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# The Effect Of Landau Damping On The Longitudinal Phase Space Instability Across the Transition Energy

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THE EFFECT OF LANDAU DAMPING ON THE  
LONGITUDINAL PHASE SPACE INSTABILITY ACROSS  
THE TRANSITION ENERGY

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As pointed out by J.M. Wang (RHIC-PG32) that the microwave ~~in~~coherent instability may play an important role in the design of RHIC machine across the transition energy region. In this note we shall examine the growth rate in the presence of the beam momentum and energy spread. We shall proceed in three sections:

(I) The equation of longitudinal motion across the transition region and derive an invariant of the longitudinal phase space

(II) Discuss the dispersion integral of the instability.

(III) Conclusion and Recommendation.

# I Equation of Synchrotron motion near transition energy. 2

$$\theta = \varphi - \varphi_s$$

$$(h \Delta\omega) \Rightarrow \dot{\theta} \equiv \frac{h \eta \omega_0^2}{E \beta^2} W \quad W = \frac{\Delta E}{\omega_0}$$

$$\dot{W} = \frac{eV}{2\pi} (\Delta \sin \varphi - \Delta \sin \varphi_s) \approx \left( \frac{eV}{2\pi} \cos \varphi_s \right) \theta$$

$$H = \frac{h \eta \omega_0^2}{2pR} W^2 + \frac{eV}{2\pi} [\cos \varphi - \cos \varphi_s + (\varphi - \varphi_s) \Delta \sin \varphi_s]$$

$$\frac{d}{dt} \left( \frac{1}{\Omega_s^2} \dot{\theta} \right) + \theta = 0$$

$$\omega_w = \frac{c}{R} = \frac{\omega_0}{\beta}$$

$$\Omega_s^2(t) = \frac{\omega_0^2 h}{E(t)} \frac{eV}{2\pi} |\eta(t) \cos \varphi_s|$$

$$= \frac{\omega_0^2 h}{E_0} \frac{eV}{2\pi} |\cos \varphi_s| \left| \frac{2\dot{\gamma}}{\gamma^4} t \right| \equiv \frac{t}{T^3} \quad |t| \ll T = \frac{\delta E}{\dot{\gamma}}$$

$$\eta(t) \equiv \frac{1}{\gamma^2} - \frac{1}{\delta^2} \sim + \frac{2\dot{\gamma}}{\gamma^3} t$$

Assumptions

(i) Acceleration is an adiabatic process

$\gamma(t)$ ;  $\eta(t)$  is a smooth function of  $t$

$$\gamma(t) = \gamma_t + \dot{\gamma} t$$

$$T^3 = 2h \frac{\omega_w^2 eV \dot{\gamma}}{2\pi E_0 \gamma_t^4} |\cos \varphi_s|$$

$$\frac{d}{dt} \left( \frac{1}{\Omega_s^2 t^3} \dot{\theta} \right) + \theta = 0$$

$$t > 0 \quad y = \int_0^t dt' \Omega_s(t') = \frac{2}{3} \left( \frac{t}{T} \right)^{3/2}$$

$$\theta = y^{2/3} \Phi \quad x \equiv t/T$$

$$\frac{d^2 \Phi}{dy^2} + \frac{1}{y} \frac{d\Phi}{dy} + \left[ 1 - \frac{\left( \frac{2}{3} \right)^2}{y^2} \right] \Phi = 0$$

$$\theta = a x \left[ \cos \psi J_{2/3}(y) + \sin \psi N_{2/3}(y) \right]$$

$$\boxed{\frac{\theta}{xa} = \cos \psi J_{2/3}(y) + \sin \psi N_{2/3}(y)}$$

$$\frac{\hbar \eta \omega_0^2}{E \beta^2} W = \dot{\theta} = \frac{\theta}{Tx} + \frac{x^{3/2}}{T} a \left[ \cos \psi \left( \frac{2}{3y} J_{2/3} - J_{5/3} \right) + \sin \psi \left( \frac{2}{3y} N_{2/3} - N_{5/3} \right) \right]$$

$$\frac{1}{ax^{3/2}} \left( \frac{2\theta}{x} - \frac{\hbar \eta \omega_0^2}{E \beta^2} W \right) = \cos \psi J_{5/3}(y) + \sin \psi N_{5/3}(y)$$

$$\begin{pmatrix} \cos \psi \\ \sin \psi \end{pmatrix} = M^{-1} \begin{pmatrix} \theta/xa \\ \left[ \frac{2\theta}{x} - \frac{\hbar \eta \omega_0^2}{E \beta^2} W \right] / x^{3/2} a \end{pmatrix}$$

