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National Laboratory

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Multiple Coulomb Scattering a) Tandem Exit b) Booster Exit

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February 1984

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U.S. Department of Energy

USDOE Office of Science (SC)

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MULTIPLE COULOMB SCATTERING AND EMITTANCE
GROWTH IN STRIPPER FOILS

- a) Tandem Exit
- b) Booster Exit

Glenn Young

February 23, 1984

Brookhaven National Laboratory

is small,

Rewriting the equation for $\langle \Theta^2 \rangle$, we have

$$\sqrt{\langle \Theta^2 \rangle} \equiv \Theta_{\text{plane}}^{\text{rms}} = \sqrt{2\pi \frac{\rho N_A}{A} \pm \frac{Z^2}{2} \ln(210 Z^{-1/3})} \quad \frac{2Z}{p} \frac{e^2}{hc} \frac{tc}{\beta c}$$

$(\beta = v/c)$

We can approximate $\frac{X_0^{-1}}{4 \alpha r_0^2 N_A} \sim \frac{Z^2 \ln(210 Z^{-1/3})}{A}$

and err by only 10% or less for our stripping foils, except for carbon where the neglect of the second coulomb log gives ~16% error for $(\Theta_{\text{plane}}^{\text{rms}})^2$. This gives a convenient formula in terms of X_0 , which is tabulated; the correction is included by multiplying by $(1+\delta)$ where δ is evaluated for each material.

For carbon, $\delta = -0.079$
 copper, $\delta = 0.00193$
 tantalum, $\delta = 0.0478$

This then works out to

$$\Theta_{\text{plane}}^{\text{rms}} = (1+\delta) Z, \frac{14.98 \text{ MeV}/c}{p \beta} \sqrt{\frac{\Delta}{X_0}}$$

where Δ is the scatterer thickness in g/cm^2 and X_0 is in g/cm^2 . X_0 is tabulated in Tsai's article, or see the Particle Data Properties booklet, $X_0 = 42.70 \text{ g}/\text{cm}^2$ carbon; $6.82 \text{ g}/\text{cm}^2$ tantalum
 $12.86 \text{ g}/\text{cm}^2$ copper

2/23/84

In the following graphs, values of $\theta_{\text{plane}}^{\text{rms}}$ are given for C, S, Cu, I and Au ions for the following cases.

- (a) Tandem exit energy/nucleon expected for 2 stage operation
5-20 $\mu\text{g}/\text{cm}^2$ carbon stripper
- (b) booster exit 300-500 MeV/nucleon i.e. matching to
performance expected for gold
10-100 mg/cm^2 copper stripper

Emitance growth can be found from

$\epsilon = \beta (\theta_0^2 + \theta_{\text{plane}}^{\text{rms}^2})$, where $\epsilon = \beta \theta_0^2$ is the emitance prior to passing through the stripper. The last graphs then give values of the β function required to limit the emitance growth at the tandem exit to $0.5 \pi \text{ mm mrad}$, and at the booster exit to $1.0 \pi \text{ mm mrad}$. Carbon is fully stripped after the tandem exit stripper. The curves for ^{32}S and ^{63}Cu at the booster exit are included for completeness only. The stripper thicknesses listed are overkill by at least a factor of 100 or more.

As usual, gold yields the most stringent requirements.

At the tandem exit, 20 $\mu\text{g}/\text{cm}^2$ carbon @ 1 MeV/A requires $\beta^* = 14 \text{ cm}$. At the booster exit, 367 MeV/A thru a 70 mg/cm^2 Copper stripper requires $\beta^* = 2.02 \text{ m}$. This line can handle all lighter beams, as they use thinner stripping foils.

NOTE ADDED

Another formula for multiple scattering of heavy ions is given by Green (U. Washington, Seattle, Annual Report 1975). It gives somewhat more optimistic numbers than the one from Jackson.

