BNL-101942-2014-TECH

AD/RHIC/30;BNL-101942-2013-IR

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December 1987

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U.S. Department of Energy

USDOE Office of Science (SC)

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AD/RHIC-30

RHIC TECHNICAL NOTE NO. 30

OPTIMIZATION OF MULTIWIRE COIL ENDS HAVING 45 DEGREE BENDS

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December 30, 1987

Optimization of Multiwire Coil Ends Having 45 Degree Bends

The current Multiwire process does not permit a change in direction of the wire other than 45 degree. The present paper answers the question of whether the bends in the flattened coil can be located along straight lines in such a way as to eliminate or reduce higher harmonics in the ends. The more general question of bends located along curves is not addressed.

Single-layer coils typically consist of a band of wires with constant spacing in the two-dimensional part of the coil or straight-section. Two such bands, with returns at each end, complete one pole of a 2m pole magnet. If when wrapped around a circular cylinder of radius r, each band of a pole subtends an angle of $\pi/3m$ radian at the axis of the cylinder, with r $\pi/3m$ of gap between the two bands of the pole, the coil will have zero third harmonic. The allowed harmonics n satisfy $n/m = k$, $k = 1,3,5, - - -$; the fundamental is $k = 1$ and the third harmonic is $k = 3$. For a dipole ($m = 1$), the third harmonic has $2n = 6$ poles, in a quadrupole ($m = 2$) the third harmonic has $2n = 12$ poles, etc. The nomenclature for n and m as given here is for consistency with reference 1; harmonics "b_i" more commonly used correspond to $i = n-1$.

Figure 1 shows half of a developed (flattened) end. The axes are z in the direction of the magnet axis and s; $z = 0$ is the end of the two dimensional part of the magnet. The band of conductors has width $a = s_o - s_i$ before bending at the first short-dashed line, width b after the first bend and width $c = z_0 - z_B$ after the second bend.It is found that

$$b/a = \sin(\pi/4)(1 + \tan(\alpha)) \qquad (1)$$

$$c/a = (1 + \tan(\alpha))/(1 + \cot(\beta)) \qquad (2)$$

Eqn(2) shows that if $\beta = \pi/2 - \alpha$, $c = a$, i.e., the spacing where the wires cross the pole ($s = 0$) is the same as on the side. If $\alpha = 22.5$ degree and $\beta = 67.5$ degree, $a = b = c$, that is, the end maintains constant wire spacing everwhere. Such an end for which $s_1 = s_2 = 0$ is half of a regular octagon.

In all that follows, it will be assumed that the point A lies on the $z = 0$ axis, i.e., that $s_1 = s_i$.

The harmonics can be computed using eqn(12) of ref. 1; for constant radius that equation becomes

$$q_n = \frac{4\ m\ Q_n}{n} \int_0^{\pi/2m} N(\theta_0)d\theta_0 \int_{C_{\theta_0}} \cos(n\ \theta)dz \qquad (3)$$

where q_n is 1/n of the n^{th} Taylor expansion coefficient of the integral of B_y, $Q_n = (\mu_0\ I/(2\ \pi))(r^n/R^{2n} + 1/r^n)$ and R is the iron radius. For a 2m pole magnet, the relation between s and θ is $s = (\pi/(2\ m) - \theta)r$, where r is the cylinder radius. Note that α and β of Fig. 1 are independent of m. For a winding with constant wire spacing, $N(\theta_0) = r/d$, where d is the spacing; if the band of wires ends at $\theta_0 = \theta_i$ as in Fig 1, eqn(3) becomes

$$q_n = - \frac{4\ m\ Q_n\ r}{n\ d} \int_0^{\theta_i} d\theta_0 \int_{C_{\theta_0}} \cos(n\ \theta)\ dz \tag{4}$$

