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Possible Benefits from Shuffling Dipoles in the RHIC

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I. Introduction

It is essential to state at the outset that there is no unique way of shuffling magnets. Many factors are involved in deciding how to do it; for example, one may take into account not just the linear machine parmeters but other things such as size and distribution of magnet errors, magnet installation schedule, allowance (or non-allowance) of "unusable" magnets and type and scope of diagonostic systems and correction systems. In addition, one may be influenced, conciously or unconciously, by the past experiences and may be inclined to emphasize some factors over others even when that is not justified by technical considerations alone. The example given in this note is just that, an example of what one can do under certain assumptions. Better ways of shuffling magnets should emerge as more data on field qualities would become available.

For the Tevatron at Fermilab, the goal of shuffling dipoles was a quite limited one and, because of that, the problem was a well-defined one.[1] We simply tried to minimize the magnitude of several isolated resonance-driving terms, these resonances arising from sextupole (b_2 and a_2) and octupole (a_3 only) components. The dimensionless figure-of-merit was the magnitude of each term relative to what one should expect from the distribution of b_2, a_2 or a_3 if the shuffling were not done. Since this involves only one particular harmonic component for each resonance, it is the simplest case of what one might call the "global" compensation.[2] (The nature of "global" and "local" compensations will be explained below.) Another example of the global compensation has been discussed recently[3] in which many harmonic components near the most important one are minimized by a particular way of shuffling. This sort of consideration becomes necessary when one is concerned about the loss of linearity in the beam motion, which may cause a reduction in the dynamic aperture of the machine, even though isolated resonances are not a direct threat to the beam stability.

In contrast to the global compensation, the "local" compensation is more appropriate when the source of field errors (or nonlinear elements) is within a relatively small area of the ring. One then tries to confine the effect of

errors within that area. If the compensation is perfect, there will be no effect outside the area although the effect may not be so small inside. This scheme has been promoted especially by Tom Collins[4] in connection with a group of special sextupoles in the SSC lattice. The difference in approach between two compensation schemes, global and local, can be seen, for example, in two different (but completely equivalent) forms for $\Delta\beta/\beta$, the error in betatron amplitude function β caused by the quadrupole component b_1 in dipoles:

a) global

$$(\Delta\beta/\beta) \text{ at } \psi = -(\nu/\pi) \sum_{n=-\infty}^{\infty} \frac{J_n}{4\nu^2 - n^2} e^{in\psi/\nu} \qquad (1)$$

$$\text{with} \quad J_n = \sum_{k=1}^{M} (\beta\theta\, b_1)_k\, e^{-in\psi_k/\nu} \qquad (2)$$

ψ = betatron phase, θ = bend angle, ν = tune.

Eq.(1) is valid at any location around the ring so that the source of error b_1, k=1 to M magnets, can be distributed all around the ring. Obviously, one tries to minimize J_n's with n near (2ν).

b) local

Here the source of error b_1 is confined to a small area. The goal is to minimize or completely eliminate $(\Delta\beta/\beta)$ at all points outside this area. For this, one must consider $\Delta\alpha$ together with $(\Delta\beta/\beta)$. Consider an arbitrary point outside and take this point as the origin of phase ψ. We then have

$$(\Delta\beta/\beta) - i(\Delta\alpha - \frac{\alpha}{\beta}\Delta\beta) = \frac{-e^{2i\pi\nu}}{2\sin(2\pi\nu)} \sum_{k=1}^{M} (\beta\theta b_1)_k e^{2i\psi_k} \qquad (3)$$

If M magnets are arranged such that the summation in Eq.(3) is zero, the errors $\Delta\alpha$ and $(\Delta\beta/\beta)$ are zero everywhere outside the M magnets.

It should be noted here that, inside the region under consideration, $\Delta\alpha$ and $(\Delta\beta/\beta)$ could be large. If the error is dipole field b_0 or a_0 instead of the quadrupole field b_1, the effect will be on the horizontal or vertical dispersion. An interesting example of this is the overpass at BØ of the main ring at Fermilab.[5] The beam line is raised by 19' near BØ in such a way that the vertical dispersion around the ring outside the overpass area is minimized to less then 0.5m but it is as large as 5m inside the overpass.

