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RHIC Arc Magnet Field Quality Tolerances

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Introduction

Field errors in the ring magnets will have detrimental effects on the operation and performance of RHIC. Establishing field quality tolerance criteria is obviously of paramount importance. Furthermore, if the field quality requirements cannot be achieved, an appropriate correction magnet system must be provided. Tolerances on linear i.e., dipole and quadrupole errors can be derived by standard methods and are not considered here. The goal of this note is rather to establish tolerances on the higher harmonics in the RHIC arc magnets.

The summary approach to designing superconducting magnets with adequate field quality consists in assuming that 2/3 of the coil aperture has "good field" and then to adopt an aperture which accommodates the largest beam size encountered. In RHIC, the required good field aperture for gold beams at 30 GeV/nucleon after 10 hours is ± 25.6 mm (when calculated at QF as $X_p \times \Delta p/p + 6\sigma_\beta$) and a coil aperture of 80 mm was adopted. Once the aperture of the magnets is fixed, it becomes possible to establish field quality tolerances.

The present note will attempt to set quantitative tolerances on the field quality of the arc dipoles and quadrupoles. The use of field harmonics is standard in superconducting magnets and tolerances will be derived for individual harmonics, both systematic and random. A comparison with expectations for RHIC magnets will lead to a better definition of the correction coil system.

Nonlinear magnet errors lead to amplitude and momentum-dependent tune shifts. These effects are dominated by systematic errors since their contributions add coherently around the ring. The operating tune range in

RHIC is limited to $\Delta\nu = 0.033$ which is free of tenth-order resonances. A criterion on amplitude and momentum dependent tune shifts is proposed which requires $\Delta\nu \lesssim 3\times10^{-3}$ (i.e., about one order of magnitude smaller than the tune range) resulting from individual systematic harmonics in the arc magnets.

Random and systematic field errors determine the dynamic aperture of the magnets, defined as the domain of initial conditions within which particle motion is stable. In order to assure adequate safety margin for storage ring operation, the criterion adopted for RHIC states that the dynamic aperture be large enough to contain betatron oscillations with amplitude as large as $6\sigma_\beta$.

The largest aperture requirement exists for gold-on-gold operation at 30 GeV/nucleon. In this energy range superconductor magnetization and iron saturation are small and the dynamic aperture is limited by nonlinear random errors. The simultaneous presence of all field harmonics is best analyzed by particle tracking. This aspect is not treated in the present note, but it can be stated that the "expected" rms errors quoted in the conceptual design report are adequate.

Proper operation of the accelerator is assured if the ensemble of all magnets satisfies these field quality criteria. Based on these ensemble tolerances for the field harmonics, a simple "global" criterion for acceptance or rejection of an <u>individual</u> magnet is also suggested.

II. The tune shift criterion.

Tolerances on systematic errors can be established by using the tuneshift criterion $\Delta\nu \lesssim 3\times10^{-3}$. This approach was taken in the SSC Conceptual Design Report[1] and is based on limiting the tune shift due to an error b_n of a particle with betatron amplitude A_β and momentum offset $\Delta p/p$. Assuming a ring made solely of regular cells, an approximate expression for the tune shift was derived in the smooth approximation, where β and X_p take on their average cell values, in the form [2-3]

$$\Delta\nu_n = b_n \langle\beta\rangle \sum_i \frac{1}{4^{i+i}} C^n_{2i+1} \; C^{2i+2}_{i+1} \; A_\beta^{2i} \; A_\delta^{n-2i-1}$$

$$\text{with } C^i_j = \frac{i!}{j!\,(i-j)!}$$

$$A_\delta = \langle X_p\rangle \,\Delta p/p$$

and the summation index ranging from $i = 0$ to integer $i = (n-1)/2$. Explicit expressions for the tune shifts are listed in Table I.[1,4]

The tuneshift due to quadrupole harmonics can be estimated by taking into account the length ratio of quadrupole to dipole.