The formula is

$$\Theta_{1/2} = K \frac{Z Z_1 Z_2}{E_1} \tau^n \quad \left(\text{Divide by } 1.175 \text{ to get } \sigma_{\text{plane}}^{\text{rms}} \right)$$

where $K = 1.92 \cdot 10^{-2}$ mrad, $Z_1 = Z$ of ion, $Z_2 = Z$ of foil,
 $Z = (Z_1^{2/3} + Z_2^{2/3})^{3/2}$ (this appears from calculating an effective screening radius),
 E_1 is the total kinetic energy of the ion, and τ and n are given by

$$\tau = \frac{41.5 \delta}{Z^2 A}$$

$\delta =$ foil thickness in $\mu\text{g}/\text{cm}^2$, $A =$ foil Atomic weight

and $n = N^{-.115} - .115$ where $N = \ln(1.03 + \tau)$. Needless to

say, this is a phenomenological formula.

PAST EXPERIENCE AT OAKRIDGE IS THAT THIS GIVES TOO SMALL VALUES BY ~ FACTOR OF TWO

To give some values to compare, consider ¹⁹⁷Au at the tandem and booster exits, passing thru 20 $\mu\text{g}/\text{cm}^2$ C and 70 $\mu\text{g}/\text{cm}^2$ Cu shippers, respectively. A comparison of the two formulae results is below.

	Tandem				Booster			
	.5 MeV/A	1 MeV/A	1.5 MeV/A	300	350	400	450	MeV/A
Jackson's	3.782	1.894	1.263	1.40	1.86	2.34	2.87	
Green's	.978	.489	.326	3.77	5.14	6.71	8.50	
Booster	300	350	400	450	300	350	400	450
Jackson's	.844	.733	.654	.590	1.40	1.86	2.34	2.87
Green's	.515	.441	.386	.343	3.77	5.14	6.71	8.50

$\Theta_{\text{plane}}^{\text{rms}}$ (mrad)

β^* value needed

46
3/1/84

CONCLUSION

- (1) Some attention has to be paid to the transport between the tandem high energy end and the stripper foil. A quad doublet has to be used to focus x and y to a small spot. β^* values somewhere in the range of 0.14 m (Jackson formula) to 2 m (Greene) formula are needed.

A measurement of emittance after the high energy stripper for the heavy beams is needed. We want a growth of less than 0.5π mm mrad so the injection into the booster works.

- (2) Stripping at the booster exit will require β^* of 2 m or larger, but again in both x and y . Again a quad doublet before the stripper is needed.

- (3) Better (smaller) values of scattering angles due to multiple Coulomb scattering could be had by using gold foils. Less thickness is needed to strip to the equilibrium charge state distribution due to the density effect in stripping.

For the tandem exit, this probably does not work as the thickness required ($10 \mu\text{g}/\text{cm}^2$) is mechanically quite fragile.

For the booster exit, foils of $20-30 \text{ mg}/\text{cm}^2$ are needed, which are quite rugged (0.02 mm thick). As the β^* required varies as the thickness (inversely) the β^* needed after the booster could be relaxed by a factor of two in both planes.

$\theta_{\text{plane}}^{\text{rms}}$ Mult Scatt5Tandem ExitCarbon Stripper (5, 10, 20 $\mu\text{g}/\text{cm}^2$)

(MeV/A)

 $\theta_{\text{plane}}^{\text{rms}}$ (mrad)

Ion	E/A	P/A	P	β	5 $\mu\text{g}/\text{cm}^2$	10 $\mu\text{g}/\text{cm}^2$	20 $\mu\text{g}/\text{cm}^2$
^{12}C (Z=6)	6.5	110.2	1322.8	.1175	0.182	0.258	0.364
	7	114.4	1372.9	.1219	0.169	0.239	0.339
	7.5	118.4	1421.3	.1261	0.158	0.224	0.316
	8	122.3	1468.1	.1302	0.148	0.210	0.296
	8.5	126.1	1513.5	.1342	0.139	0.197	0.279
^{32}S (Z=16)	3.5	80.8	2586.4	.0864	0.338	0.478	0.676
	4	86.4	2765.4	.0924	0.296	0.418	0.591
	4.5	91.7	2933.5	.0979	0.263	0.372	0.526
	5	96.6	3092.6	.1032	0.237	0.335	0.473
	5.5	101.4	3244.0	.1082	0.215	0.304	0.430
^{63}Cu (Z=29)	2	61.1	3847.6	.0654	0.544	0.769	1.088
	2.5	68.3	4302.4	.0731	0.435	0.616	0.871
	3	74.8	4713.6	.0804	0.363	0.513	0.725
	3.5	80.8	5092.0	.0864	0.311	0.440	0.622
	4	86.4	5444.3	.0924	0.272	0.385	0.544
^{127}I (Z=53)	.5	30.5	3876.6	.0328	1.968	2.783	3.936
	1	43.2	5483.1	.0463	0.986	1.394	1.971
	1.5	52.9	6716.3	.0567	0.657	0.929	1.314
	2	61.1	7752.4	.0654	0.493	0.698	0.986
^{197}Au (Z=79)	.5	30.5	6013.3	.0328	1.891	2.674	3.782
	1	43.2	8505.3	.0463	0.947	1.339	1.894
	1.5	52.9	10418.2	.0567	0.631	0.893	1.263

