BNL-101922-2014-TECH

AD/RHIC/10;BNL-101922-2013-IR

Reduction of Iron Saturation in Cosine Theta Dipoles

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July 1985

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USDOE Office of Science (SC)

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RHIC Technical Note No. 10

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July 29, 1985

Introduction

It is possible to reduce or eliminate iron saturation effects in high field cosine theta dipoles by putting a large gap between the coil and the iron, but part or most of the of the field added by the iron will be lost. With a small gap between coil and iron and with sufficient iron on the midplane, iron saturation always occurs first at the poles. Thus to delay saturation onset, it may be sufficient to remove iron near the poles.

The RHIC dipole as presently designed is an ideal candidate for a study of possible improvement, since it attains a quench field greater than 4.5 T and has a small gap, about 5 mm, between coil and iron. The coil designed for it[1], which has 4 blocks of 16, 9, 6 and 3 turns, respectively, is close to perfect at low field, as shown in Table 1.

Table 1
Harmonic Content of the RHIC Coil at ∞ mu, $10^{-4}B_0$

radius	b_2	b_4	b_6	b_8	b_{10}	b_{12}	b_{17}	b_{16}	b_{18}
25 mm	.08	0.	0.	0.	.02	-.05	-.11	-.13	.08
32 mm	.14	0.	0.	0.	.22	-1.00	-3.62	6.60	6.93

At infinite mu, the algebraic sum of the harmonics at 32 mm given in Table 1 is ~3.78, which is close to the desired value[2] of 2.

The iron contributes substantial sextupole and decapole at both intermediate and high fields[3] as shown in Table 2. The moderately large b_2 at 3.44 T can be nulled to first approximation by external trim sextupoles,

Table 2
Harmonics at 25 mm, 10^{-4} BO

Bo,T	∞μ	0.141	0.395	3.44	4.10
I,kA		0.177	0.496	4.37	5.39
TF,T/kA	0.805	0.798	0.798	0.780	0.749
$b_2{}^1$	0.09	-20.	0.	7.2	20.
$b_4{}^1$	0.0	- 0.4	-0.4	0.7	-1.2
$b_6{}^1$	0.0	- 0.1	-0.1	-0.4	-0.5

and the b_4 and b_6 terms add algebraically to 0.1 at r = 32 mm. The possible benefits of a shaped iron aperture may arise at higher fields, where b_4 and b_6 don't cancel and b_2 requires substantial correction.

The aperture chosen for study has a 3 mm gap between coil and iron at the midplane rather than 5 in order to get a larger iron benefit.

In a preliminary report[4], it has been shown that a basic elliptical aperture with a half height about 5 mm greater than the midplane half-width of 52.84 mm reduces the b_2 shift due to iron saturation at 4.1 T to essentially zero. The ellipse introduces at low field, a large b_2 and a smaller b_4 term which must be compensated for by a matching coil design. The combination of new coil and elliptical aperture has a b_4 saturation shift larger than the original circular design, and to compensate for this shift, the ellipse is perturbed by a bump at 54 degrees of maximum height -0.9 mm. This perturbed ellipse plus a new matching coil has essentially zero shift in both b_2 and b_4 due to saturation, but has a $b_6$saturation shift larger than in the original circular design.

The present paper shows that the b_6 shift can be controlled by an additional bump at 64.3 degree. As might be anticipated, the resulting aperture plus matching coil introduces a b_8 shift, but continuing the trend that each new higher harmonic shift introduced was smaller than the preceeding, the b_8 shift introduced is small enough to be tolerated. Another difference between the present work and the earlier is that sharper bumps are used.

Procedure

It is a supposition that the perturbation should be placed precisely at the harmonic pole positions nearest the fundamental pole, which in a dipole magnet are given by eq. 1.

$$\Theta_n = \pi \left(1/2 - 1/(n+1)\right), \; n = 2, 4, 6, -- \qquad (1)$$

What one would like to achieve by choice of the perturbations is complete separation of the saturation effects, i.e., a bump at 90 degree (the ellipse elongation) would change only the sextupole saturation shift, and the one at 54 degree would change only the decapole saturation shift, etc.

An alternative method of looking at the problem is to assume that an aperture shape with matching coil exists which has the property that it provides a pure dipole field at low field and at the highest field prior to midplane iron saturation becoming the predominant influence on field shape. With this viewpoint, the perturbations can be regarded as a set of basis functions which can be used in a variational process. Use of an arbitrary set of orthogonal functions for basis functions may require evaluation of the effects of a large number of perturbations and it appears that the more general approach can be bypassed by the use of the perturbations mentioned above.