Table I. Tune shift expressions

n	$\Delta\nu_n$
1	$b_1\langle\beta\rangle/2$
2	$b_2\langle\beta\rangle\ A_\delta$
3	$b_3\langle\beta\rangle\ (\frac{3}{2}\ A_\delta^2 + \frac{3}{8}\ A_\beta^2)$
4	$b_4\langle\beta\rangle\ (2\ A_\delta^2 + \frac{3}{2}\ A_\beta^2)\ A_\delta$
5	$b_5\langle\beta\rangle\ (\frac{5}{2}\ A_\delta^4 + \frac{15}{4}\ A_\delta^2\ A_\beta^2 + \frac{5}{16}\ A_\beta^4)$
6	$b_6\langle\beta\rangle\ (3\ A_\delta^4 + \frac{15}{2}\ A_\delta^2\ A_\beta^2 + \frac{15}{8}\ A_\beta^4)\ A_\delta$
7	$b_7\langle\beta\rangle\ (7\ A_\delta^2 + \frac{105}{8}\ A_\delta^4\ A_\beta^2 + \frac{105}{16}\ A_\delta^2\ A_\beta^4 + \frac{35}{128}\ A_\beta^6)$
8	$b_8\langle\beta\rangle\ (4\ A_\delta^6 + 21\ A_\delta^4\ A_\beta^2 + \frac{35}{2}\ A_\delta^2\ A_\beta^4 + \frac{35}{16}\ A_\beta^6)\ A_\delta$
9	$b_9\langle\beta\rangle\ (\frac{9}{2}\ A_\delta^8 + \frac{63}{2}\ A_\delta^6\ A_\beta^2 + \frac{315}{8}\ A_\delta^4\ A_\beta^4 + \frac{315}{32}\ A_\delta^2\ A_\beta^6 + \frac{63}{256}\ A_\beta^8)$
10	$b_{10}\langle\beta\rangle\ (5\ A_\delta^8 + 45\ A_\beta^6\ A_\beta^2 + \frac{315}{4}\ A_\delta^4\ A_\beta^4 + \frac{525}{16}\ A_\delta^2\ A_\beta^6 + \frac{315}{128}\ A_\beta^8)\ A_\delta$

III. Systematic magnet tolerances

Using the tune shift formulas given above, tolerances on systematic harmonics were calculated and are listed in Table II. The tolerances depend obviously on the operating conditions.

Expected systematic errors are also shown in Table II. Systematic errors are caused, in addition to the coil cross section design, by superconductor magnetization at low fields and iron saturation at high fields. Random field harmonics due to the coil fabrication will have a bias as is observed with the FNAL Tevatron magnets.[5,6] One must expect that the systematic errors, i.e., the average of all random errors, will be about 30% of the rms errors and that they can exist in the non-allowed harmonics. In fact, it is here assumed that the theoretical coil design of a 5-block dipole and 3-block quadrupole is sufficiently flexible to make $b_4' = b_6' = b_8' = b_{10}' = 0$ in the dipoles and $b_5' = b_9' = 0$ in the quadrupoles. Estimated systematic harmonics shown in Table II for the mid field range (~30 GeV/amu) are solely due to fabrication errors.

Comparing tolerances with expected errors indicates that colliding Au-Au beam operation at 30 GeV/nucleon represents the most critical situation (note that degraded colliding beam operation at lower energies is acceptable). The momentum and betatron aperture requirements are largest at 30 GeV/nucleon and consequently the tolerances on magnetic field quality are here most severe. Injection and top energy operation at design intensity should not present tolerance problems provided that b_2, b_3 and b_4 correction coils in the arcs are available. In fact, the magnet aperture would seem adequate to allow a 5-fold increase of the beam intensity.[7]