$\theta_{\text{plane}}^{\text{rms}}$ Mult Scatt6Booster ExitCopper Strippes (10, 40, 70, 100 mg/cm^2)

Ion	E/A	P/A ($\frac{\text{MeV}}{c \cdot A}$)	p ($\frac{\text{GeV}}{c}$)	β	10 mg/cm^2	40 mg/cm^2	70 mg/cm^2	100 $\frac{\text{mg}}{\text{cm}^2}$
^{32}S Z=16	300	805.5	25.78	0.6541	0.397	0.794	1.051	1.255
	350	880.1	28.2	0.6868	0.346	0.692	0.915	1.094
	400	951.4	30.44	0.7145	0.308	0.616	0.815	0.974
	450	1020.2	32.65	0.7385	0.278	0.554	0.736	0.879
	500	1086.97	34.78	0.7593	0.254	0.508	0.672	0.803
^{63}Cu Z=29	300	805.5	50.75	0.6541	0.366	0.732	0.968	1.157
	350	880.1	55.4	0.6868	0.318	0.636	0.841	1.006
	400	951.4	59.94	0.7145	0.283	0.566	0.749	0.895
	450	1020.2	64.27	0.7385	0.256	0.502	0.677	0.810
	500	1087.0	68.48	0.7593	0.233	0.466	0.616	0.737
^{127}I Z=53	300	805.5	102.3	0.6541	0.331	0.662	0.876	1.047
	350	880.1	111.8	0.6868	0.289	0.578	0.765	0.914
	400	951.4	120.8	0.7145	0.257	0.514	0.680	0.813
	450	1020.2	129.6	0.7385	0.232	0.464	0.614	0.734
	500	1087.0	138.0	0.7593	0.212	0.424	0.561	0.670
^{197}Au Z=79	300	805.5	158.7	0.6541	0.319	0.638	0.844	1.009
	350	880.1	173.4	0.6868	0.277	0.554	0.733	0.876
	400	951.4	187.4	0.7145	0.247	0.494	0.654	0.781
	450	1020.2	200.9	0.7385	0.223	0.446	0.590	0.705
	500	1087.0	214.1	0.7593	0.203	0.406	0.537	0.642

^{12}C Tandem Exit

$\theta_{\text{plane}}^{\text{rms}}$ Mult Scatt

GRAPHIC CORPORATION Buffalo, New York
Friedman J. S. A.
SEMI-LOGARITHMIC
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AD-0830-00