II. Special Considerations for the RHIC

One obvious difference between the RHIC and the Tevatron is in the number of dipoles, 144 in the regular arc sections of the RHIC compared with almost 800 for the Tevatron. Calculations which we regarded as impractical because of the required computing time for the Tevatron may not be so for the RHIC. Another difference (which may be more relevant to the shuffling) is that, for the Tevatron, the fluctuations in quadrupole components b_1 and a_1 were reduced down to 0.5×10^{-4} (at 1", rms) by moving the collared coil relative to the surrounding yoke. Since the effect of (b_1,a_1) was negligible, we concentrated on minimizing the effects of nonlinear field components. For the RHIC, the situation seems to be the other way around; the linear effects due to b_1 and a_1 on betatron amplitudes and dispersions may reduce the effective aperture of the ring more than nonlinear effects arising from higher multipole components such as b_2 and a_2. Therefore, it is assumed here that

(i) In shuffling dipoles in the regular arc sections, only the effects of b_1 on β_x, β_y and X_p (horizontal dispersion), and the effects of a_1 on Y_p (vertical dispersion) are taken into account. The effect of b_2 is controlled only to the extent that it is no more than one would expect from statistical arguments.

The choice of the number of dipoles to be shuffled each time will undoubtedly depend on the schedule of magnet construction and tunnel preparation. It may even change during the course of the project as it did for

the Tevatron. Here we take eight or twelve as a reasonable choice covering four or six regular cells. With less than eight magnets, it will be difficult to balance the errors (particularly when some errors are abnormal) while more than six cells would cover too much phase advance.

(ii) Two cases are considered, one with eight dipoles and the other with twelve in each group to be shuffled.

Problems associated with magnet errors in the insertions are rather special. They may be compensated for by special shunts or separate power supplies. Even if it becomes necessary to shuffle insertion magnets, it should be done independently from the shuffling of regular dipoles. It is expected that the effect of errors in regular quadrupoles is much less than that of dipole errors. Again, any shuffling of quadrupoles should be done separately.*

(iii) All regular quadrupoles are assumed to be free of errors. Insertions are assumed to be perfect.

III. Calculations for Shuffling

Since the purpose of this note is simply to demonstrate how shuffling can be done to minimize various effects of magnet errors, a precise quantitative estimate of these effects is not an essential requirement. In order to simplify the computation, all magnets (quadrupoles and dipoles) in the arc sections are treated as a thin lens. Moreover, each insertion is represented by a matrix that matches all linear parameters with the phase advance of 636^{0} in both directions. The cell length is 29.622m and the bend angle is 38.85mr per dipole. Shufflings are done for $\nu_x = \nu_y = 28.8$ corresponding to phase advance of 91^{0}/cell but the performance is checked for $\nu_x = \nu_y = 28.4$ to see that it is not degraded by a small change in tune.

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* The most important error in regular quadrupoles is the fluctuation in the integrated gradient field. It may be difficult to shuffle quadrupoles unless one is certain of the average over the entire ring. At the same time, it does not seem practical to postpone the installation untill all of them are built and measured.

The distribution of b_1, a_1 and b_2 is all taken to be Gaussian around the mean zero with the rms values

$$\langle b_1 \rangle = 2.1\times10^{-4}/25\text{mm}, \quad \langle a_1 \rangle = 4.3\times10^{-4}/25\text{mm}, \quad \langle b_2 \rangle = 4.6\times10^{-4}/(25\text{mm})^2$$

Using these numbers, one can estimate the expected value of various errors due to 144 dipoles:

$$\left\langle \frac{\Delta\beta}{\beta} \right\rangle_x, \left\langle \frac{\Delta\beta}{\beta} \right\rangle_y = \frac{1}{2\sin|2\pi\nu|} \frac{1}{\sqrt{2}} \langle b_1 \rangle \theta_B \beta_{x,y} (144)^{\frac{1}{2}} = 0.0322, \tag{4}$$

$$\langle \Delta X_p/\sqrt{\beta_x} \rangle = \frac{1}{2\sin|\pi\nu|} \frac{1}{\sqrt{2}} \langle b_1 \rangle \theta_B X_p \beta_x (144)^{\frac{1}{2}} = 0.0109\text{m}^{\frac{1}{2}}, \tag{5}$$

$$\langle Y_p/\sqrt{\beta_y} \rangle = \frac{1}{2\sin|\pi\nu|} \frac{1}{\sqrt{2}} \langle a_1 \rangle \theta_B X_p \beta_y (144)^{\frac{1}{2}} = 0.0224\text{m}^{\frac{1}{2}} \tag{6}$$