The form for the perturbation used in the earlier report[4] consists of an ellipse for the sextupole and a bump normal to the ellipse with magnitude proportional to $\theta^{1.225} \sin^2 2\theta$ for the decapole. The mathematical form used for the bumps should have zero magnitude and slope at $\theta = 0$ and $\theta = \pi/2$ and should have peak magnitude at the angles given by eq. (1). A more general form which satisfies these requirements is given by eq. (2),

$$D_n = A_n G_n \theta^k \sin^m 2\theta, \; m = 2, 3, --- \qquad (2)$$

with k in (2) satisfying eq. (3),

$$k = -2\, m\, \theta_n \,\mathrm{ctn}\, 2\, \theta_n \qquad (3)$$

where θ_n in eq.(3) is given by eq. (1). G_n is a normalizing constant such that A_n is the peak amplitude of the bump. This form has one bump in the quadrant. All of the harmonics have more than one pole in the quadrant, i.e. b_2 at 30 degree, b_4 at 18 degree and b_6 at 38.6 degree. The latter, in particular, may be close enough to the pole to be affected by saturation. However, it is not obvious whether a bump there should be of the same sign as the one at 64.3 degree or more generally, what the magnitude of it should be compared to the pole-most bump. If these additional bumps are independent of the pole-most bump, a more powerful minimization process, such as MINUIT must be used, rather than the Newton-Raphson process actually used, since the number of independent variables is greater than the number of parameters being optimized. The form of eq. (2) cannot be used for bumps at angles less than 45 degrees, because k of eq. (3) is then negative, giving an infinity in D_n at $\theta = 0$.

One additional form for the b_4 bump was studied. Equation (4) is an ellipse if n = 2, and closely resembles an ellipse if n is close to 2. For n = 1.9

$$(x/a)^n + (y/b)^n = 1 \qquad (4)$$

there is a bump peaked at about 49 degree of amplitude -1 mm if a = 5.284 cm and b = 5.784, typical values for the RHIC aperture. Forty-nine degrees is close enough to 54 that this form, termed "2+δ", is suitable for a b_4 bump.

Three forms were studied in detail for use as b_4 bumps: $\sin^2 2\theta$, $\sin^4 2\theta$ and "2+δ". The b_4 shift due to saturation at 4.1 T is denoted by Δb_4, and the change in Δb_4 due to a bump is denoted by $\Delta^2 b_4$. A typical ellipse was modified by a -1 mm bump caused by each of the three forms and the harmonic shifts obtained. In all cases, the aperture was used with a matching coil which at low field results in all b_i, $i < 18$ being less than .05 at 2.5 cm radius. The $\Delta^2 b_4$ for the three forms are 2.81, 3.32 and 2.18, respectively for a 1 mm bump. Thus the $\sin^4 2\theta$ form gives the largest b_4 saturation shift per mm of bump. Each bump also introduces saturation shifts in the other harmonics. Table 3 gives $\Delta^2 b_i$, i = 2, 6 and 8, normalized to $\Delta^2 b_4$.

Table 3

bump type	$\Delta^2 b_2$	$\Delta^2 b_6$	$\Delta^2 b_8$
$\sin^2 2\theta$	-.68	-.15	-.011
$\sin^4 2\theta$	-1.07	-.11	-.021
2 + δ	-1.33	-.14	-.0046

The data of Table 3 shows that $\Delta^2 b_6$ is a minimum for the $\sin^4 2\theta$ bump, but that the $\sin^4 2\theta$ bump has the largest $\Delta^2 b_8$. The 2 + δ bump causes the least $\Delta^2 b_8$, and only 27% more $\Delta^2 b_6$ than the $\sin^4 2\theta$ form. It thus appears that the 2 + δ form is the best. Unfortunately, the 2 + δ form gives the least $\Delta^2 b_4$ per mm of bump. It was initially tried with δ = -.39 (n = 1.61 in eq (4)), and the roughly -4 mm bump was too close to the coil (about 1/2 mm) for practical construction, so this form was dropped. Of the two remaining, the $\sin^4 2\theta$ form is more suitable, since it requires the smallest bump and has the least $\Delta^2 b_6$.

Two forms, $\sin^2 2\theta$ and $\sin^4 2\theta$ were studied for Δb_6 correction. The $\Delta^2 b_6$ obtained were .29 and .36 per mm, respectively. The effects on the other $\Delta^2 b_i$, normalized to $\Delta^2 b_6$ are given in Table 4.

Table 4

bump type	$\Delta^2 b_2$	$\Delta^2 b_4$	$\Delta^2 b_8$
$\sin^2 2\theta$	-1.17	-11.8	-.10
$\sin^4 2\theta$	-1.75	-3.06	-.11

The $\sin^4 2\theta$ bump gives the lesser $\Delta^2 b_4$ and about the same $\Delta^2 b_8$. Possibly a $\sin^n 2\theta$ with $n > 4$ would be better, but none were tried.