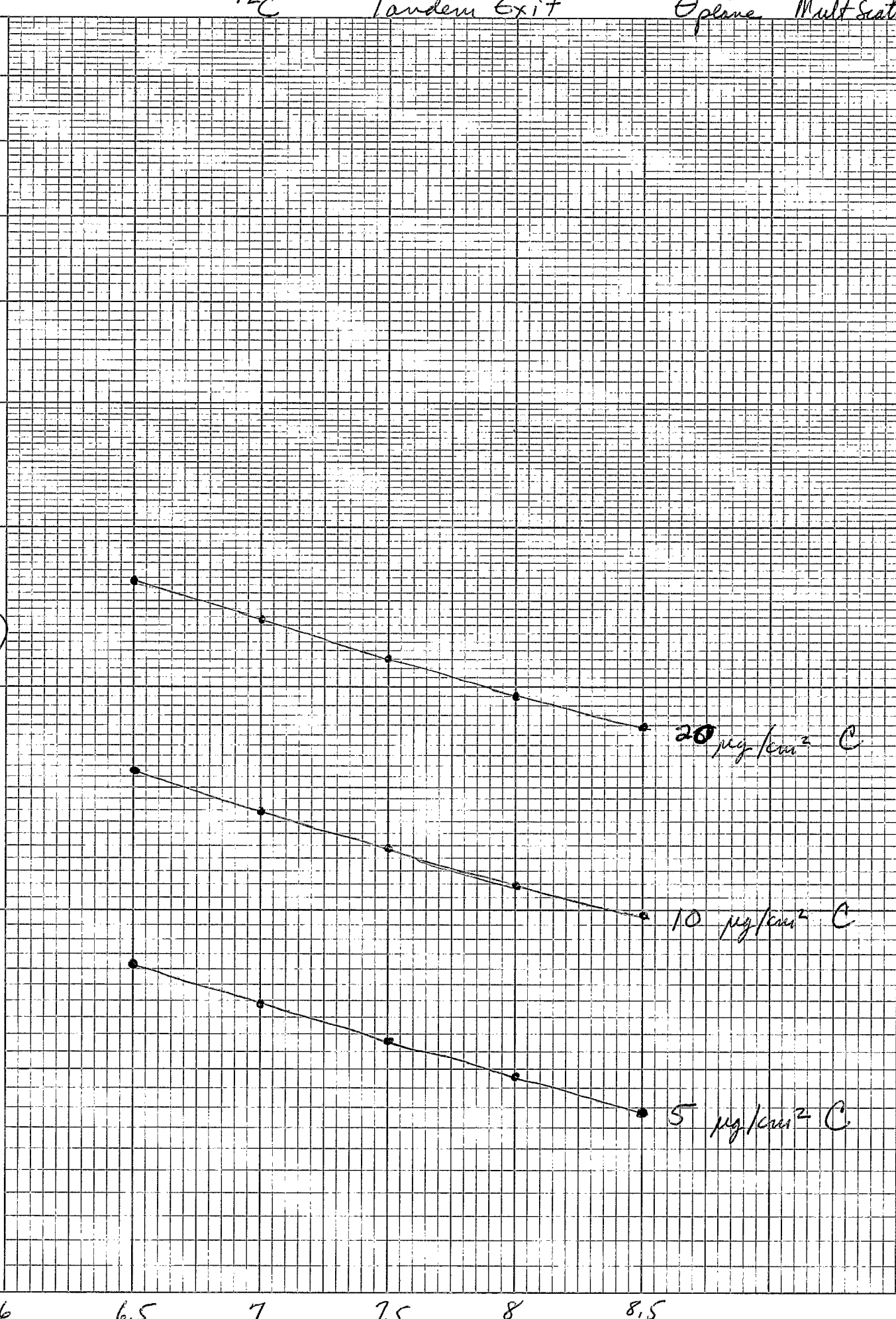
$\theta_{\text{plane}}^{\text{rms}}$ (mrad)

0.4⁴

0.3³

0.2²

0.1¹



20 $\mu\text{g}/\text{cm}^2$ C

10 $\mu\text{g}/\text{cm}^2$ C

5 $\mu\text{g}/\text{cm}^2$ C

MeV/A

32S

Tandem Exit

θ_{plane}^{rms}

Multi Scatt

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θ_{plane}^{rms}
(mrad)