$$M \equiv \begin{pmatrix} J_{2/3} & N_{2/3} \\ J_{5/3} & N_{5/3} \end{pmatrix}$$

$$\cos^2 \psi + \sin^2 \psi = 1 \equiv \alpha_{\theta\theta} \theta^2 + 2\alpha_{\theta W} \theta W + \alpha_{WW} W^2 = \text{invariant}$$

$$\alpha_{\theta\theta} = \left[ \left( \frac{3}{2} \gamma N_{5/3} - 2N_{2/3} \right)^2 + \left( 2J_{2/3} - 1.5 \gamma J_{5/3} \right)^2 \right] / a^2 (\det)^2 \times^5$$

$$\alpha_{\theta W} = \frac{h T \eta \omega_0^2}{a^2 (\det)^2 \times E \beta^2} \left[ N_{2/3} \left( \frac{3}{2} \gamma N_{5/3} - 2N_{2/3} \right) - J_{2/3} \left[ 2J_{2/3} - \frac{3}{2} \gamma J_{5/3} \right] \right]$$

$$\alpha_{WW} = \frac{h^2 T^2 \eta^2 \omega_0^4}{a^2 (\det)^2 \times E^3 \beta^2} \left[ J_{2/3}^2 + N_{2/3}^2 \right]$$

Asymptotically:

$$\alpha_{\theta\theta} = 1 / \hat{\theta}^2 \quad \hat{\theta} = \left( \frac{2 A h^2 \omega_0^2 \gamma T^2}{\pi \epsilon_0 \beta^2 \gamma_t^4} \right)^{1/2} \left( \frac{t}{T} \right)^{1/4}$$

$$\alpha_{WW} = 1 / \hat{W}^2 \quad \hat{W} = \frac{1}{h} \left( \frac{A \epsilon_0 \beta^2 \gamma_t^4}{2 \pi h^2 \omega_0^2 \gamma T^2} \right)^{1/2} \left( \frac{T}{t} \right)^{1/4}$$

$$a^2 = \frac{2 A h^2 \omega_0^2 \gamma T^2}{3 \epsilon_0 \beta^2 \gamma_t^4}$$

$$[A] \equiv \text{eV sec.}$$

Gaussian Bunch.

$$\xi_0 \approx \sqrt{3}$$

$$\rho(\theta, W) = N \frac{\xi_0^2 [\alpha_{\theta\theta} \alpha_{WW} - \alpha_{\theta W}^2]^{1/2}}{\pi} e^{-\xi_0^2 (\alpha_{\theta\theta} \theta^2 + 2 \alpha_{\theta W} \theta W + \alpha_{WW} W^2)}$$

$$= N g(\theta) g(W)$$

$$g(W) = \frac{\xi_0 \sqrt{\alpha_{WW}}}{\sqrt{\pi}} e^{-\xi_0^2 \alpha_{WW} \left( W + \frac{\alpha_{\theta W}}{\alpha_{WW}} \theta \right)^2}$$

$$g(\theta) = \frac{\xi_0}{\sqrt{\pi}} \left[ \frac{\alpha_{\theta\theta} \alpha_{WW} - \alpha_{\theta W}^2}{\alpha_{\theta W}} \right]^{1/2} e^{-\xi_0^2 \left[ \frac{\alpha_{\theta\theta} \alpha_{WW} - \alpha_{\theta W}^2}{\alpha_{\theta W}} \right] \theta^2}$$

## II Coherent Instability.

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a Coasting beam.

Small perturbation in current  $I = I_0 + I_1 e^{i n [(\theta - \Delta \Omega t) - \omega t]}$

$$\rightarrow \omega = \omega_0 + \omega_1 e^{i n [(\theta - \Delta \Omega t) - \omega t]}$$

$$\Rightarrow \text{Voltage change / turn} = - I_1 Z e^{i n [(\theta - \Delta \Omega t) - \omega_0 t]}$$

$$\frac{\partial E}{\partial t} = - \frac{\partial E}{\partial \theta} \omega + e U_s \frac{\omega}{2\pi}$$

$$\Rightarrow \frac{\partial \omega}{\partial t} = - \frac{\partial \omega}{\partial \theta} \omega - \frac{\partial \omega}{\partial E} \frac{\partial E}{\partial t} = - \frac{\partial \omega}{\partial \theta} \omega_0 + \frac{e \eta \omega_0^2 Z}{\beta^2 E \cdot 2\pi} I_1 e^{i(n\theta - \omega_0 t)}$$

$$\omega_1 \Delta \Omega = i e \eta \omega_0^2 I_1 \left( \frac{Z}{n} \right) / 2\pi \beta^2 E$$

