# Optimization of Multiwire Coil Ends Having 45 Degree Bends 

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## Optimization of Multiwire Coil Ends Having 45 Degree Bends

The current Multiwire process does not permit a change in direction of the wire other than 45 degree. The present paper answers the question of whether the bends in the flattened coil can be located along straight lines in such a way as to eliminate or reduce higher harmonics in the ends. The more general question of bends located along curves is not addressed.

Single-layer coils typically consist of a band of wires with constant spacing in the two-dimensional part of the coil or straight-section. Two such bands, with returns at each end, complete one pole of a 2 m pole magnet. If when wrapped around a circular cylinder of radius $r$, each band of a pole subtends an angle of $\pi / 3 \mathrm{~m}$ radian at the axis of the cylinder, with $\mathrm{r} \pi / 3 \mathrm{~m}$ of gap between the two bands of the pole, the coil will have zero third harmonic. The allowed harmonics n satisfy $\mathrm{n} / \mathrm{m}=\mathrm{k}, \mathrm{k}=1,3,5$, - - ; the fundamental is $k=1$ and the third harmonic is $k=3$. For a dipole ( $m=1$ ), the third harmonic has $2 \mathrm{n}=6$ poles, in a quadrupole ( $\mathrm{m}=2$ ) the third harmonic has $2 \mathrm{n}=12$ poles, etc. The nomenclature for n and m as given here is for consistency with reference 1 ; harmonics " $b_{i}$ " more commonly used correspond to $\mathrm{i}=\mathrm{n}-1$.

Figure 1 shows half of a developed (flattened) end. The axes are $z$ in the direction of the magnet axis and $s ; z=0$ is the end of the two dimensional part of the magnet. The band of conductors has width a $=s_{o}-s_{i}$ before bending at the first short-dashed line, width $b$ after the first bend and width $c=z_{0}-z_{B}$ after the second bend. It is found that

$$
\begin{align*}
& \mathrm{b} / \mathrm{a}=\sin (\pi / 4)(1+\tan (\alpha))  \tag{1}\\
& \mathrm{c} / \mathrm{a}=(1+\tan (\alpha)) /(1+\cot (\beta) \tag{2}
\end{align*}
$$

Eqn(2) shows that if $\beta=\pi / 2-\alpha, c=a$, i.e., the spacing where the wires cross the pole ( $s=0$ ) is the same as on the side. If $\alpha=22.5$ degree and $\beta$ $=67.5$ degree, $a=b=c$, that is, the end maintains constant wire spacing everwhere. Such an end for which $s_{1}=s_{2}=0$ is half of a regular octagon.

In all that follows, it will be assumed that the point A lies on the $z=0$ axis, i.e., that $s_{1}=s_{i}$.

The harmonics can be computed using eqn(12) of ref. 1 ; for constant radius that equation becomes

$$
\begin{equation*}
\mathrm{q}_{\mathrm{n}}=\frac{4 \mathrm{~m} \mathrm{Q}_{\mathrm{n}}}{\mathrm{n}} \int_{0}^{\pi / 2 \mathrm{~m}} N\left(\theta_{0}\right) \mathrm{d} \theta_{0} \iint_{\theta_{0}} \cos (\mathrm{n} \theta) \mathrm{dz} \tag{3}
\end{equation*}
$$

where $q_{n}$ is $1 / n$ of the $n^{\text {th }}$ Taylor expansion coefficient of the integral of $B_{y}, Q_{n}=\left(\mu_{0} I /(2 \pi)\right)\left(r^{n} / R^{2 n}+I / r^{n}\right)$ and $R$ is the iron radius. For a $2 m$ pole magnet, the relation between $s$ and $\theta$ is $s=(\pi /(2 \mathrm{~m})-\theta) r$, where r is the cylinder radius. Note that $\alpha$ and $\beta$ of Fig. 1 are independent of m. For a winding with constant wire spacing, $N\left(\theta_{0}\right)=r / d$, where $d$ is the spacing; if the band of wires ends at $\theta_{0}=\theta_{i}$ as in Fig 1, eqn(3) becomes

$$
\begin{equation*}
q_{n}=-\frac{4 m Q_{n}^{r}}{n d} \int_{0}^{\theta_{\mathrm{i}}} \frac{\mathrm{i}}{d} \theta_{0} \int C_{\theta_{0}} \cos (n \theta) d z \tag{4}
\end{equation*}
$$