0.7

0.6

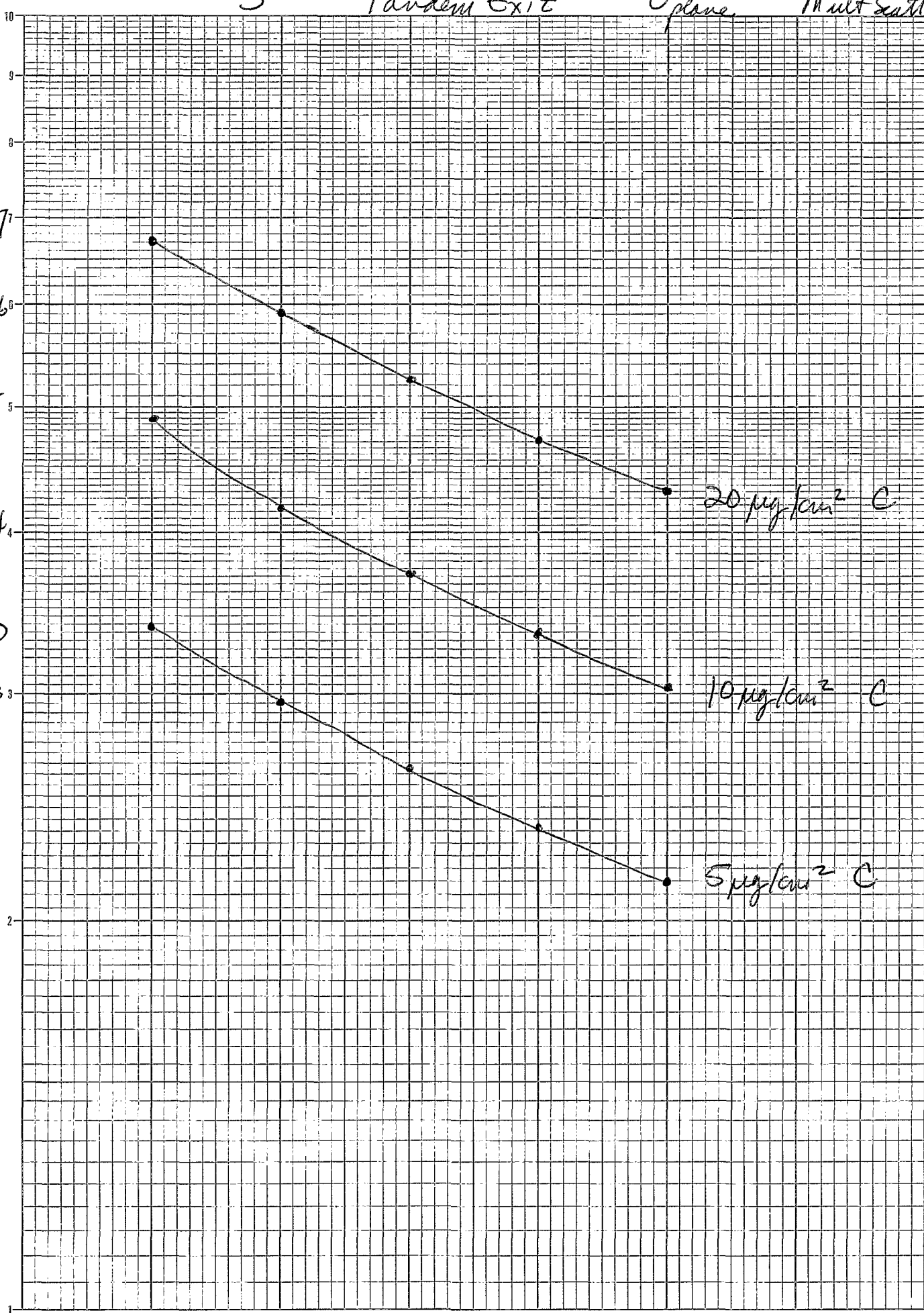
0.5

0.4

0.3

0.2

0.1



20 $\mu\text{g}/\text{cm}^2$ C

10 $\mu\text{g}/\text{cm}^2$ C

5 $\mu\text{g}/\text{cm}^2$ C

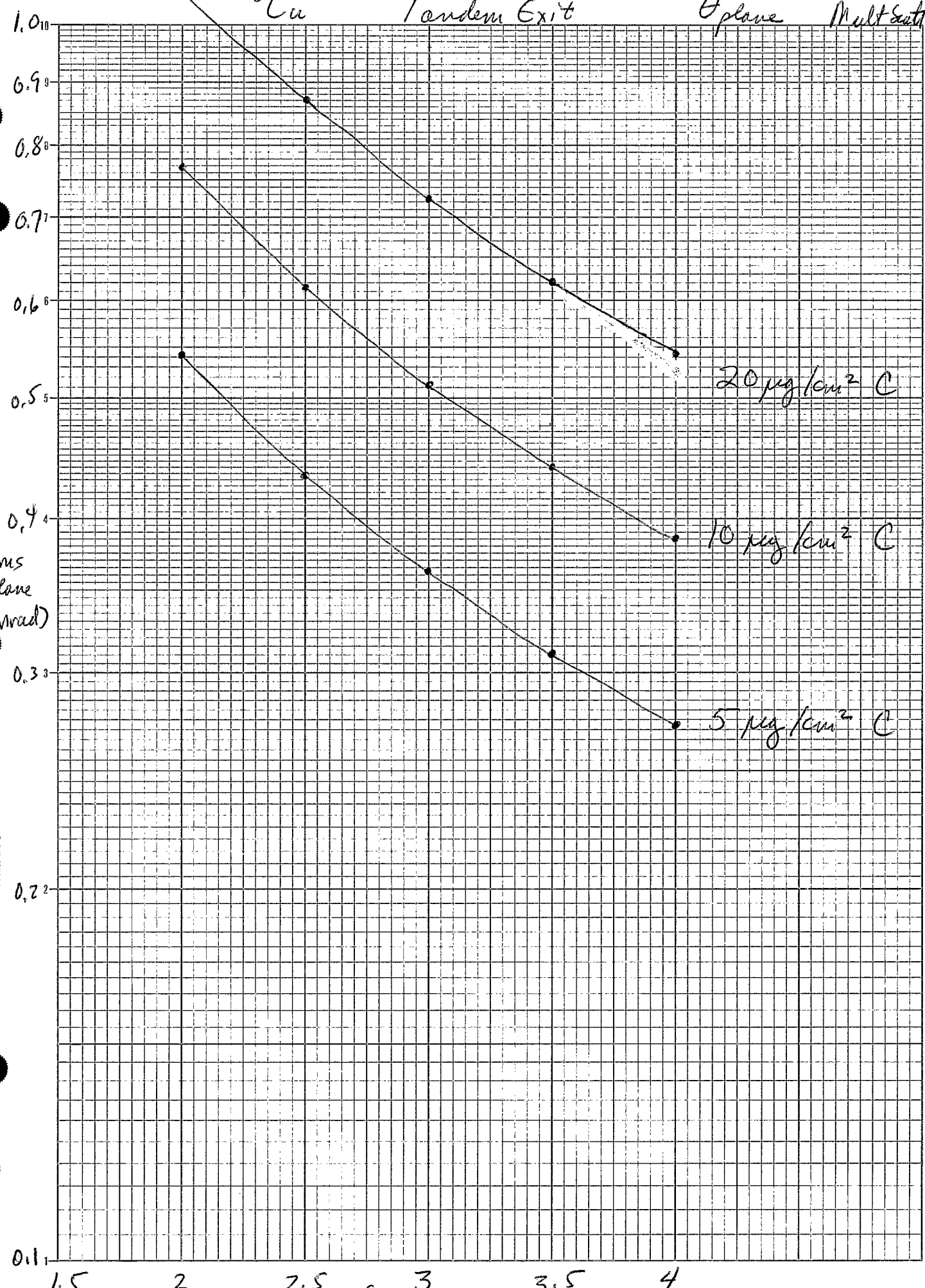
MeV

^{63}Cu

Tandem Exit

$\theta_{\text{plane}}^{\text{rms}}$

Mult Scatt



AD-0030-00
1 CYCLE X 70 DIVISIONS
SEMI-LOGARITHMIC
GRAPHIC CONTROLS CORPORATION
Buffalo, New York
Printed in U.S.A.