The latter part of this double integral (with respect to z) has 3 parts: $0 \le z \le z_1$, $z_1 \le z \le z_2$ and $z = z_2$, where z_1 and z_2 lie on the first and second lines of bends as shown in Fig 1 for a typical wire beginning at θ_0. On the first part, $\theta = \theta_0$ and

$$z_1 = r(\theta_i - \theta_0)\tan\alpha \tag{5}$$

The second line of bends intersects the s axis at s_2 or θ_2, and has the equation

$$z = r(\theta_2 - \theta)\tan\beta \tag{6}$$

and on the middle segment, the typical wire has the equation

$$z = r(\theta - \theta_0(1 + \tan\alpha) + \theta_i \tan\alpha) \tag{7}$$

Combining (6) and (7) with the elimination of θ gives

$$z_2 = r \tan(\beta)(\theta_2 - \theta_0 + (\theta_i - \theta_0)\tan\alpha)/(1 + \tan\beta) \tag{8}$$

Equation (4) then becomes

$$q_n = - \frac{4\ m\ Q_n\ r}{n\ d} \int_0^{\theta_i} \left\{ \int_{z_1}^{z_2} \cos\{n[\frac{z}{r} + \theta_0(1 + \tan\alpha) - \theta_i \tan\alpha]\}dz + \int_0^{z_1} \cos(n\ \theta_0)dz \right\} d\theta_0 \tag{9}$$

where z_1 and z_2 are given by eqn (5) and (8), resp. Note that the third segment of the typical wire is independent of z and does not contribute to the integral. Eqn (9) can be evaluated in closed form; the result is

$$q_n = \frac{4\ m\ Q_n\ r^2}{n^3\ d} \left\{ (\cos(n\ \theta_i) - 1)(\tan(\alpha) - 1) + \left(\frac{1 + \tan\beta}{1 + \tan\alpha}\right) \left[\cos\left(\frac{n(\theta_i + \theta_2 \tan\beta)}{1 + \tan\beta}\right) - \cos\left(\frac{n(\theta_2 \tan\beta - \theta_i \tan\alpha)}{1 + \tan\beta}\right) \right] \right\} \tag{10}$$

There are three independent parameters in eqn (10): α, β, and θ_2. There are, however, constraints on them. Firstly, b and c of eqn (1) and (2) must be greater than or equal to a. Secondly, the distance between bends must be greater than about 3 d. It is intended to use 15 mil (bare) wire in the RHIC corrector, for which d = 24 mil. The distance between points A and B of Figure 1 is given by

$$\overline{AB} = r\ (\theta_2 - \theta_i)\ \sin(\beta)/\sin(3\pi/4 - \beta) \tag{11}$$

and the distance from the z axis to B is given by

$$s_B = r\,[\pi/2m - (\theta_2 \tan(\beta) + \theta_i)/(1 + \tan\beta)] \qquad (12)$$

The constraints are $\overline{AB} \geq 3\ d$ and $s_B \geq 1.5\ d$. Since $\theta_i = \pi/3m$, these constraints result in an end shape which changes with m, if the constraint is in force. Using these numbers, (11) and (12) can be rewritten

$$\theta_2 \geq \frac{\pi}{3\ m} + \frac{3\ d \sin(3\pi/4 - \beta)}{r \sin\beta} \qquad (13)$$

$$\theta_2 \leq \left(\frac{\pi}{2\ m} - \frac{1.5\ d}{r} - \frac{\pi/3m}{1 + \tan\beta}\right)(1 + \cot\beta) \qquad (14)$$

It is found by a numerical survey that infinitely many roots of $q_n = 0$ for the third harmonic (k = 3) exist. The same is not true for k = 3 and k = 5 simultaneously, and it is found that q_n for k = 5 is minimized by a configuration such that $s_B = 1.5$ d, i.e., the equality in eqn(14) holds, and such that a = c. Configurations for the quadrupole, octupole and decapole coils of the RHIC corrector which are in agreement with these findings have their parameters listed in Table 1.

Table 1

m	β	α	θ_i	θ_2	A_1	A_3	A_5
2	46.23	43.77	30.	57.39	-.8415	.0006	1.130
4	47.83	42.17	15.	27.16	-.8112	-.0007	1.475
5	49.04	40.96	12.	20.89	-.7895	-.0008	1.732

The quantities A_k, k = 1,3,5 are the angular dependent part of eqn (10), the part in curly brackets.