Equation of continuity:  $\frac{\partial I}{\partial t} = - \frac{\partial I}{\partial \theta} \omega_0 - \frac{\partial \omega}{\partial \theta} I_0$

$$I_1 \Delta \Omega = \omega_1 I_0$$

$$\Delta \Omega^2 = i \frac{e \eta \omega_0^2 I_0}{2\pi \beta^2 E} \left( \frac{Z}{n} \right)$$

$$e^{-i n \Delta \Omega t}$$

$$: \begin{cases} \Delta \Omega_i > 0 & \text{unstable} \\ \Delta \Omega_i < 0 & \text{stable.} \end{cases}$$

$$\text{Growth} = e^{\int n \Delta \Omega_i dt}$$

high energy spread

$$1 = - \Delta \Omega^2 \int \frac{g'(\omega)}{\delta \Omega - \omega} d\omega$$



b Bunched Beam with energy spread [Wang - Pellegrini]

$$1 = -(\Delta\Omega)^2 \int \frac{\left(\frac{dg}{d\omega}\right)}{(\delta\Omega - \omega)} \cdot d\omega$$

normal modes  
 $\leftrightarrow$  (TDA RPA equations)  
 (phonon)

$$\int g \, d\omega = 1$$

$$\Delta\Omega^2 = i \frac{e^2 \eta \omega_0^2 I_{peak}}{2\pi \beta^2 E} \left(\frac{Z}{n}\right)$$

$$\propto \left[ \frac{\left(\frac{N}{B}\right) \cdot e^2}{A} \right]$$

$$= i \frac{e^2 \eta \omega_0^3 N}{(2\pi)^2 \beta^2 E} G(\theta) \left(\frac{Z}{n}\right)$$

see p. 4

$$\zeta \equiv \frac{Z}{n} = \zeta_r + i \zeta_i$$

$$\langle n \rangle = \frac{R}{b} \approx 2.0 \times 10^4 \quad (\text{RHIC})$$

RHIC-28

	P,	<sup>2</sup> H	C	S	Cu	I	Au
$\frac{N}{B} [10^9]$	$10^3$	$10^2$	22	64	4.5	2.6	1.1
e	1	1	6	16	29	53	79
A	1	2	12	32	63	127	197

50	66	51.2	60.1	57.5	34.8
			3.8		3.4

$$\frac{Z}{n} = 2 + 5 \times 10^{-4}$$

Growth

### III Conclusions and Recommendation

From the following table and figure, the following

conclusions can be drawn:

(1) less instability for higher harmonic number  $h$

(2) The phase space area should be at least

$0.4 \text{ evsec/amu}$  before the transition.

(3) Calculation indicates that there is an interesting correlation between the total growth

$$G = e^{\int \Omega_i dt} \quad (\Omega_i > 0)$$

and the parameters  $\left| \frac{Z}{n} \right| * N_B$ .

$^{197}_{79}\text{Au}$

$\gamma_T = 26.4$

$\langle n \rangle = 2,034 \times 10^4$

$V = 1. \text{MV}$

$\bar{\beta} = 29.5 \text{ m}$

$\phi_s = 5.45^\circ$

$\bar{x}_p = 1.0$

$\dot{\gamma} = 3.2 / \text{sec}$

$R = 610,168 \text{ m}^m, a = 0.03 \text{ m} \left[ \begin{matrix} \text{tube} \\ \text{radius} \end{matrix} \right]$

$A_0$ [evsec]	H	$Re \left[ \frac{Z}{n} \right]$	$N_B$ [ $10^9$ ]	$G = e^{\int \Omega_i dt}$	$\left  \frac{Z}{n} \right  \times N_B$	
0.2	12x57	0	1.15	1.17	1.38	
		5	4.15	19.7	5.91	
		1	1.15	1.29	1.80	
			2.30	3.05	3.59	
		3	0.58	1.29	1.87	
			1.15	2.97	3.72	
				2.30	162	7.43
	6x57	1	1.15	1.42		
				2.30	4.59	
		3	0.58	1.39		
			1.15	4.05		
			2.30	6.92		
		5	1.15	43.76		
0.4	12x57	5	1.15	1.56		
			2.30	6.88		
	6x57	5	1.15	1.75		
			2.30	11.63		
0.4	12x57	5	4.5	2.91		

$^{63}_{29}\text{Cu}$