The latter part of this double integral ( with respect to $z$ ) has 3 parts: $0 \leq z \leq z_{1}, z_{1} \leq z \leq z_{2}$ and $z=z_{2}$, where $z_{1}$ and $z_{2}$ lie on the first and second lines of bends as shown in Fig 1 for a typical wire beginning at $\theta_{0}$. On the first part, $\theta=\theta_{0}$ and

$$
\begin{equation*}
z_{1}=r\left(\theta_{i}-\theta_{0}\right) \tan \alpha \tag{5}
\end{equation*}
$$

The second line of bends intersects the $s$ axis at $s_{2}$ or $\theta_{2}$, and has the equation

$$
\begin{equation*}
\mathrm{z}=\mathrm{r}\left(\theta_{2}-\theta\right) \tan \beta \tag{6}
\end{equation*}
$$

and on the middle segment, the typical wire has the equation

$$
\begin{equation*}
z=r\left(\theta-\theta_{0}(1+\tan \alpha)+\theta_{\mathbf{i}} \tan \alpha\right) \tag{7}
\end{equation*}
$$

Combining (6) and (7) with the elimination of $\theta$ gives

$$
\begin{equation*}
\mathrm{z}_{2}=\mathrm{r} \tan (\beta)\left(\theta_{2}-\theta_{0}+\left(\theta_{\mathbf{i}}-\theta_{0}\right) \tan \alpha\right) /(1+\tan \beta) \tag{8}
\end{equation*}
$$

Equation (4) then becomes

$$
\begin{gather*}
q_{n}=-\frac{4 m Q_{n}}{n d} \int_{0}^{\theta_{n}}\left\{\int_{z_{1}}^{z_{2}} \cos \left\{n\left[\frac{z}{r}+\theta_{0}(1+\tan \alpha)-\theta_{i} \tan \alpha\right]\right\} d z\right. \\
 \tag{9}\\
\left.+\int_{0}^{z_{1}} \cos \left(n \theta_{0}\right) d z\right\} d \theta_{0}
\end{gather*}
$$

where $z_{1}$ and $z_{2}$ are given by eqn (5) and (8), resp. Note that the third segment of the typical wire is independent of $z$ and does not contribute to the integral. Eqn (9) can be evaluated in closed form; the result is

$$
\begin{align*}
\mathrm{q}_{\mathrm{n}}= & \frac{4 \mathrm{~m}_{\mathrm{n}} \mathrm{r}^{2}}{\mathrm{n}^{3} \mathrm{~d}}\left\{\left(\cos \left(\mathrm{n} \theta_{\mathrm{i}}\right)-1\right)(\tan (\alpha)-1)+\left(\frac{1+\tan \beta}{1+\tan \alpha}\right)[ \right. \\
& \left.\left.\cos \left(\frac{\mathrm{n}\left(\theta_{\mathrm{i}}+\theta_{2} \tan \beta\right)}{1+\tan \beta}\right)-\cos \left(\frac{\mathrm{n}\left(\theta_{2} \tan \beta-\theta_{i} \tan \alpha\right)}{1+\tan \beta}\right)\right]\right\} \tag{10}
\end{align*}
$$

