
13.	300 μA	110 μs	20.59 x 10^{10}	6.02 x 10^{10}	2.933 x 10^{10}

In order to decide between device arrangements, and optimal pulse length or source current, we need to look at Space Charge Limit of Booster.

I.1 Space Charge Limit of Booster-Effect of Tandem Injection Scheme

Now, the maximum number of particles that can be injected into a given device, under restriction of space charge is[1] (using non-relativistic standard notation);

$$N_A^{SC} = \left(\frac{4A}{3r_o} \right) \left(B_F \epsilon^{max} \frac{\delta\nu}{F} \right) \left(\frac{\beta}{Q} \right)^2 \qquad (1)$$

In equation (1) one can identify three parts. The first part is simply fundamental constants. The second part represents machine parameters into which beam is injected. The third part represents dynamics of beam prior to injection. Hence β^2/Q^2 represents the previous dynamical history of beam coming into the device of interest. Because of the very low energy of the beam, no image current effects are expected and F = 1. The bunching factor B_F is defined as the ratio of average current to peak current. We propose $B_F = 0.3$ at injection, which may be more feasible after rf capture than the value of $B_F = 0.5$ in the CDR.[1] Finally ϵ^{max} is the largest emittance of the beam at injection. We are assuming "round" beam with the same emittance value in the vertical and horizontal direction. We identify ϵ^{max} with the 95% contour of the beam, assuming a gaussian distribution equal to the vertical betatron acceptance of the Booster, i.e. 50 π mm·rad. Since $r_o = 1.535 \times 10^{-15}$ m, we have

$$N^{SC} = 40.84 \times 10^{12} \times A\ (\beta/Q)^2\ \delta\nu$$

where Q is final charge state injected into Booster.

In general, for a non-relatistic problem, β is a function of an effective charge Q^{eff}. Let us rewrite N^{SC}. Now,

$$m_o c^2 A\ (\gamma - 1) = (Q^{eff} + 1)Ve$$

where Q^{eff} is general charge state and V is the Tandem terminal voltage.

$$\gamma = 1 + \frac{\left(Q^{eff} + 1\right)Ve}{m_o c^2 A}$$

Now $\gamma = (1 - \beta^2)^{-1/2} \approx 1 + \beta^2/2$ in the non-relativistic limit.

$$\beta^2 = \frac{2\left(Q^{eff} + 1\right)Ve}{m_o c^2 A}$$

$$N_A^{SC} = 8.8 \times 10^{10} \times \delta\nu \times V \times \left(\frac{Q^{eff} + 1}{Q^2} \right) \qquad (2)$$

where V is MVolt. Equation (2) is much more transparent than the traditional formula.(1) It shows clearly the effect of the acceleration device (Tandem or Tandem plus Linac) prior to Booster injection and the charge state Q at injection.

I.2 Stripping Foil Efficiencies

As part of the front end analysis, it will be critical to analyze the stripping foil efficiencies. A theoretical analysis of stripping foil efficiencies is an extensive subject in itself and should be pursued in a separate publication. We note that at ultra-relativistic energies, where full stripping is possible, the theoretical analysis is sound. At Tandem energies, where the most loosely bound electrons are picked up or stripped, theory is also in good shape. However, at intermediate energies where shell crossing occurs much work remains.[5,8] The theory of electron pick up or stripping, that is relevant to this discussion, is divided into two parts. For the first part we need a quartum-mechanical calculation to determine the total ionization cross sections, and the shell lifetimes for the appropriate states. In order to estimate equilibrium fractions of projectile shell variances in any thickness dx of the solid, it is necessary to develop a sequential chain of pick up or stripping that is similar in spirit to nuclear decay chains. Equilibrium conditions are readily derived from these chains.

On the experimental side detailed and reliable measurements of stripping foil efficiencies exist at both Tandem energies[6] and at energies where full stripping is possible.[7] However, as with theory, at intermediate energies (^{197}Au say, $50 \lesssim E_{LAB} \lesssim 150$ MeV) almost no knowledge of stripping foil efficiencies exist.[8] Unfortunately this is the energy range appropriate to the introduction of an intermediate Linac.

At the lower energy domain, Betz et al. have developed a simple formula[9] for predicting the charge states after tranversing a foil of "equilibrium" length. By equilibrium length, we mean the limiting foil thickness that produces a Gaussian distribution of partially stripped charge states, for a given bombarding energy.

According to Betz[8] for low bombarding energies $\overline{q} = Z[1 + (Z^{-\alpha}/\beta')^{-1/k}]^{-k}$ where $\alpha = .45$, $\beta = v/c$, $\beta' = 3.6/300$, $k = .16$.

Thus for ^{197}Au, $\overline{q} = 79\ [1 + .0167\beta^{-1.6667}]^{-.6}$ where $\overline{q}$ is the final charge state for a given β (charge equilibrium).