To find the proper aperture shape, a set of three linear equations in the three unknown bump sizes must be solved. Since the saturation shifts are not strictly linear in bump size, more than one iteration may be required. The first iteration is to pick an approximate ellipse elongation or bump. The perturbation matrix does not have to be obtained using coils exactly matched to the aperture, as was done for Table 3 and 4; an inaccurate matrix may increase the number of iterations required.

Results

In the present case, the first iteration was to pick a 5.4 mm ellipse bump. Perturbation of this ellipse gave the following set of linear equations (e, A and B denote the b_2, b_4 and b_6 bumps).

i	e	A	B	$-\Delta b_i$	
2	-8.6	4.20	-1.24	-5.43	
4	.2	-3.48	-2.28	2.85	(5)
6	.085	.37	.70	.53	

The Δb_i are the saturation shifts of the e = 5.4 mm ellipse. The second iteration reduced the Δb_i to -1.78, -.23 and .27, and the third iteration to 0.22, .02 and -.06, with $\Delta b_8 = -.27$ (not controlled). The resulting configuration, termed "EL5H", has e, A and B equal to 4.7, -1.87 and 1.446 mm, respectively.

Obtaining a matching coil with integer turns for EL5H was not easy. The best found, termed "I", was a 16, 9, 7, 4 turns per block configuration.

The dimensions of coil "I" are given in Table 5.

Table 5

Turns	Theta Start	Theta end
16	.14	26.9844
9	28.7666	43.8666
7	48.1564	59.9008
4	68.2134	74.9245

The coil inner and outer radii are the same as the reference design coil(1). The estimated harmonic content of the EL5H/I combination at low field and 25 mm radius is given in Table 6.

Table 6

i	2	4	6	8	10	12	14
b_i	-1.6	-.25	-.58	.03	.07	-.20	.06

These harmonics are not bad, but they do not satisfy the $\Delta B/B = 2 \times 10^{-4}$ at 3.2 cm requirement(2) stated at the beginning, mainly because of the b_6 term (the b_2 term is compensated for by external sextupole magnets).

It was pointed out in the previous paper(4) that small amounts of b_i due to the integer turn constraint can be eliminated by small changes in e, A and B, without greatly changing the saturation shifts. The matrix for doing so is obtained from the shift due to an added bump at low field. For the EL5H/I iron-coil configuration, the appropriate set of linear equations is (6):

i	e	A	B	$-b_i$	
2	18.615	-8.56	6.76	1.6	
4	.575	9.08	5.4	.25	
6	.005	- .55	-1.48	.58	(6)

The solution of this set is Δe = .406, ΔA = -.300 and ΔB = -.502, giving 5.106, -1.57 and .944 for e, A and B, respectively. This new iron configuration, termed "EL5I" was run with the "I" coil giving the harmonics at 25 mm listed in Table 7.

Table 7

B,T	b_2	b_4	b_6	b_8	b_{10}	b_{12}	b_{14}	b_{16}	b_{18}
.382	- .10	-.15	.12	-.08	.06	-.20	.07	-.15	(.05)
3.29	-1.74	.17	.13	-.12	.06	-.20	.07	-.15	
4.13	-1.36	-.02	-.17	-.28	.07	-.20	.07	-.16	
32mm									
.382	- .17	-.39	.51	-.54	.70	-3.84	2.19	-7.90	(4.25)

Besides b_2, the largest saturation shift is in b_6; Δb_6 = -.29, compared to -.06 for EL5H from which EL5I was derived. Fortunately, b_6 starts off positive, so it's magnitude is always small.

The final row in Table 7 is the low-field b_i computed at 32mm radius instead of 25, for comparison with Table 1. The algebraic sum of the harmonics (at 32 mm radius (omitting b_2) is -5.02, almost as good as the reference design value of -3.78. Of course, harmonics higher than b_{18} may be present. The values of the harmonics b_{10} through b_{16} given in Table 7 are identical to those for the "I" coil computed with ∞ μm iron with a circular aperture by the analytic program used for coil design. The low-field b_{18} term in parenthesis is from that program, rather than from MDP, which does not calculate harmonics above b_{17}.

At 4.13T, the useful aperture would appear to be about 30 mm, where the algebraic sum of the harmonics, thru b_{18} excluding b_2 is -3.83.

It should be noted that MDP computes the field on the assumption that the coils are constant current density, but in fact, the current density varies more nearly as 1/r. An approximation to the 1/r variation can be achieved by subdividing the radial sector blocks radially, with equal current in each radial step. Only a few radial subdivisions are needed, five were used. The result is a change in the low-field b_2 (2.5 cm) of Table 6 from -.10 to -1.45, a change in b_4 from -.15 to + .01, a change in b_6 from .12 to .14, and a change in the transfer function from 8.135G/A to 8.142. All higher harmonics were unchanged. Figure 1 shows one quadrant of EL5I with the subdivided "I" coil.