There are three independent parameters in eqn (10): $\alpha, \beta$, and $\theta_{2}$. There are, however, constraints on them. Firstly, b and c of eqn (1) and (2) must be greater than or equal to a. Secondly, the distance between bends must be greater than about 3 d . It is intended to use 15 mil (bare) wire in the RHIC corrector, for which $\mathrm{d}=24 \mathrm{mil}$. The distance between points A and B of Figure 1 is given by

$$
\begin{equation*}
\overline{\mathrm{AB}}=\mathrm{r}\left(\theta_{2}-\theta_{i}\right) \sin (\beta) / \sin (3 \pi / 4-\beta) \tag{11}
\end{equation*}
$$

and the distance from the z axis to B is given by

$$
\begin{equation*}
\mathrm{s}_{\mathrm{B}}=\mathrm{r}\left[\pi / 2 \mathrm{~m}-\left(\theta_{2} \tan (\beta)+\theta_{\mathbf{i}}\right) /(1+\tan \beta)\right] \tag{12}
\end{equation*}
$$

The constraints are $\overline{A B} \geq 3 \mathrm{~d}$ and $\mathrm{s}_{\mathrm{B}} \geq 1.5 \mathrm{~d}$. Since $\theta_{\mathrm{i}}=\pi / 3 \mathrm{~m}$, these constraints result in an end shape which changes with m, if the constraint is in force. Using these numbers, (11) and (12) can be rewritten

$$
\begin{align*}
& \theta_{2} \geq \frac{\pi}{3 \mathrm{~m}}+\frac{3 \mathrm{~d} \sin (3 \pi / 4-\beta)}{\mathrm{r} \sin \beta}  \tag{13}\\
& \theta_{2} \leq\left(\frac{\pi}{2 \mathrm{~m}}-\frac{1.5 \mathrm{~d}}{\mathrm{r}}-\frac{\pi / 3 \mathrm{~m}}{1+\tan \beta}\right)(1+\cot \beta) \tag{14}
\end{align*}
$$

It is found by a numerical survey that infinitely many roots of $q_{n}=0$ for the third harmonic ( $k=3$ ) exist. The same is not true for $k=3$ and $k$ $=5$ simultaneously, and it is found that $q_{n}$ for $k=5$ is minimized by a configuration such that $s_{B}=1.5 \mathrm{~d}$, i.e., the equality in eqn(14) holds, and such that $a=c$. Configurations for the quadrupole, octupole and decapole coils of the RHIC corrector which are in agreement with these findings have their parameters listed in Table 1.

Table 1

| m | $\beta$ | $\alpha$ | $\theta_{\mathrm{i}}$ | $\theta_{2}$ | $\mathrm{~A}_{1}$ | $\mathrm{~A}_{\mathbf{3}}$ | $\mathrm{A}_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 2 | 46.23 | 43.77 | 30. | 57.39 | -.8415 | .0006 | 1.130 |
| 4 | 47.83 | 42.17 | 15. | 27.16 | -.8112 | -.0007 | 1.475 |
| 5 | 49.04 | 40.96 | 12. | 20.89 | -.7895 | -.0008 | 1.732 |

The quantities $A_{k}, k=1,3,5$ are the angular dependent part of eqn (10), the part in curly brackets.

For comparison, the harmonics of the 2-D part of the winding can be computed using eqn(4), which for $\theta=\theta_{0}$ and $z=1$, becomes

$$
\begin{equation*}
q_{n}=\frac{4 m Q_{n} r^{2}}{n^{3} d}\left\{-\frac{n}{r} \sin \left(n \theta_{i}\right)\right\} \tag{15}
\end{equation*}
$$

The curly bracket term in eqn (15) is termed $C_{k}$; the coefficient of it is the same as in eqn(10) for easy comparison. Table 2 gives $C_{k}$ for the coils given in Table 1.