For arrangement A in figure 1, at $\beta = .0478$ $\overline{q} = 36.3118$. Experimentally[1,4] $\overline{q} = 33$. Let us modify the above formula with simple linear regression;

$$Q = 20 + m\overline{q} \text{ where } m = .358$$

Please note in above formula $\overline{q} = \overline{q}(\beta)$ only, there is no dependence on initial charge state. We will also assume $S_F = .17$ for ^{197}Au is true for other β values when charge distribution is a Gaussian. This is a reasonable assumption. If the distribution is not Gaussian, i.e., fully stripped, then these assumptions are not valid, and need later Thieberger report.[7] For ^{12}C we take $S_F = 0.9$.

With this simple formula the experimental results at Tandem energies are reproduced. The simple error in the formula, of approximately 3 charge states, can probably be extrapolated with confidence to higher charge states, say 42 for ^{197}Au. We note, Thieberger et al.[7] have measured the yield of many different fully stripped ions as a function of bombarding energy. For ^{197}Au, below ~ 170

MeV/A there are no fully stripped ions whereas above 500 MeV/A 80% are fully stripped.

Section II - Arrangement A

In Table 2 the space charge limits for Arrangement A of Figure 1 are shown.

For arrangement A	$Q^{eff} = 5$ (^{12}C)	$Q^{eff} = 13$ (^{197}Au)
(By measurement)	$Q = 6$ (^{12}C)	$Q = 33$ (^{197}Au)

and V = 15 MV

Table 2 - Space-Charge Limits of Booster for Arrangement A.

^{12}C		^{197}Au	
$\delta\nu$	N_A^{SC}	$\delta\nu$	N_A^{SC}
.1	2.20×10^{10}	.1	0.170×10^{10}
.3	5.6×10^{10}	.3	0.509×10^{10}
.5	11.0×10^{10}	.5	0.848×10^{10}

In Tables 3 and 4 the number of particles injected into the booster are given. The label 1-13 in these tables corresponds to the Tandem currents and pulse lengths of Table 1. At this point we see the critical role of the booster injection scheme in determining particle numbers. For a given pulse length (indicated by label), and given injection velocity, the injected beam can achieve a given number of revolutions (column 3). At present, eight turns can be accommodated easily in betatron phase space.[10] To go beyond this requires further theoretical and experimental work.

Table 3 - Number of Ions Injected into Booster. ^{12}C

	$N^T T^T S^T$	$N^T T^T S^T S^F$	No. of Revolutions	No. injected in 8 turns	No. injected in 32 turns*
1.	1.17×10^{10}	1.058×10^{10}	15	$.561 \times 10^{10}$	--- $\times 10^{10}$
2.	1.606×10^{10}	1.446×10^{10}	21	$.550 \times 10^{10}$	--- $\times 10^{10}$
3.	2.92×10^{10}	2.63×10^{10}	37	$.568 \times 10^{10}$	1.137×10^{10}
4.	7.301×10^{10}	6.572×10^{10}	93	$.565 \times 10^{10}$	1.136×10^{10}
5.	1.606×10^{10}	1.446×10^{10}	15	$.771 \times 10^{10}$	--- $\times 10^{10}$
6.	2.209×10^{10}	1.989×10^{10}	21	$.757 \times 10^{10}$	--- $\times 10^{10}$
7.	4.02×10^{10}	3.618×10^{10}	37	$.782 \times 10^{10}$	1.564×10^{10}
8.	10.00×10^{10}	8.99×10^{10}	93	$.773 \times 10^{10}$	1.547×10^{10}
9.	2.92×10^{10}	2.63×10^{10}	15	1.402×10^{10}	--- $\times 10^{10}$
10.	4.02×10^{10}	3.618×10^{10}	21	1.378×10^{10}	--- $\times 10^{10}$
11.	7.30×10^{10}	6.57×10^{10}	37	1.421×10^{10}	2.840×10^{10}
12.	18.26×10^{10}	16.43×10^{10}	93	1.413×10^{10}	2.827×10^{10}
13.	6.02×10^{10}	5.42×10^{10}	21	2.065×10^{10}	--- $\times 10^{10}$

* 50% Stacking Efficiency.