For comparison, the harmonics of the 2-D part of the winding can be computed using eqn(4), which for $\theta = \theta_0$ and z = 1, becomes

$$q_n = \frac{4\ m\ Q_n\ r^2}{n^3\ d} \{- \frac{n}{r} \sin(n\ \theta_i)\} \qquad (15)$$

The curly bracket term in eqn (15) is termed C_k; the coefficient of it is the same as in eqn(10) for easy comparison. Table 2 gives C_k for the coils given in Table 1.

Table 2

m	r,mm	C_1	C_3	C_5	$L_e = A_1/C_1$	A_5/L
2	51.69	-.0335	0.	.1675	25.11	.045
4	46.76	-.0741	0.	.3704	10.95	.135
5	42.21	-.1026	0.	.5129	7.70	.225

The quantity $L_e = A_1/C_1$ is the effective length of the end in mm, assuming iron over the ends the same as in the straight section. Since C_5 is the fifth harmonic per mm, it should be compared with A_5/L, the fifth harmonic per mm of effective length; in all cases, the ends are better than the straight section. This being the case, it may be worthwhile to standardize the various angles so that the ends will scale with m. If m θ_2

is held constant at, say, 100 degree (which increases s_B to 1.96d or 47 mil in the decapole), then β and α are independent of m and are found to be 50.45 degree and 39.55 degree, resp. Likewise the A_k are independent of m and are -.7653,-.0002 and 2.023, resp. for k = 1,3 and 5. Since the straight section is unchanged, only L_e and A_5/L_e of Table 2 are different; they are given in Table 3.

Table 3

m	L_e,mm	A_5/L_e
2	22.84	.089
4	10.33	.106
5	7.46	.271

The fifth harmonics per unit effective length are still only about 1/2 the 2-D values (C_5 of Table 2). An end having these parameters is sketched in Figure 2. From eqn(1), the wire spacing in the intermediate region is 29% greater than in the straight section, or 31 mil.

Although the ends above are optimized with respect to harmonics, they are not the shortest posible ends; that distinction is reserved for ends which obey the equality in eqn(13) ($\overline{AB}$ = 3 d) and have a = c. All such ends (for a given m) have the same physical length, regardless of α. How good are short ends? To discuss this, it is better to use as a measure of "goodness" the magnitude of the unwanted harmonics, viz. k = 3 and 5 at a reference radius. A typical reference radius is about 2/3 the coil radius; in the RHIC Corrector, the reference radius is x_0 = 25 mm. The magnitude is $M_k = n\ q_n\ x_0^{n-1}$. Two relative measures of quality suggest themselves: the first is the integrated ratio $RI_k = 2\ M_k(\text{end})/(L_s\ M_1(S))$ where $M_1(S)$ is the magnitude of the fundamental in the straight-section which has length L_s. The second is $RM_k = [M_k(\text{end})/L_e]/M_1(S)$ or the magnitude of the unwanted harmonic per unit effective length in the end divided by the magnitude of the fundamental in the straight-section. This second ratio is an indication of the size of the "bump" in the unwanted harmonics at the end. Table 4 gives these two ratios (times 10^4) for two short ends, α = 22.5 and α = 45 degree, and for the optimized ends.

Table 4
Field Magnitude Ratios times 10^4

type	m	α	L_e, mm	L_p, mm	RI_3	RI_5	RM_3	RM_5
short	2	45.0	16.2	28.4	11.3	0.	174.0	0.4
"	4	"	8.0	13.5	0.7	0.	22.7	0.
"	5	"	6.1	10.1	0.4	0.	16.8	0.
short	2	22.5	15.2	28.4	1.3	-.5	21.8	-7.8
"	4	"	7.4	13.5	-.1	0.	-3.3	-.1
"	5	"	5.6	10.1	-.1	0.	-6.3	-.1
opt.	2	39.55	22.8	37.0	0.	-.2	0.	-2.4
"	4	"	10.3	16.7	0.	0.	0.	0.
"	5	"	7.5	12.1	0.	0.	0.	0.