Table 2

|  |  |  |  | $\mathrm{L}_{\mathrm{e}}=$ |  |  |
| :--- | ---: | :---: | :---: | :---: | ---: | ---: |
| m | $\mathrm{r}, \mathrm{mm}$ | $\mathrm{C}_{1}$ | $\mathrm{C}_{3}$ | $\mathrm{C}_{5}$ | $\mathrm{~A}_{1} / \mathrm{C}_{1}$ | $\mathrm{~A}_{5} / \mathrm{L}$ |
| 2 | 51.69 | -.0335 | 0. | .1675 | 25.11 | .045 |
| 4 | 46.76 | -.0741 | 0. | .3704 | 10.95 | .135 |
| 5 | 42.21 | -.1026 | 0. | .5129 | 7.70 | .225 |

The quantity $L_{e}=A_{1} / C_{1}$ is the effective length of the end in mm, assuming iron over the ends the same as in the straight section. Since $C_{5}$ is the fifth harmonic per mm, it should be compared with $A_{5} / L$, the fifth harmonic per mm of effective length; in all cases, the ends are better than the straight section. This being the case, it may be worthwhile to standardize the various angles so that the ends will scale with m. If m $\theta_{2}$
is held constant at, say, 100 degree (which increases $s_{B}$ to 1.96 d or 47 mil in the decapole), then $\beta$ and $\alpha$ are independent of $m$ and are found to be 50.45 degree and 39.55 degree, resp. Likewise the $A_{k}$ are independent of $m$ and are $-.7653,-.0002$ and 2.023 , resp. for $k=1,3$ and 5. Since the straight section is unchanged, only $L_{e}$ and $A_{5} / L_{e}$ of Table 2 are different; they are given in Table 3.

Table 3

| m | $\mathrm{L}_{\mathrm{e}}, \mathrm{mm}$ | $\mathrm{A}_{5} / \mathrm{L}_{\mathrm{e}}$ |
| :---: | ---: | :--- |
|  |  |  |
| 2 | 22.84 | .089 |
| 4 | 10.33 | .106 |
| 5 | 7.46 | .271 |

The fifth harmonics per unit effective length are still only about $1 / 2$ the $2-\mathrm{D}$ values ( $\mathrm{C}_{5}$ of Table 2). An end having these parameters is sketched in Figure 2. From eqn(1), the wire spacing in the intermediate region is $29 \%$ greater than in the straight section, or 31 mil.

Although the ends above are optimized with respect to harmonics, they are not the shortest posible ends; that distinction is reserved for ends which obey the equality in eqn(13) ( $\overline{\mathrm{AB}}=3 \mathrm{~d}$ ) and have $a=c$. All such ends (for a given m) have the same physical length, regardless of $\alpha$. How good are short ends? To discuss this, it is better to use as a measure of "goodness" the magnitude of the unwanted harmonics, viz. $k=3$ and 5 at a reference radius. A typical reference radius is about $2 / 3$ the coil radius; in the RHIC Corrector, the reference radius is $x_{0}=25 \mathrm{~mm}$. The magnitude is $M_{k}=$ $\mathrm{n} \mathrm{q}_{\mathrm{n}} \mathrm{x}_{\mathrm{o}}{ }^{\mathrm{n}-1}$. Two relative measures of quality suggest themselves: the first is the integrated ratio $R I_{k}=2 M_{k}(e n d) /\left(L_{S} M_{1}(S)\right)$ where $M_{1}(S)$ is the magnitude of the fundamental in the straight-section which has length $L_{s}$. The second is $R M_{k}=\left[M_{k}\right.$ (end) $\left./ L_{e}\right] / M_{1}(S)$ or the magnitude of the unwanted harmonic per unit effective length in the end divided by the magnitude of the fundamental in the straight-section. This second ratio is an indication of the size of the "bump" in the unwanted harmonics at the end. Table 4 gives these two ratios (times $10^{4}$ ) for two short ends, $\alpha=22.5$ and $\alpha=45$ degree, and for the optimized ends.

