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Particle Tracking on the BNL Relativistic Heavy Ion Collider

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Summary

Tracking studies including the effects of random multipole errors as well as the effects of random and systematic multipole errors have been made for RHIC. Averages of aperture determinations for ten independent sets of random multipole errors are consistent with there being little or no degradation of the dynamic aperture when systematic multipole errors are included. Coupling between the horizontal and vertical betatron motion enlarges the beam but does not cause instability. Initial results for operating at an off diagonal working point are discussed.

Introduction

RHIC, the Relativistic Heavy Ion Collider being designed at the Brookhaven National Laboratory in the USA is a storage ring collider for heavy ions having energies ranging from 30 to 100 GeV/amu. Specifications for RHIC require full performance for the species Au^{79+} with relaxed performance for more massive ions as well as at energies less than 30 GeV/amu for all ions. The design of this accelerator has been described elsewhere (1). Briefly, RHIC has a circumference of 3.833 km and has two independent storage rings that use superconducting magnets. The ion beams interact at six crossing points at which the beta functions can be varied from 3 to 6 meters. The time averaged luminosity is 4.4×10^{26} cm^{-2}/sec for Au on Au at 100 GeV/amu.

Intrabeam scattering is pronounced in RHIC and causes beam growth. A conservative estimate indicates that $\pm 6\sigma$ of aperture is needed if a beam lifetime of ten hours is to be achieved. This requirement is most severe at 30 GeV/amu where σ = 3mm and sets the requirement at a minimum half aperture of 18 mm for betatron motion. At higher energies the beam size decreases, and at lower energies the specification for a ten hour lifetime is relaxed.

Procedure

The magnetic fields of the superconducting magnets have higher order components that impact on the stability of the particle motion. In this paper the results of tracking with PATRICIA(2) are reported when the effects of systematic and random field multipoles are included. The effects of higher order field components are introduced as thin lens kicks associated with each magnet element. The magnetic field is represented as :

$$B = B_0 + \Delta B = B_0(1 + \sum_n c_n r^n)$$

where: $c_n = b_n + ia_n$ with b_n and a_n being the normal and skew multipoles,
$r = x + iz$, and
n = multipole order. (n = 1,2,.. denotes quadrupole, sextupole,...

The kick r' given to a particle is $r' = \Delta B \cdot \ell / B_0 \rho$ with ℓ, B_0, and ρ being the element length, central bending field, and radius of curvature, respectively. The kick for the dipoles is divided equally and placed at each end of the dipole; the kick associated with a quadrupole is placed at the center of the quadrupole. At each element the kick r' is evaluated, and the particle coordinates are changed appropriately: $x = x$, $z = z$, $x' = x' + Re(r')$, and $z' = z' + Im(r')$.

In the simulations, rms values of the random multipole errors of Table I are used to generate a set of multipole errors for each magnet in the lattice. The random multipole errors are selected from a Gaussian distribution that is truncated at $\pm 3\sigma$. (RHIC magnets having larger deviations from the average will be rejected and reworked).

Most studies have been made with a test particle having equal emittances in the horizontal and vertical planes and with the initial conditions $x \neq 0$, $z \neq 0$, $x' = z' = 0$. Typical runs were made for 1000 traversals of the lattice. The amplitude of the test particle was increased in fixed steps until an amplitude was reached where the particle did not survive the specified number of traversals. To reduce spurious results due to isolated regions of stability, the amplitude step was small (~0.36 mm in a focusing quadrupole of the arc), and the required number of traversals of the lattice was large (1000). The maximum initial betatron amplitude for which particle motion is stable was determined when there were no limits imposed by the dimensions of the vacuum chamber--dynamic aperture.

Studies

Effects of random multipoles

The dynamic aperture with sextupoles to correct chromaticity to zero but with no multipole errors is ~47 mm at $\Delta P/P = 0\%$ when two sextupole families are used. The dynamic aperture when random multipoles are present has been reported in an earlier work(3). In those studies, the amplitude of a test particle having the initial coordinates mentioned above and with equal emittances $\varepsilon_x = \varepsilon_z$ was gradually increased to determine the maximum stable initial amplitude. This procedure was repeated for all ten sets of random multipole errors. The average of these determinations was considered to be the dynamic aperture. The dependence of this dynamic aperture on the momentum deviation $\Delta P/P$ is shown in Fig. 1. Random multipole errors are attributed to construction errors; thus these results are independent of the beam energy.

Effects of systematic multipoles

Although RHIC is principally a collider for operation at energies between 30 and 100 GeV/amu, consideration is being given to operation as a collider below 30 GeV/amu and to internal target operation at energies as low as 7 GeV/amu. Systematic multipoles due to magnetization effects in the superconductor and saturation in the iron change with magnet excitation. These multipoles are listed in Table I in the columns labelled "7" and "100". It is seen that the magnitude of the multipoles changes with excitation and that the sign of b_2 in the dipoles and b_5 in the quadrupoles changes between operation at 7 and 100 GeV/amu.