Table 4 - Number of Ions Injected into Booster. ^{197}Au

	$N^T S^T T^T$	$N^T T^T S^T S^F$	No. of Revolutions	No. injected in 8 turns	No. injected in 32 turns*
1.	.569 x 10^{10}	.096 x 10^{10}	5	.096 x 10^{10}	--- x 10^{10}
2.	.780 x 10^{10}	.132 x 10^{10}	7	.132 x 10^{10}	--- x 10^{10}
3.	1.423 x 10^{10}	.243 x 10^{10}	14	.138 x 10^{10}	--- x 10^{10}
4.	3.56 x 10^{10}	.605 x 10^{10}	35	.138 x 10^{10}	.276 x 10^{10}
5.	.780 x 10^{10}	.132 x 10^{10}	5	.132 x 10^{10}	--- x 10^{10}
6.	1.074 x 10^{10}	.182 x 10^{10}	7	.182 x 10^{10}	--- x 10^{10}
7.	1.96 x 10^{10}	.333 x 10^{10}	14	.190 x 10^{10}	--- x 10^{10}
8.	4.893 x 10^{10}	.830 x 10^{10}	35	.189 x 10^{10}	.379 x 10^{10}
9.	1.423 x 10^{10}	.243 x 10^{10}	5	.243 x 10^{10}	--- x 10^{10}
10.	1.960 x 10^{10}	.333 x 10^{10}	7	.333 x 10^{10}	--- x 10^{10}
11.	3.558 x 10^{10}	.586 x 10^{10}	14	.335 x 10^{10}	--- x 10^{10}
12.	8.89 x 10^{10}	1.51 x 10^{10}	35	.345 x 10^{10}	.690 x 10^{10}
13.	2.933 x 10^{10}	.499 x 10^{10}	7	.499 x 10^{10}	--- x 10^{10}

* 50% Stacking Efficiency.

The results of Tables 3 and 4 are obvious. The injection efficiency into the booster considerably reduces the available number of particles. It is fortunate this depletion is smallest for the heavier or slower ions. The fifth column in Tables 3 and 4 show the number of particles to be injected after 32 turns in the booster, assuming a 50% stacking efficiency. To obtain 32 turns with arrangement A, a minimum pulse of 200 μS is needed for ^{12}C, and a minimum pulse of 500 μS is needed for ^{197}Au.

Comparing Tables 2, 3 and 4 it seems that for this mode of operation, the Tandem could provide increased numbers of particles by increasing the pulse length. However the injection mechanism into the booster dilutes this advantage. For different pulse lengths, the final number of particles injected are often constant, for a given source current.

We note, while it is desirable to increase source current and injection efficiency for the heavier masses, we are near the space charge limit for ^{197}Au when the source current ~ 200 μA and the pulse length 110 μS!

On entry into the booster the ions acquire energy

W/A (^{12}C) = 1.737 GeV/A [fully stripped]
W/A (^{197}Au) = 321.4 MeV/A [charge 33+] (γ = 1.3453; β = .6689)

At this point we ignore ^{12}C because these ions are fully stripped already. At 321.4 MeV/A the ^{197}Au beam, on entering foil S_B, will result in .5 of the ions being fully stripped. In Table 5, the final number of particles reaching AGS are given, where once again the fourth column represents 50% stacking efficiency in the booster.

Thus to reach the required intensity of 2.2 x 10^9 ions/bunch, according to the CDR, we need a source current of at least 180 μA and a pulse length of 110 μsec. This corresponds to the case of 8 turns efficiently injected in one single rf bucket. If the booster has three rf buckets then 530 μA and 110 μsec are required. A future upgrade with twice the number of ions per bunch could be quite questionable with the present Tandem as the source, unless a h = 1 rf system is adopted for the Booster.

Table 5 - Final Particle Numbers into AGS. ^{197}Au

	Tandem Source	Tandem Pulse	No. injected into AGS after 8 turns in booster	No. injected into AGS after* 32 turns in booster
1.	80 μA	80 μs	$.048 \times 10^{10}$	$--- \times 10^{10}$
2.	80 μA	110 μs	$.062 \times 10^{10}$	$--- \times 10^{10}$
3.	80 μA	200 μs	$.062 \times 10^{10}$	$--- \times 10^{10}$
4.	80 μA	500 μs	$.062 \times 10^{10}$	$.138 \times 10^{10}$
5.	110 μA	80 μs	$.062 \times 10^{10}$	$--- \times 10^{10}$
6.	110 μA	110 μs	$.062 \times 10^{10}$	$--- \times 10^{10}$
7.	110 μA	200 μs	$.093 \times 10^{10}$	$--- \times 10^{10}$
8.	110 μA	500 μs	$.093 \times 10^{10}$	$.189 \times 10^{10}$
9.	200 μA	80 μs	$.121 \times 10^{10}$	$--- \times 10^{10}$
10.	200 μA	110 μs	$.166 \times 10^{10}$	$--- \times 10^{10}$
11.	200 μA	200 μs	$.166 \times 10^{10}$	$--- \times 10^{10}$
12.	200 μA	300 μs	$.166 \times 10^{10}$	$.345 \times 10^{10}$
13.	300 μA	110 μs	$.249 \times 10^{10}$	$--- \times 10^{10}$

* 50% Stacking Efficiency.

Section III - Arrangement B

The advantage of this system is that the Linac provides higher energy ions to the stripping foil S_F; thus increasing the maximum charge state that can be reached before injection into booster. It is because the input charge state into the Booster will be increased that the maximum energy attained by the Booster will be higher and thus stripping efficiency at S_B will also be higher.