Table 4
Field Magnitude Ratios times $10^{4}$

| type | m | $\alpha$ | $\mathrm{L}_{\mathrm{e}}, \mathrm{mm}$ | $\mathrm{L}_{\mathrm{p}}, \mathrm{mm}$ | $\mathrm{RI}_{3}$ | $\mathrm{RI}_{5}$ | $\mathrm{RM}_{3}$ | $\mathrm{RM}_{5}$ |
| :---: | :---: | :---: | ---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |
| short | 2 | 45.0 | 16.2 | 28.4 | 11.3 | 0. | 174.0 | 0.4 |
| $"$ | 4 | $"$ | 8.0 | 13.5 | 0.7 | 0. | 22.7 | 0. |
| $"$ | 5 | $"$ | 6.1 | 10.1 | 0.4 | 0. | 16.8 | 0. |
| short | 2 | 22.5 | 15.2 | 28.4 | 1.3 | -.5 | 21.8 | -7.8 |
| " | 4 | $"$ | 7.4 | 13.5 | -.1 | 0. | -3.3 | -.1 |
| $"$ | 5 | $"$ | 5.6 | 10.1 | -.1 | 0. | -6.3 | -.1 |
|  |  |  |  |  |  |  |  |  |
| opt. | 2 | 39.55 | 22.8 | 37.0 | 0. | -.2 | 0. | -2.4 |
| $"$ | 4 | $"$ | 10.3 | 16.7 | 0. | 0. | 0. | 0. |
| $"$ | 5 | $"$ | 7.5 | 12.1 | 0. | 0. | 0. | 0. |

The physical length $L_{p}$ is the maximum value for $z_{2}$, obtained when $\theta_{0}=0$ in eqn (8). The short end with $\alpha=22.5$ (equal wire spacing everywhere) is probably acceptable, although there is a sizable bump in the 12 pole term of the quadrupole, $\mathrm{RM}_{3}$; the optimized end is preferred.

The 2-d part of the RHIC Corrector dipole winding is not a single-layer 60 degree ( $\pi / 3$ radian) winding. It has 6 layers in 3 , approximately equal pairs, with angles $\theta_{i}$ and radii as given in Table 5 .

Table 5

| layer no. | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $r, m m$ | 60.41 | 61.10 | 61.79 | 62.47 | 63.16 | 63.84 |
| $\theta_{i}$ | 78.75 | 78.43 | 62.87 | 62.74 | 33.85 | 33.49 |

Optimization is most conveniently done by adjusting the straightsection lengths of two of the three double layers, similar to what is presently done with cable-wound dipoles. Since the Corrector dipoles do not occupy a large fraction of the ring, it is sufficient to minimize only the first unwanted harmonic, $k=n=3$, but there is no reason not to optimize both. The three pairs of straight-sections will have incremental lengths at one end $\ell_{i}$, $i=1,2,3$, one of which will be zero, chosen so that the other two will be greater than zero. The set of linear equations to be solved for the $\ell_{i}$ is

$$
\begin{equation*}
\text { [U] } \vec{\ell}=-\overrightarrow{\mathrm{E}} \tag{16}
\end{equation*}
$$

where [U] is one of the three $2 \times 2$ subsets of a $2 \times 3$ matrix, an element of which is $U_{i k}=\Sigma M_{k}$ where the sum is of the 2 values of $M_{k}$ in the $i{ }^{\text {th }}$ double layer and $\bar{M}_{k}$ is calculated using $q_{n}$ given by eqn (15). This $M_{k}$ will hereafter be termed $M S_{k}$. Likewise, $E_{k}=\Sigma M_{k}$, where the sum is over the 6 layers and $M_{k}$ is calculated using $\mathrm{q}_{\mathrm{n}}$ given by eqn (10); this $\mathrm{M}_{\mathrm{k}}$ will be termed $\mathrm{ME}_{\mathrm{k}}$.

In principle, each of the three double layers could have it's own end configuration, but adequate designs can be obtained using the same configuration for all three pairs. By the "same configuration" is meant short end $(\overline{A B}=3 \mathrm{~d})$, long end $\left(s_{B}=1.5 d\right)$ or $\theta_{2}$ equal to a fixed value. The setting $u p$ and solving of eqn (16) is done in a computer program "AUTOEND" which gives the two $\ell_{i}$ for each of the three solutions, and in addition calculates $L_{e}$ and $L_{p}$ for that solution which has both $l_{i}$ greater than or equal to zero. The effective length is now given by

$$
\begin{equation*}
\mathrm{L}_{\mathrm{e}}=\left[\sum_{i=1}^{6}\left(\ell_{\mathrm{i}} \mathrm{MS}_{1, i}+\mathrm{ME}_{1, i}\right)\right] /\left[\sum_{i=1}^{6} \mathrm{MS}_{1, i}\right] \tag{17}
\end{equation*}
$$

The physical length $L_{p}$ is now the maximum value of $z_{2}\left(\theta_{0}=0\right)+\ell_{i}$ for the six layers.