The impact of these systematic multipoles on the dynamic aperture has been determined by tracking. The study has been made in the presence of random multipoles with the chromaticity set to zero by two families of sextupoles in the arc cells. The procedure is the

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	DIPOLES				QUADRUPOLES		
	Systematic		Random		Systematic		Random
n	7	100	σb_n	σa_n	7	100	$\sigma b_n=\sigma a_n$
1			8.30E-3	1.68E-2	-8.00E+2	-8.00E+3	1.60E-2
2	-1.60	1.04	7.36E-1	2.08E-1			5.76E-1
3			8.32E+0	1.41E+1			1.47E+1
4	2.56E+2	1.20E+3	5.63E+2	1.54E+2			4.35E+2
5			5.12E+3	9.22E+3	-2.05E+3	6.14E+3	1.23E+4
6	0.0	2.05E+5	3.28E+5	8.19E+4			3.28E+5
7			3.28E+6	4.92E+6			9.83E+6
8			1.97E+8	6.55E+7			2.62E+8
9			2.62E+9	0.0	0.0	2.62E+9	7.86E+9

Table I. Magnet multipoles in units of m^{-n} for *RHIC* dipoles quadrupoles.

same as that described for the study made with random multipoles with the exception that the complex multipole coefficient $c_n = b_n(\text{systematic}) + b_n(\text{random}) + i\, a_n(\text{random})$. As before, the quadrupole term (n=1) has been assumed to be zero.

Studies were made to establish the influence of the systematic multipoles at 7 and 100 GeV/amu. As before the aperture was determined for ten sets of random multipoles to simulate ten possible machines. The results were averaged; the dynamic aperture thus obtained, as well as its rms deviation, were determined at three momenta: $\Delta P/P = 0$ and $\pm$ 0.5%. The results are also plotted in Fig. 1. The average values of the dynamic aperture with and without systematic multipoles agree rather well. The value for the 100 GeV/amu multipoles is low at $\Delta P/P = -0.5\%$. Extension of the tracking studies to $\Delta P/P = \pm 1\%$ reveals a nonlinear chromaticity that causes the tune to leave the region desired for RHIC operation. The tune dependence on momentum is more pronounced for the 100 GeV/amu multipoles than for the 7 GeV/amu multipoles, and this dependence is thought to be responsible for the low value of the dynamic aperture at $\Delta P/P = -0.5\%$ when the 100 GeV/amu multipoles are present. Tracking at $\Delta P/P = \pm 1\%$ has been postponed until the tunes at these momenta have been corrected.

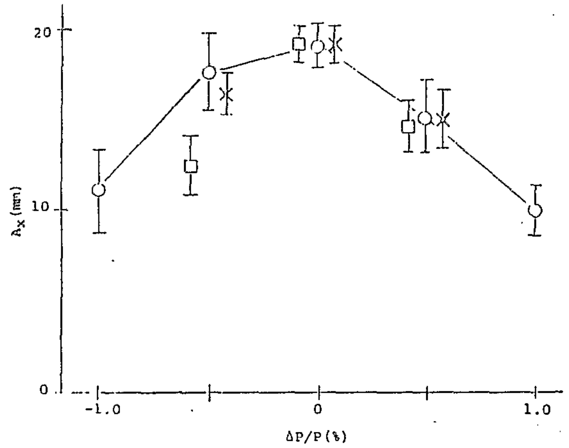

Fig. 1 Dynamic aperture of RHIC in the presence of random and systematic multipoles. Initial conditions: $\varepsilon_{0x} = \varepsilon_{0y}$, $\beta_x = 50.1\text{m}$, $\beta_y = 8.5\text{m}$, and $A_x = \text{SQRT}(\varepsilon_x\beta_x)$. Values are averages for ten independent sets of random multipoles. Error bars indicate rms deviations from the average.   denotes random multipoles only, X includes systematic multipoles for 7 GeV/amu, and   includes systematic multipoles for 100 GeV/amu.

Coupling

The nominal operating point (Q_x,Q_x) is (28.823, 28.826) and lies within the *region* $28.800 \le Q \le 28.833$ that is free of resonances through order ten. However, operation near the principal diagonal with $Q_x \sim Q_y$ produces coupling between the horizontal and vertical betatron motion that results in the transfer of emittance back and forth between the horizontal and vertical planes. For tracking with equal initial emittances ε_0, the initial conditions $x,z \neq 0$, $x'=z'=0$ results in the emittance in one plane being $\varepsilon_0 \le \varepsilon \le 2\varepsilon_0$ and the emittance in the other plane being $0 \le \varepsilon \le \varepsilon_0$. Use of a different set of initial conditions $x,z' \neq 0$, $x'=z=0$ reverses the emittance transfer. The particle motion is stable throughout the repeated emittance transfers, however the emittance transfer increases the physical aperture necessary to contain a beam having a given initial emittance. Alternate operating points have been explored with the goal of reducing coupling. A split of one unit between the horizontal and vertical tunes has been tried, but the coupling changed little.

Conclusion

The dynamic apertures with and without systematic multipoles agree at $\Delta P/P=0$ and 0.5% and at -0.5% for the 7GeV/amu systematic multipoles. The dynamic aperture at $\Delta P/P = -0.5\%$ for the 100 GeV/amu multipoles is lower than that for the other two cases. Initial runs at $\Delta P/P = \pm 1\%$ reveal a chromaticity that moves the tune outside the region contemplated for operation. As the shift is largest for the 100 GeV/amu multipoles, it is thought to be responsible for the low dynamic aperture at $\Delta P/P = -0.5\%$. Tracking at $\Delta P/P = \pm 1\%$ has been postponed until the tune dependence has been corrected. Coupling enlarges the beam. Efforts to reduce coupling by splitting the tunes are still in an early stage, but studies for a tune split of one unit show little if any reduction of coupling.

References

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