The physical length L_p is the maximum value for z_2, obtained when $\theta_0 = 0$ in eqn (8). The short end with $\alpha = 22.5$ (equal wire spacing everywhere) is probably acceptable, although there is a sizable bump in the 12 pole term of the quadrupole, RM_3; the optimized end is preferred.

The 2-d part of the RHIC Corrector dipole winding is not a single-layer 60 degree ($\pi/3$ radian) winding. It has 6 layers in 3, approximately equal pairs, with angles θ_i and radii as given in Table 5.

Table 5

layer no.	1	2	3	4	5	6
r, mm	60.41	61.10	61.79	62.47	63.16	63.84
θ_i	78.75	78.43	62.87	62.74	33.85	33.49

Optimization is most conveniently done by adjusting the straight-section lengths of two of the three double layers, similar to what is presently done with cable-wound dipoles. Since the Corrector dipoles do not occupy a large fraction of the ring, it is sufficient to minimize only the first unwanted harmonic, $k = n = 3$, but there is no reason not to optimize both. The three pairs of straight-sections will have incremental lengths at one end ℓ_i, $i = 1,2,3$, one of which will be zero, chosen so that the other two will be greater than zero. The set of linear equations to be solved for the ℓ_i is

$$[U]\ \vec{\ell} = -\vec{E} \tag{16}$$

where [U] is one of the three 2 x 2 subsets of a 2 x 3 matrix, an element of which is $U_{ik} = \Sigma M_k$ where the sum is of the 2 values of M_k in the i^{th} double layer and M_k is calculated using q_n given by eqn (15). This M_k will hereafter be termed MS_k. Likewise, $E_k = \Sigma M_k$, where the sum is over the 6 layers and M_k is calculated using q_n given by eqn (10); this M_k will be termed ME_k.

In principle, each of the three double layers could have it's own end configuration, but adequate designs can be obtained using the same configuration for all three pairs. By the "same configuration" is meant short end ($\overline{AB} = 3$ d), long end ($s_B = 1.5$d) or θ_2 equal to a fixed value. The setting up and solving of eqn (16) is done in a computer program "AUTOEND" which gives the two ℓ_i for each of the three solutions, and in addition calculates L_e and L_p for that solution which has both ℓ_i greater than or equal to zero. The effective length is now given by

$$L_e = \left[\sum_{i=1}^{6} (\ell_i\ MS_{1,i} + ME_{1,i}) \right] / \left[\sum_{i=1}^{6} MS_{1,i} \right] \tag{17}$$

The physical length L_p is now the maximum value of $z_2(\theta_0{=}0) + \ell_i$ for the six layers.

Figure 3 is the output generated by AUTOEND for four configurations; each configuration has five lines of print. The first of the five lines gives m, α and a parameter T2 which controls how θ_2 is generated: T2 = -1 generates short ends, T2 = 0 generates long ends and T2 > 0. is equal to a fixed value of θ_2. The next three lines give the calculated straight-section lengths ℓ_i, i = 1,2 or 3 which make the integrated harmonics k = 3 and 5 equal to zero (in each line, the missing ℓ_i is held at zero). The final line is the effective length and the physical length in meters. The surprising thing is that the listed value of α, selected (by cut and try) to minimize the positive lengths, results in a second layer also having zero length. Of the four cases, the shortest physical length results from the "long end" case, i.e. $s_B = 1.5d$ in all six layers. Figure 4 shows the developed inner layer of each pair of the three double layers for this case, which has an effective length of 62.7 mm and physical length of 99.4 mm.

The straight section lengths of the four harmonic coils in the RHIC Corrector are determined from the requirement that the effective length of each coil be 0.5 meter. This means $L_{ss} = 0.5 - 2\ L_e$ and the overall physical length is then $L_o = L_{ss} + 2\ L_p$. These lengths are given in Table 6.