Figure 3 is the output generated by AUTOEND for four configurations; each configuration has five lines of print. The first of the five lines gives $\mathrm{m}, \alpha$ and a parameter T 2 which controls how $\theta_{2}$ is generated: $\mathrm{T} 2=-1$ generates short ends, $\mathrm{T} 2=0$ generates 10 ng ends and $\mathrm{T} 2>0$. is equal to a fixed value of $\theta_{2}$. The next three lines give the calculated straightsection lengths $\ell_{i}$, $i=1,2$ or 3 which make the integrated harmonics $k=3$ and 5 equal to zero (in each line, the missing $l_{i}$ is held at zero). The
final line is the effective length and the physical length in meters. The surprising thing is that the listed value of $\alpha$, selected (by cut and try) to minimize the positive lengths, results in a second layer also having zero length. Of the four cases, the shortest physical length results from the "long end" case, i.e. $s_{B}=1.5 \mathrm{~d}$ in all six layers. Figure 4 shows the developed inner layer of each pair of the three double layers for this case, which has an effective length of 62.7 mm and physical length of 99.4 mm.

The straight section lengths of the four harmonic coils in the RHIC Corrector are determined from the requirement that the effective length of each coil be 0.5 meter. This means $L_{s s}=0.5-2 L_{e}$ and the overall physical length is then $L_{o}=L_{s s}+2 \mathrm{~L}_{\mathrm{p}}$. These lengths are given in Table 6.

Table 6

| m | type | $\mathrm{L}_{\mathrm{SS}}, \mathrm{m}$ | $\mathrm{L}_{\mathrm{O}}, \mathrm{m}$ |
| :--- | :--- | :--- | :--- |
|  |  | 0.3746 | 0.5733 |
| 1 | long | 0.4544 | 0.5284 |
| 2 | opt. | 0.4794 | 0.5128 |
| 4 | opt. | 0.4850 | 0.5092 |
| 5 | opt. | 0.4696 | 0.5264 |
| 2 | short $(\alpha=22.5)$ | 0.4852 | 0.5122 |
| 4 | $\operatorname{short}(\alpha=22.5)$ | 0.4888 | 0.5090 |

## References

(1) F.E. Mills and G.H. Morgan, "A Flux Theorem for the Design of Magnet Coil Ends", Particle Accelerators 5, pp. 227 - 235 (1973)


```
M,A,T2 = 1 32.7145 -1.00.00
L(I), L(2) = 0.06995 0.00000
L(I), L(3) = 0.06995 0.00000
L(2), L(3) = -0.07396 -0.06972
EFF LENGTH, PHYS LENGTH = 0.06632 0.15488
M,A,T2 = 1 44.9500 0.0000
L(I), L(2) = 0.00350 0.00000
L(I), L(3) = 0.00350 0.00000
L(2), L(3) = -0.00370 -0.00349
EFF LENGTH, PHYS LENGTH = 0.06271 0.09937
M,A,T2 = 1 43.7718 98.0000
L(1),L(2) = 0.06050 0.00000
L(1), L(3) = 0.06050 0.00000
L(2), L(3) = -0.06397-0.06030
EFF LENGTH, PHYS LENGTH = 0.07965 0.15479
M,A,T2 = 1 42.2531 90.0000
L(1), L(2) = 0.07261 0.00000
L(1), L(3) = 0.07261 0.00000
L(2), L(3) = -0.07677-0.07237
EFF LENGTH, PHYS LENGTH = 0.08105 0.16271
```

FIG. 3


FIG. 4


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