where, on the right hand side of each equation, θ_B=0.03885 (bend angle), $\beta_x = \beta_y$ = 22.1m and X_p=0.99m at each dipole (regarded as a thin lens). As the measure of deviations from linearlity in betatron oscillations, we use the distortion functions $(B_3^2 + A_3^2)^{\frac{1}{2}}$, $(B_s^2 + A_s^2)^{\frac{1}{2}}$ and $(B_d^2 + A_d^2)^{\frac{1}{2}}$ defined by Tom Collins.[4] There are two more pairs of functions, $\bar{B}$ and $\bar{A}$, and B_1 and A_1 but their expected values are not much different from that of $(B_d^2+A_d^2)^{\frac{1}{2}}$. Expected values are, for ν = 28.8,

$$\langle (B_3^2+A_3^2)^{\frac{1}{2}} \rangle = \frac{1}{16} \frac{1}{\sin|3\pi\nu_x|} \langle b_2 \rangle \theta_B (\beta_x^3/\beta_o)^{\frac{1}{2}} (144)^{\frac{1}{2}} = 2.34\text{m}^{-1}, \tag{7}$$

$$\langle (B_s^2+A_s^2)^{\frac{1}{2}} \rangle = \frac{1}{16} \frac{1}{\sin|\pi(\nu_x+2\nu_y)|} \langle b_2 \rangle \theta_B (\beta_x\beta_y^2/\beta_o)^{\frac{1}{2}} (144)^{\frac{1}{2}} = 2.34\text{m}^{-1}, \tag{8}$$

$$\langle (B_d^2+A_d^2)^{\frac{1}{2}} \rangle = \frac{1}{16} \frac{1}{\sin|\pi(\nu_x-2\nu_y)|} \langle b_2 \rangle \theta_B (\beta_x\beta_y^2/\beta_o)^{\frac{1}{2}} (144)^{\frac{1}{2}} = 3.79\text{m}^{-1} \tag{9}$$

where the reference value of β is taken to be β_o = 1m.

One random set of (b_1,a_1,b_2) was generated for 144 dipoles and the calculations were always made for this particular set. The comparison is made between the unique, optimally shuffled arrangement of this set and 1,000 randomly arranged rings using the same set of (b_1,a_1,b_2). As the figure-of-merit, a simple expression

$$\text{F.M.} \equiv |\Sigma a_1 \exp(i\psi_y)|^2 + |\Sigma b_1 \exp(i\psi_x)|^2 + |\Sigma b_1 \exp(2i\psi_x)|^2 + |\Sigma b_1 \exp(2i\psi_y)|^2 \qquad (10)$$

evaluated at dipole locations was initially used with the supplementary condition that

$$|\Sigma b_2 \exp(3i\psi_x)| \quad \& \quad |\Sigma b_2 \exp(i\psi_\pm)| \qquad (\psi_\pm \equiv \psi_x \pm 2\psi_y) \qquad (11)$$

do not exceed the expected rms values. The summations here are over eight or twelve dipoles of each group so that one is trying to minimize the effect of each group outside the four or six cells under consideration. In shuffling magnets in the second group, it might be better to include the predetermined sums over the first group. Then for the third shuffling, the sums would include the results from the two previous groups, and so on. However, this is not necessarily the optimum procedure since the "inside" region in which the minimization is not done at all covers larger and larger fraction of the entire ring. For the best overall result, it is not obvious what the largest number of groups should be in the summation. It was then realized that, for a given arrangement of all magnets, linear lattice parameters $(\beta_x,\beta_y,X_p,Y_p)$ can be calculated rapidly at all locations around the ring so that the figure-of-merit could be more directly related to these parameters. Results presented in the next section have been obtained with the figure-of-merit

$$\text{F.M.} \equiv \Sigma(\Delta\beta_x/\beta_x)^2 + \Sigma(\Delta\beta_y/\beta_y)^2 + \Sigma(\Delta X_p/\sqrt{\beta_x})^2 + \Sigma(Y_p/\sqrt{\beta_y})^2 \qquad (12)$$

where the summations are at all (6x25) quadrupole locations, "inside" as well as "outside" regions. Each quantity to be summed is calculated exactly for a given arrangement of dipoles with M dipoles, 2M dipoles, 3M dipoles,