The disadvantages are; a) there is a transmission loss on entering Linac? b) higher velocity on entering booster makes injection more inefficient.

Let us say $Am_o c^2 (\gamma_L - 1) = Am_o c^2 (\gamma_T - 1) + Q_T V_L$; where γ_L is γ factor for Linac, γ_T for Tandem, V_L accelerating voltage for Linac, and V_T accelerating voltage for Tandem.

$$\gamma_L = \gamma_T + \frac{Q_T V_L e}{m_o c^2 A}$$

$$\beta_L^2 \approx \beta_T^2 + \frac{2 Q_T V_L e}{m_o c^2 A}$$

But $$\beta_T^2 = \frac{2 Q_T V_T e}{m_o c^2 A} \quad \frac{(Q_T + 1)}{Q_T}$$ from which $$\beta_L^2 = \beta_T^2 \left[1 + \frac{V_L}{V_T} \frac{Q_T}{Q_T + 1} \right]$$

Here after transvering Linac New Space Charge Limit for Booster is:

$$N_B^{SC} = N_A^{SC} \left[1 + \frac{V_L}{V_T} \frac{Q_T}{Q_T + 1} \right] \left[\frac{Q}{Q_L} \right]^2$$

where Q is charge on entering booster in arrangement A and Q_L is charge for arrangement B. Obviously as V_L is increased, for a given foil thickness, Q_L increases also. Hence foil stripping knowledge is very important here.

Let us work out N_B^{SC} in Table 6, taking into account acceleration voltage and stripping foil final charge states. We assume S_F remains ~ .17 for charge states given here and concentrate on our worst case ^{197}Au.

Table 6 - Booster Space Charge. ^{197}Au ($\delta\nu$ = 0.3) Q_T=13 Q=33 V_T=15MV

Linac V_L	γ-Tandem & Linac	β-Tandem & Linac	Q_L from stripping Formula (Betz)	N_B^{SC}
0 MV	1.0011	.0478	~ 33	0.509×10^{10}
20 MV	1.0025	.0703	~ 37	0.906×10^{10}
40 MV	1.0039	.0881	~ 41	1.15×10^{10}
60 MV	1.0053	.1027	~ 42	1.48×10^{10}
80 MV	1.0067	.1155	~ 43*	1.75×10^{10}
100 MV	1.0082	.1269	~ 44*	2.06×10^{10}
200 MV	1.0152	.1726	~ 46*	3.51×10^{10}

* Unreliable extension of Betz formula.

Before we work out final particle numbers in Booster and AGS, let us look at final Booster energies and subsequent foil stripping efficiency in S_B from reference 7) for ^{197}Au.

Table 7 - Full Stripping Yield Efficiency in S_B. ^{197}Au

Linac V_L	Booster Final Energy MeV/A	Full Stripping Yield Efficiency in S_B
0 MV	321.4 MeV/A	~ .5
20 MV	391.4 MeV/A	~ .65
40 MV	465.4 MeV/A	~ .74
60 MV	484.4 MeV/A	~ .78
80 MV	503.66 MeV/A	~ .8 (max)
100 MV	523.06 MeV/A	~ .8 (max)
200 MV	562.42 MeV/A	~ .8 (max)

Before working out N_{AGS}, we also need the number of revolutions, for various pulse lengths.

Table 8 - Number of Revolutions in Booster. ^{197}Au

	80 μS	110 μS	200 μS	500 μS
20 MV	8	11	21	52
40 MV	10	14	26	65
60 MV	12	16	30	76
80 MV	13	19	34	86
100 MV	15	20	37	94
200 MV	20	28	51	128

Comparing Tables 6, 7 and 8 is interesting for the Linac increases dramatically the space-charge limits for the Booster, but the increased velocity coupled to the present stacking scheme decreases the number of particles that can be injected, relative to Tables 4 and 5. Overall, arrangement A is presently more efficient, but for the increased ion sources of the future,[2] this arrangement might well become optimal if the current is large enough. (See discussion in conclusion.)

Table 9 - Number of Particles Injected into Booster and AGS for Arrangement B. ^{197}Au