Table 6

m	type	L_{ss}, m	L_o, m
1	long	0.3746	0.5733
2	opt.	0.4544	0.5284
4	opt.	0.4794	0.5128
5	opt.	0.4850	0.5092
2	short(α=22.5)	0.4696	0.5264
4	short(α=22.5)	0.4852	0.5122
5	short(α=22.5)	0.4888	0.5090

References

(1) F.E. Mills and G.H. Morgan, "A Flux Theorem for the Design of Magnet Coil Ends", Particle Accelerators 5, pp. 227 - 235 (1973)

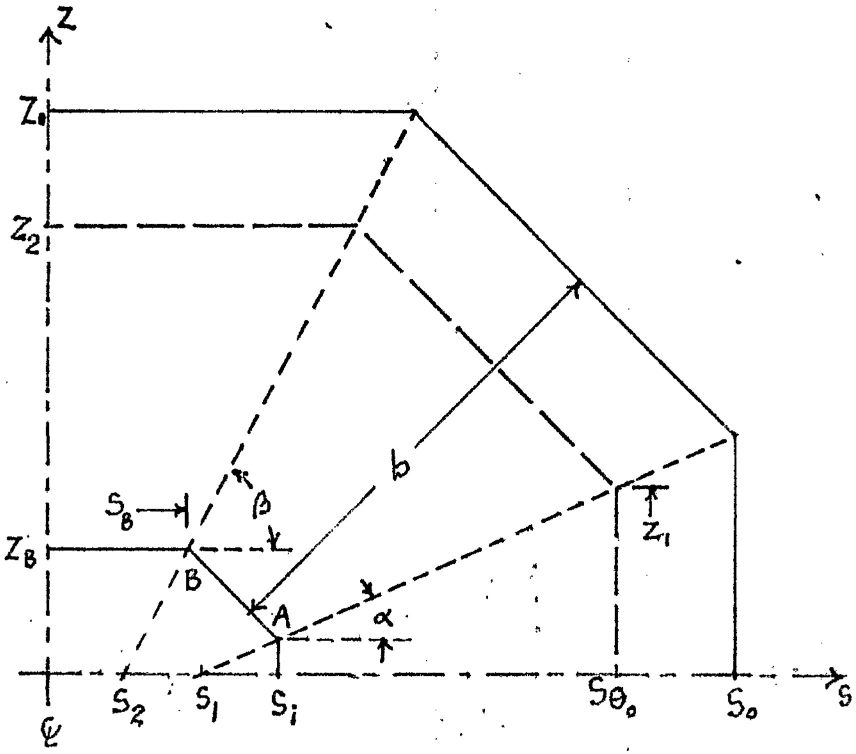

FIG. 1

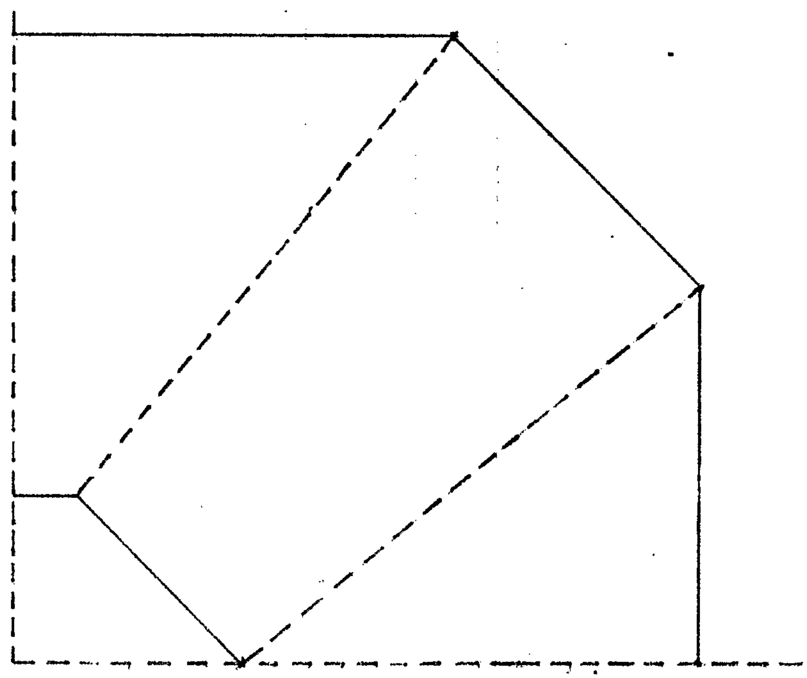

FIG. 2

```
M,A,T2 =  1   32.7145  -1.0000
L(1), L(2) =  0.06995  0.00000
L(1), L(3) =  0.06995  0.00000
L(2), L(3) = -0.07396 -0.06972