Table II. Systematic b_n Tolerances

Ion	Au		Au		Au		Au		p	
β^* (m)	6		6		3		3		3	
E_{kin} (GeV/amu)	11		30		100		100		300	
N_B	5.5×10^9 5×Design		1.1×10^9		1.1×10^9		5.5×10^9 5×Design		8×10^{11} 8×Design	
$A_\beta=6\sigma_\beta$ (mm)	16		18		9.6		12.2		5.6	
$\Delta p/p$ (%)	±0.36		±0.5		±0.26		±0.36		±0.13	
	Tol.*	Exp.	Tol.	Exp.	Tol.	Exp.	Tol.	Exp.	Tol.	Exp.
Dipole										
b'_2	0.17	−3	0.13	±1.4†	0.25	6.5	0.18	6.5	0.48	28
b'_3	0.14		0.10	±0.4	0.35		0.21		1.1	
b'_4	0.27	<1	0.15	±0.6	1.0	−4.7	0.44	−4.7	6.0	−8.8
b'_5	0.29		0.16	±0.1	1.9		0.67		19	
b'_6	0.46		0.19	±0.2	4.5	−0.5	1.2	−0.5	85	−1.3
b'_7	0.57		0.21	±0.1	9.4		2.0		300	
b'_8	0.81		0.25	±0.1			3.3			
b'_9	1.0		0.28	±0.1						
b'_{10}	1.4		0.32	±0.0						
Quadrupole										
b'_5	2.2	−0.2	1.2	±0.3	15	+0.6	5.1	±0.6	145	+2
b'_9	8.0	<0.1	2.1	±0.1	340	<0.1	42	<0.1		

*Tol. = Tolerance. Exp. = Expected field harmonics
†Range of expected systematic construction errors

IV. Random magnet tolerances

The tune shift criterion can also be used to obtain tolerances on random magnet errors, which are larger by a factor $\sqrt{N}$, with N the number of magnets in the ring. In RHIC

$$\sqrt{N} = 12$$

leading to the tolerances on individual harmonics shown in Table III. Operation at 30 GeV/nucleon is most sensitive to random errors and tolerances are obtained for this case. Further investigations are necessary to decide if the expected sextupole error (b_2) is acceptable or if shuffling as was done for the FNAL Tevatron is required.[8,9]

In the production phase of the magnets, it will be useful to have a single "global" acceptance criterion instead of a multitude of tolerances. A simple yet meaningful measure of the nonlinear random errors (b_2 and higher) in an individual magnet can be defined as[10]

$$\delta_{D,V} = \{ \sum(\Delta b_n')^2\}^{1/2}$$

$$\delta_{D,H} = \{ \sum_n(a_n')^2\}^{1/2}$$

Measurements of the field coefficients must be taken in the absence of magnetization and saturation effects, i.e., at about 1T which corresponds to ~30 GeV/nucleon. The field coefficients a_n and Δb_n represent integral values, i.e., the ends must be included. The terms which have a built-in value, and for which a systematic trim coil exists, such as b_2 and b_4 in dipoles, have to be analyzed by taking $\Delta b_n = b_n - b_{n,average}$. For the others $\Delta b_n = b_n$ with b_n the actually measured value. The expected rms value of these quantities is

Table III. Random b_n Tolerances (rms)

Harmonic n	Tolerance b_n	Expected b_n, a_n
Dipole		
2	1.6	4.6, 1.3
3	1.2	1.3, 2.2
4	1.8	2.2, 0.6
5	1.9	0.5, 0.9
6	2.3	0.8, 0.2
7	2.5	0.2, 0.3
8	3	0.3, 0.1
9	3.4	0.1, 0.
10	3.8	
Quadrupole		
5	14	1.2
9	25	0.3

$$\epsilon_{D,V} = 5.3\times10^{-4}$$

$$\epsilon_{D,H} = 2.8\times10^{-4}$$

assuming Gaussian distribution of the magnet errors and allowing a 5% rejection rate, one finds the acceptance criterion for the individual magnet

$$\delta_{D,V} \leq 10^{-3}$$

$$\delta_{D,H} \leq 5\times10^{-4}$$

Magnets which do not conform should be returned for rework.

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