Label	Linac Voltage MV	No. Injected into Booster in 8 turns	No. Injected* into Booster in 32 turns	No. Injected into AGS after 8 turn from injection Booster	No. Injected* into AGS after 32 turn from injection Booster
1.	20	$.0487 \times 10^{10}$	--- $\times 10^{10}$	$.0318 \times 10^{10}$	--- $\times 10^{10}$
2.	20	$.0487 \times 10^{10}$	--- $\times 10^{10}$	$.0318 \times 10^{10}$	--- $\times 10^{10}$
3.	20	$.0487 \times 10^{10}$	--- $\times 10^{10}$	$.0318 \times 10^{10}$	--- $\times 10^{10}$
4.	20	$.0487 \times 10^{10}$	$.0930 \times 10^{10}$	$.0318 \times 10^{10}$	$.061 \times 10^{10}$
5.	20	$.0668 \times 10^{10}$	--- $\times 10^{10}$	$.0431 \times 10^{10}$	--- $\times 10^{10}$
6.	20	$.0662 \times 10^{10}$	--- $\times 10^{10}$	$.0431 \times 10^{10}$	--- $\times 10^{10}$
7.	20	$.0662 \times 10^{10}$	--- $\times 10^{10}$	$.0431 \times 10^{10}$	--- $\times 10^{10}$
8.	20	$.0662 \times 10^{10}$	$.128 \times 10^{10}$	$.0431 \times 10^{10}$	$.083 \times 10^{10}$
9.	20	$.122 \times 10^{10}$	--- $\times 10^{10}$	$.0792 \times 10^{10}$	--- $\times 10^{10}$
10.	20	$.122 \times 10^{10}$	--- $\times 10^{10}$	$.0792 \times 10^{10}$	--- $\times 10^{10}$
11.	20	$.122 \times 10^{10}$	--- $\times 10^{10}$	$.0792 \times 10^{10}$	--- $\times 10^{10}$
12.	20	$.122 \times 10^{10}$	$.232 \times 10^{10}$	$.0792 \times 10^{10}$	$.151 \times 10^{10}$
13.	20	$.181 \times 10^{10}$	--- $\times 10^{10}$	$.117 \times 10^{10}$	--- $\times 10^{10}$
1.	100	$.0268 \times 10^{10}$	--- $\times 10^{10}$	$.021 \times 10^{10}$	--- $\times 10^{10}$
2.	100	$.0268 \times 10^{10}$	--- $\times 10^{10}$	$.021 \times 10^{10}$	--- $\times 10^{10}$
3.	100	$.0268 \times 10^{10}$	$.0526 \times 10^{10}$	$.021 \times 10^{10}$	$.0421 \times 10^{10}$
4.	100	$.0268 \times 10^{10}$	$.0526 \times 10^{10}$	$.021 \times 10^{10}$	$.0421 \times 10^{10}$
5.	100	$.0356 \times 10^{10}$	--- $\times 10^{10}$	$.029 \times 10^{10}$	--- $\times 10^{10}$
6.	100	$.0362 \times 10^{10}$	--- $\times 10^{10}$	$.029 \times 10^{10}$	--- $\times 10^{10}$
7.	100	$.0362 \times 10^{10}$	$.0936 \times 10^{10}$	$.029 \times 10^{10}$	$.0574 \times 10^{10}$
8.	100	$.0362 \times 10^{10}$	$.0705 \times 10^{10}$	$.029 \times 10^{10}$	$.0574 \times 10^{10}$
9.	100	$.0624 \times 10^{10}$	--- $\times 10^{10}$	$.05 \times 10^{10}$	--- $\times 10^{10}$
10.	100	$.0624 \times 10^{10}$	--- $\times 10^{10}$	$.05 \times 10^{10}$	--- $\times 10^{10}$
11.	100	$.0624 \times 10^{10}$	$.124 \times 10^{10}$	$.05 \times 10^{10}$	$.1 \times 10^{10}$
12.	100	$.0624 \times 10^{10}$	$.124 \times 10^{10}$	$.05 \times 10^{10}$	$.1 \times 10^{10}$
13.	100	$.0712 \times 10^{10}$	--- $\times 10^{10}$	$.056 \times 10^{10}$	--- $\times 10^{10}$

* 50% Stacking Efficiency.

Section IV - Arrangement C

Here we have simply from (2), $Q_{eff} = Q$.

Where $Q = 5$ for ^{12}C
$= 13$ for ^{197}Au

In Table 10 we show the space-charge limits for arrangement C.

Table 10 - Space Charge Limits, No Linac or Foil S_F.

^{12}C $\delta\nu$	N^{sc}	^{197}Au $\delta\nu$	N^{sc}
.1	3.17×10^{10}	.1	1.09×10^{10}
.3	9.51×10^{10}	.3	3.27×10^{10}
.5	15.8×10^{10}	.5	5.45×10^{10}

Obviously this is larger than Table 2, and in fact because now $S_F = 1$ the number of particles reaching the booster is larger than Table 3 and 4 also. However, these particles now have reduced charge states. Tables 11 and 12 show the number of particles of charge state 5 and 13 reaching booster.

Table 11 - Number of ^{12}C Ions Reaching Booster.