EFF LENGTH, PHYS LENGTH = 0.06632 0.15488
M,A,T2 =  1   44.9500   0.0000
L(1), L(2) =  0.00350  0.00000
L(1), L(3) =  0.00350  0.00000
L(2), L(3) = -0.00370 -0.00349
EFF LENGTH, PHYS LENGTH = 0.06271 0.09937
M,A,T2 =  1   43.7718  98.0000
L(1), L(2) =  0.06050  0.00000
L(1), L(3) =  0.06050  0.00000
L(2), L(3) = -0.06397 -0.06030
EFF LENGTH, PHYS LENGTH = 0.07965 0.15479
M,A,T2 =  1   42.2531  90.0000
L(1), L(2) =  0.07261  0.00000
L(1), L(3) =  0.07261  0.00000
L(2), L(3) = -0.07677 -0.07237
EFF LENGTH, PHYS LENGTH = 0.08105 0.16271
```

FIG. 3

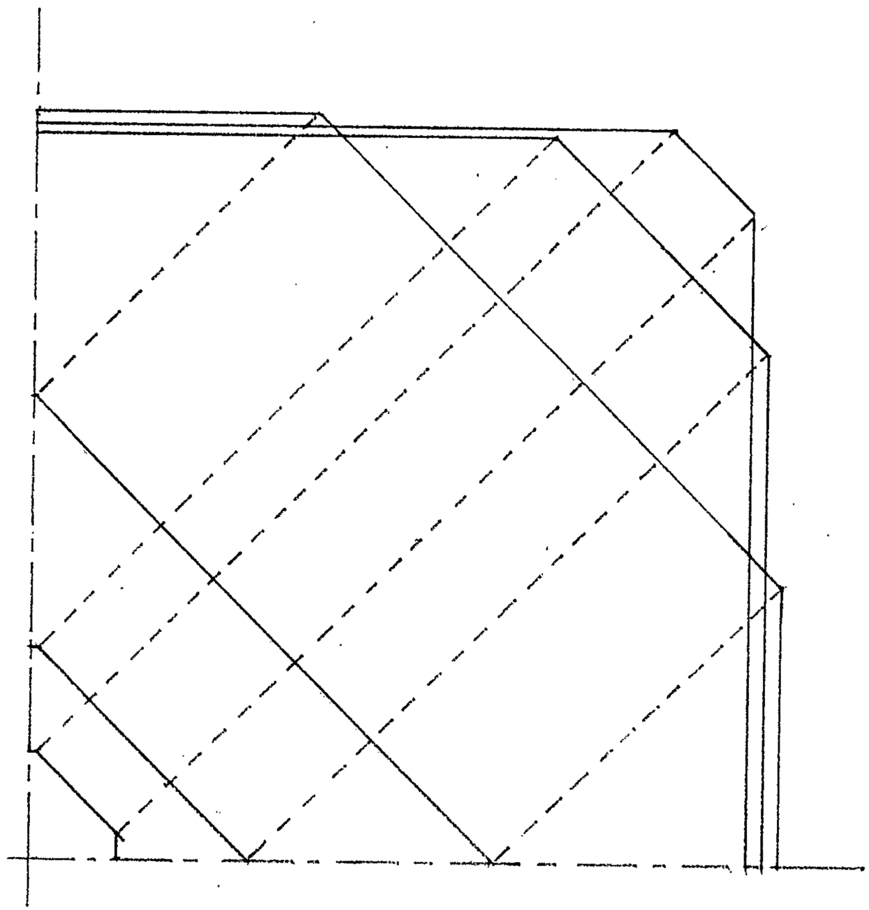

FIG. 4