and so on where M = 8 or 12. For shuffling the last M dipoles, (144-M) dipoles are already determined and the ring is entirely "inside". The local nature of balancing is thus shifted gradually to the global nature. In order to find the final "optimum" arrangement, approximately 1,000 random cases were studied. Although the figure-of-merit does not include the effect of sextupole component b_2, quantities such as $(B_3^2 + A_3^2)$ summed at all 150 quadrupole locations were monitored to prevent large nonlinear effect in the selected "optimum" arrangement. It is of course possible to add nonlinear distortion effects arising from the skew sextupole component a_2 for this monitoring as long as one is not too greedy.

IV. Results

Seven quantities, four of them linear and three nonlinear, are calculated to test the performance of the shuffling.

I. $\frac{1}{\sqrt{150}} \{\Sigma(\Delta\beta_x/\beta_x)^2\}^{\frac{1}{2}}$ II. $\frac{1}{\sqrt{150}} \{\Sigma(\Delta\beta_y/\beta_y)^2\}^{\frac{1}{2}}$

III. $\frac{1}{\sqrt{150}} \{\Sigma(\Delta X_p/\sqrt{\beta_x})^2\}^{\frac{1}{2}}$ IV. $\frac{1}{\sqrt{150}} \{\Sigma(Y_p/\sqrt{\beta_y})^2\}^{\frac{1}{2}}$

V. $\frac{1}{\sqrt{150}} \{\Sigma(B_3^2+A_3^2)\}^{\frac{1}{2}}$ V. $\frac{1}{\sqrt{150}} \{\Sigma(B_s^2+A_s^2)\}^{\frac{1}{2}}$

VII. $\frac{1}{\sqrt{150}} \{\Sigma(B_d^2 +A_d^2)\}^{\frac{1}{2}}$ (Summations are over 150 quadrupole locations.)

The shuffled arrangement is compared with 1,000 randomly arranged cases with eight or twelve dipoles as a unit. The tune used to find the optimum arrangement is 28.8 in both horizontal and vertical directions (91^o/regular cell) but the same arrangement is used with the tune of 28.4 (89^o/regular cell) to see the tune dependence of the performance. In comparing the performance, the "rank" of 0 means the shuffled case is better than any of 1,000 cases and 1,000 means worse than any.

Table 1. M = 8 (eight dipoles shuffled each time); $\nu = 28.8$

	expected	average of 1,000 cases	largest of 1,000 cases	shuffled arrangement	rank
I.	0.0322	0.0330	0.0742	0.0074	0
II.	0.0322	0.0319	0.0616	0.0069	0
III.	0.0109	0.0115	0.0291	0.0021	0
IV.	0.0224	0.0248	0.0619	0.0044	0
V.	2.34	2.31	4.59	1.69	221
VI.	2.34	2.39	5.10	1.97	365
VII.	3.79	3.48	8.51	1.14	8

M = 8; $\nu = 28.4$

Shuffled arrangement		
	I.	0.0140
	II.	0.0063
	III.	0.0019
	IV.	0.0108
	V.	3.27
	VI.	3.22 *
	VII.	1.20

* With this tune, the expected value of V and VI is 3.79 and it is 2.34 for VII.

Table 2. M = 12 (twelve dipoles shuffled each time); ν = 28.8

	expected	average of 1,000 cases	largest of 1,000 cases	shuffled arrangement	rank
I.	0.0322	0.0328	0.0665	0.0087	0
II.	0.0322	0.0334	0.0777	0.0070	0
III.	0.0109	0.0112	0.0303	0.0025	0
IV.	0.0224	0.0277	0.0662	0.0057	0
V.	2.34	2.36	5.06	1.26	45
VI.	2.34	2.23	5.12	1.14	30
VII.	3.79	4.33	11.9	1.19	4

M = 12; ν = 28.4

Shuffled arrangement

I. 0.0097
II. 0.0070
III. 0.0034
IV. 0.0165
V. 2.90
VI. 3.49
VII. 1.16

Conclusion

With the Gaussian distribution, it seems possible to achieve an improvement of factor four to five over the statistically expected values without too much sacrifice in the nonlinear distortion. There is no difference in the performance between M=8 and M=12 and the tune dependence of the performance is acceptable when the change in tune is less than $\sim$ 0.5.

References

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