	$N^T S^T T^T S^F$	No. of Revolutions	No. injected in 8 turns	No. injected in* 32 turns
1.	1.17×10^{10}	15	$.622 \times 10^{10}$	--- $\times 10^{10}$
2.	1.606×10^{10}	21	$.611 \times 10^{10}$	--- $\times 10^{10}$
3.	2.92×10^{10}	37	$.631 \times 10^{10}$	1.262×10^{10}
4.	7.30×10^{10}	93	$.627 \times 10^{10}$	1.256×10^{10}
5.	1.606×10^{10}	15	$.856 \times 10^{10}$	--- $\times 10^{10}$
6.	2.209×10^{10}	21	$.841 \times 10^{10}$	--- $\times 10^{10}$
7.	4.02×10^{10}	37	$.869 \times 10^{10}$	1.735×10^{10}
8.	10.0×10^{10}	93	$.859 \times 10^{10}$	1.735×10^{10}
9.	2.92×10^{10}	15	1.560×10^{10}	--- $\times 10^{10}$
10.	4.02×10^{10}	21	1.529×10^{10}	--- $\times 10^{10}$
11.	7.30×10^{10}	37	1.579×10^{10}	3.158×10^{10}
12.	18.26×10^{10}	93	1.573×10^{10}	3.141×10^{10}
13.	6.02×10^{10}	21	2.294×10^{10}	--- $\times 10^{10}$

* 50% Stacking Efficiency.

Table 12 - Number of ^{197}Au Ions Reaching Booster.

$N^T S^T T^T S^F$	No. of Revolutions	No. injected in 8 turns	No. injected in* 32 turns
1. .569 x 10^{10}	5	.569 x 10^{10}	--- x 10^{10}
2. .780 x 10^{10}	7	.780 x 10^{10}	--- x 10^{10}
3. 1.423 x 10^{10}	14	.813 x 10^{10}	--- x 10^{10}
4. 3.56 x 10^{10}	35	.813 x 10^{10}	1.626 x 10^{10}
5. .780 x 10^{10}	5	.780 x 10^{10}	--- x 10^{10}
6. 1.074 x 10^{10}	7	1.074 x 10^{10}	--- x 10^{10}
7. 1.96 x 10^{10}	14	1.136 x 10^{10}	--- x 10^{10}
8. 4.893 x 10^{10}	35	1.117 x 10^{10}	2.237 x 10^{10}
9. 1.423 x 10^{10}	5	1.423 x 10^{10}	--- x 10^{10}
10. 1.960 x 10^{10}	7	1.960 x 10^{10}	--- x 10^{10}
11. 3.558 x 10^{10}	14	2.03 x 10^{10}	--- x 10^{10}
12. 8.89 x 10^{10}	35	2.03 x 10^{10}	4.066 x 10^{10}
13. 2.933 x 10^{10}	7	2.93 x 10^{10}	--- x 10^{10}

* 50% Stacking Efficiency.

Let us concentrate on the partially stripped ^{197}Au. After the booster this ion is sent through stripping foil S_B with kinetic energy 56.76 MeV/A or β = .1116. At this energy (assuming Betz formula, which is now probably unreliable) a charge of 41 is attained. In Table 13 the number of charge 41 ^{197}Au ions reaching the AGS is shown.

Table 13 - Number of Charge 41 States Reaching AGS. ^{197}Au

	Number Injected into AGS after 8 turns in Booster	Number Injected into* AGS after 32 turns in Booster
1.	.0967 x 10^{10}	--- x 10^{10}
4.	.138 x 10^{10}	.276 x 10^{10}
5.	.131 x 10^{10}	--- x 10^{10}
8.	.187 x 10^{10}	.380 x 10^{10}
9.	.243 x 10^{10}	--- x 10^{10}
12.	.345 x 10^{10}	.686 x 10^{10}
13.	.498 x 10^{10}	--- x 10^{10}

* 50% Stacking Efficiency.

It is very interesting to note that for ^{12}C ions, arrangement C produces higher beam currents in the AGS than the present arrangement A. (Compare Table 3 and 11.) In particular the space-charge limit for C is higher than A, so higher source currents can be accommodated easily.

On a more theoretical note, the beam depletion rate for charge ~ 41 ^{197}Au ions in the AGS vacuum needs to be worked out. However, this calculation would emphasize our lack of knowledge of stripping or pick-up efficiencies in this energy range.

Section V - Arrangement D

In this case, the space charge limit at the Booster is simply given by,

$$N^{SC} = N_C^{SC} \left[1 + \frac{V_L}{V_T} \frac{Q_T}{Q_T + 1} \right] \tag{3}$$

Table 14 - Space Charge Limit for Arrangement D. ^{197}Au ($\delta\nu$=0.3) V_T=15MV Q_T=13

Linac V_L	N^{SC}
0 MV	3.27×10^{10}
20 MV	7.32×10^{10}
40 MV	11.4×10^{10}
60 MV	15.4×10^{10}
80 MV	19.5×10^{10}
100 MV	23.5×10^{10}
200 MV	43.8×10^{10}

This arrangement obviously has the highest space charge limit of all as shown in Table 14. With a final charge state of 13^+, the energy with booster alone is 56.76 MeV/A. Because of the velocity from Linac, the number of particles stored in this arrangement, for both the booster and the AGS, will be identical with Table 9, except in Table 9 the stripping foil efficiency (.17) of foil S_F was included. However, the final charge state in the AGS will be quite different.