Summary

A procedure is presented for systematically perturbing the circular iron aperture of a cosine-theta dipole in such a way as to postpone saturation effects up to the point where insufficient iron is present on the midplane. The resulting configuration has little loss in field compared to the alternate procedure of putting a large annulus between the coils and the iron. The necessary saturable iron program used was MDP, but the procedure could be performed with any program. The perturbation functions used to modify the iron shape provide good separation of effects so that only one perturbation per harmonic is needed, thus minimizing the amount of computation required.

A coil and aperture configuration with low-field harmonic content approaching the stringent RHIC requirements is presented which requires no b_4 or higher order correction coils up to 4.1 T, with a good field aperture at low field of 32 mm and at 4.1 T, 30 mm. The b_2 correction required is largest at an intermediate field, and has a maximum amplitude of about -3.2. The transfer function of the configuration is about 2% higher than that of the reference design.

References

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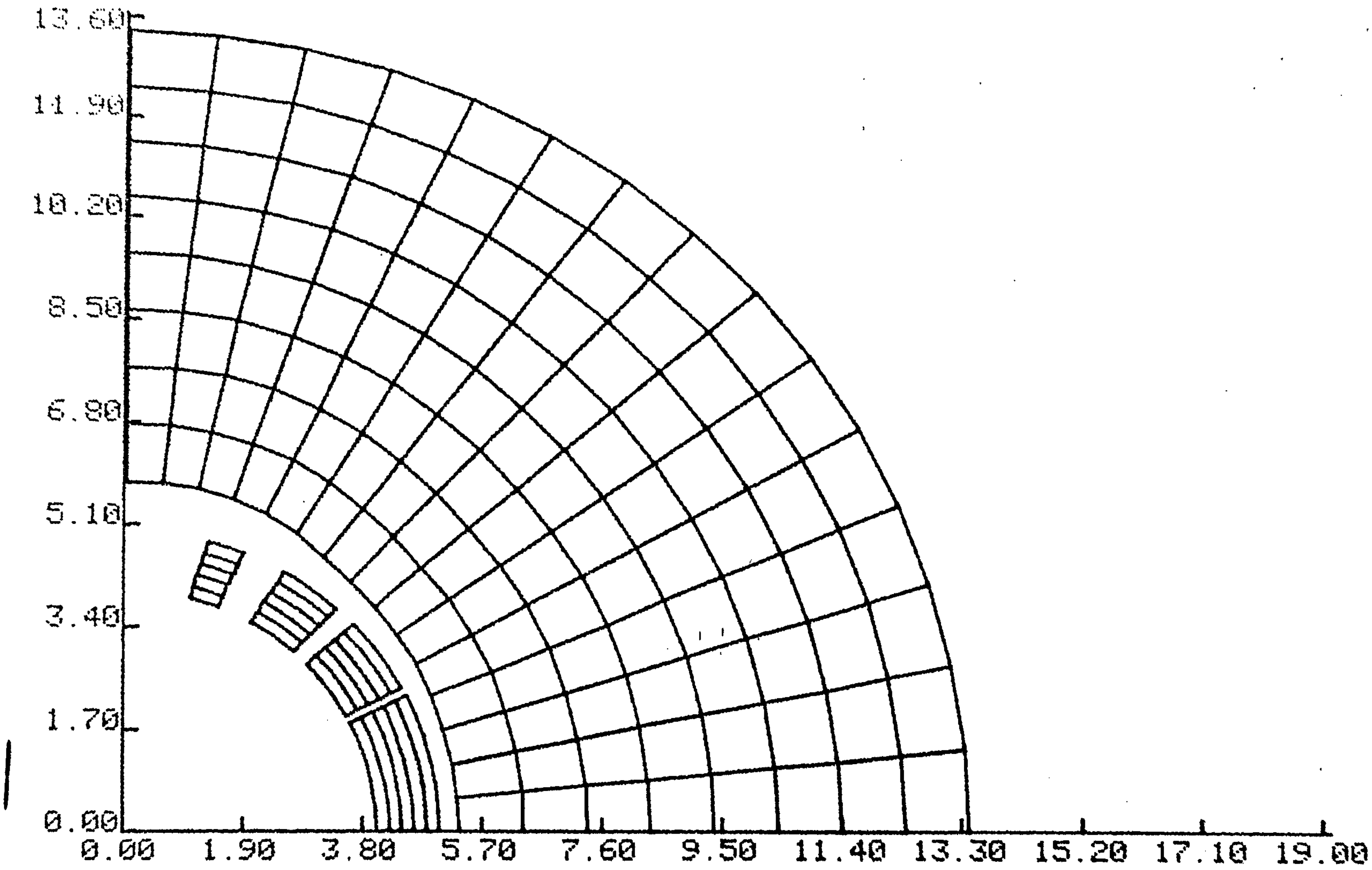

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