Table 15 - Maximum Charge State of ^{197}Au Ions in the AGS.

Linac V_L	Energy of Booster & Linac	γ Total	β Total	Maximum Charge State in AGS
20 MV	59.09 MeV/A	1.0635	.340	46
40 MV	60.39 MeV/A	1.0649	.344	46
60 MV	61.69 MeV/A	1.0663	.347	47
80 MV	62.99 MeV/A	1.0677	.350	47
100 MV	64.39 MeV/A	1.0692	.354	47
200 MV	70.91 MeV/A	1.0762	.370	47

Section VI - Conclusions and Suggestions for Future (Focus on ^{197}Au)

1. The stated initial collider ^{197}Au intensities can be achieved with existing pre-collider devices if;

   An 180 μA minimum source combined with a 110 μS pulse can be accommodated in 8 turns in the booster. This appears to be a very realistic goal for both the Tandem and the booster (which is why it is in yellow book!)

2. Estimates of the space charge limit of the booster strongly suggest a larger number of ^{197}Au particles can be accommodated with the injection scheme A. The increased number of particles can be accommodated in two ways; (See Tables 2 and 4):

   a. Increase source current to 200 μA (pulse length 110 μS)
   b. Increase efficiency of storing particles in booster.

   Table 4 is very illuminating. The easiest way of increasing number of particles from Tandem is to increase pulse length. The injection efficiency into the booster dilutes this simple solution (Table 4). If 50% stacking efficiency can be obtained for 32 turns in booster then for arrangement A this can only be achieved for 500 μS pulse lengths, 500 μS and 200 μA pulses are near the space charge limit.

3. After the booster (for arrangement A in these notes) only 50% of accelerated particles will be fully stripped following stripping foil S_B. This problem introduces role for a Linac whose additional kinetic energy prior to foil S_F would allow 80% of fully stripped ^{197}Au to ions to enter AGS. However other problems effect results.

4. Table 6 clearly shows advantage of additional Linac (arrangement B of notes), with a Linac the space-charge limit of booster is increased one order of magnitude over existing arrangement. Table 9, however shows the disadvantages with existing Tandem source and stacking scheme for Booster. The loss of particles from bunching in Linac together with stripping foil characteristics show that, for the present 8 turn stacking scheme the particle numbers in booster are approximately a factor of 3 less for a 20 MV Linac and a factor of 5 less for a 100 MV Linac (over arrangement A). Even with the advantages of the extra stripping foil charge values, and hence extra energy in booster, the final number of fully stripped particles entering AGS is approximately a factor of 2 less than existing scheme. Table 9 clearly shows that with existing stacking scheme, a 20 MV Linac is more efficient at putting fully stripped ^{197}Au ions into AGS than a 100 MV Linac!!.

   The Linac does not work as things stand!

5. How to make use of increased space-charge limit when Linac is present?

   It does not appear reasonable that the stacking efficiency in the booster can be increased to absorb a large (~ 1 ms) pulse from Tandem source completely. Assuming that stacking in betatron phase space will dilute the possibility of using long pulse lengths to increases particle intensity, it makes sense to consider Linac in conjunction with completely new ion source only. Not merely 100 μA but several mA can be accommodated if a Linac is utilized.

A Linac and a new high intensity source go together and offer the possibility of developing ^{238}U beams as well? This development needs to be expanded in a separate manuscript.

6. Arrangements C and D are interesting for they focus on our lack of knowledge of stripper foil efficiencies at intermediate energies. For light heavy ions, i.e., ^{12}C, arrangement C is more efficient than our present arrangement A, for it allows for increased space-charge limits at the Booster. Of course, for light ions there is no problem of partially charged ions entering the AGS, because of stripping in foil S_B. Arrangement D, represents a further extension of this space-charge advantage.

7. Work to be done in the future

*a) A computer program needs to be written, incorporating all we know of Tandem, Linac, Booster, foils, etc. In particular a numerical study from Source to Booster stacking, including future design options is needed. This program needs to be updated in a continuous fashion as more knowledge becomes available and future needs are clearer. Eventually, this program will become part of a more general control program for RHIC. In this connection, Emittance growth in stripper foils[11)] also needs to be included, as well as beam depletion via the AGS vacuum.

b) For the near future (arrangement A of notes) the Tandem source should be increased to (200-250)μA of ^{197}Au. (Table 2 of this manuscript.)

*c) The "stacking method" in the booster is critically important for it dilutes the possibility of using long pulse lengths to increase intensity. The stacking efficiency in the booster needs to be attacked with theoretical and experimental methods, and related to recent and future injection schemes.

d) If a 50% stacking efficiency can be utilized in Booster for 32 turns, then a 500 μS pulse is needed. In this case the reliability of the Tandem for these long pulses is almost unknown; the foil lifetimes for long pulses also needs to be investigated.

*e) In view of the fact that it is so hard to fully strip ^{197}Au, it will be very easy to pick up electrons. The passage of fully stripped ^{197}Au from foil S_B to AGS needs to be investigated in detail. In addition the role of vacuum in AGS on fully stripped ^{197}Au needs to be investigated in full theoretical and experimental detail.

f) Let us consider U beams for both the present and future preinjection options. At present, the source of Uranium is only 10 nA.[4] Assuming charge states (arrangement A) through foils S_T and S_F, are the same as ^{197}Au, the kinetic energy after booster is 229.8 MeV/A. At this energy, published results[7] indicate zero U ions will be fully stripped and hence using source available now is useless. At this point a new Linac makes sense, but this Linac would have to allow U beams to be accelerated to ~ 830 MeV/A to achieve 80% full stripping. In addition, of course, a larger ion source is necessary and detailed stripping foil efficiencies are also required. A detailed report on the Uranium problem will be written shortly.

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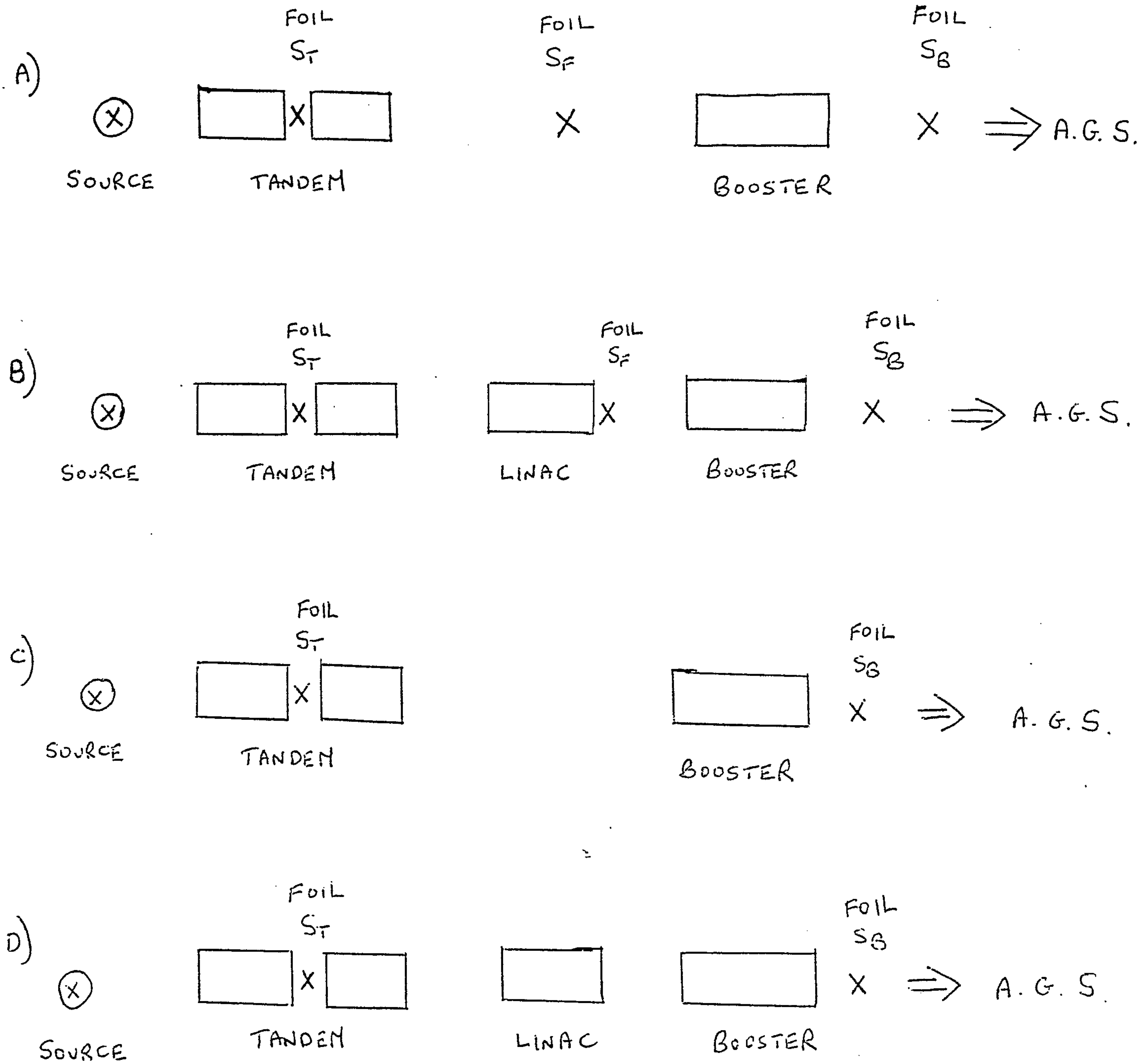
FIGURE 1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