$\theta_{\text{plane}}^{\text{rms}}$
(mrad)

20 $\mu\text{g}/\text{cm}^2$ C

10 $\mu\text{g}/\text{cm}^2$ C

5 $\mu\text{g}/\text{cm}^2$ C

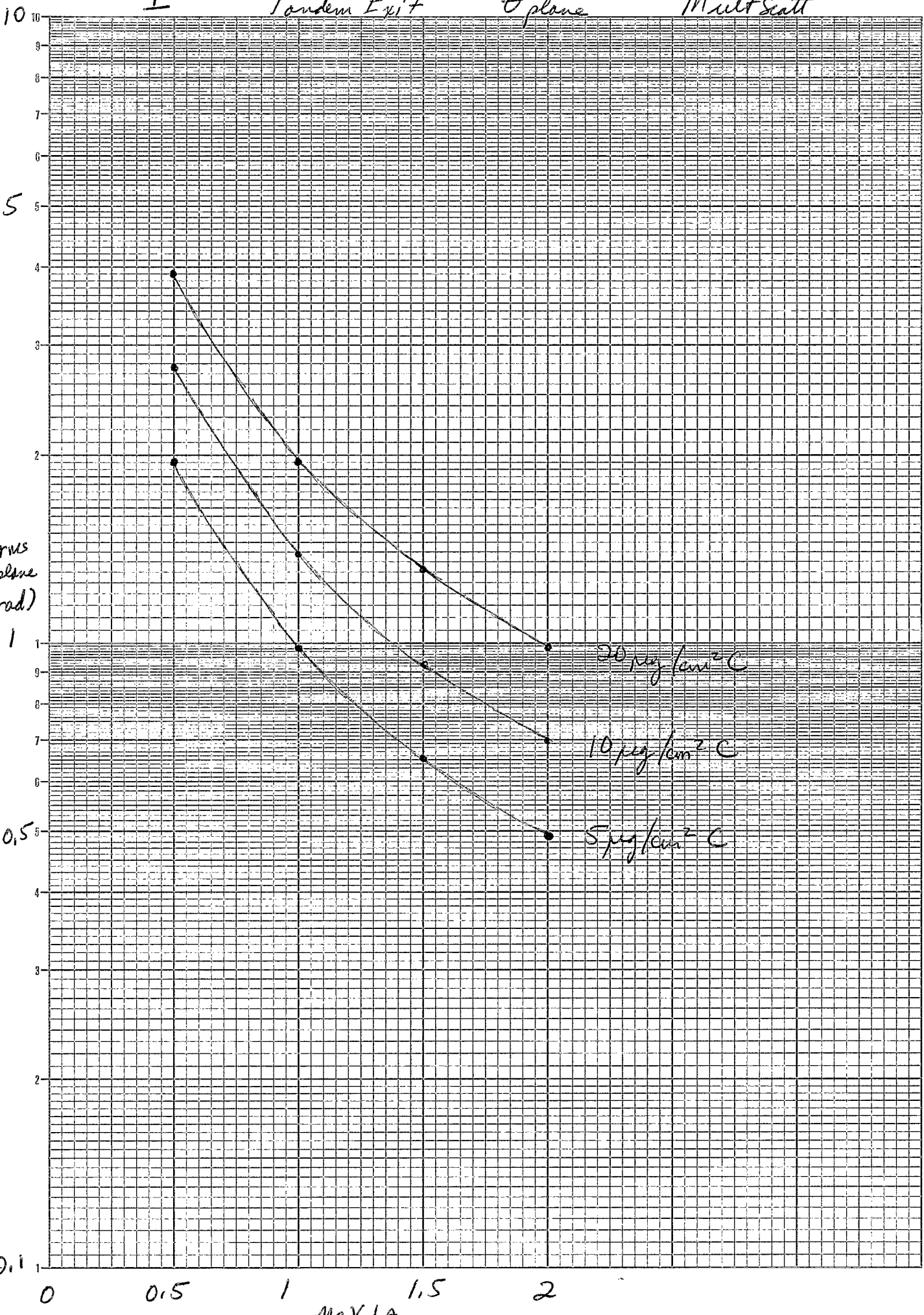
(MeV/A)

12/7

Tandem Exit

θ_{plane}^{rms}

Multiscatt



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θ_{plane}^{rms} (mrad)

GRAPHIC PAPERS GRAPHIC CONTROLS CORPORATION Buffalo, New York, U.S.A.

0.5

0.1

MeV/A

197 Au

Tandem Exit

θ_{plane}^{rms}

Mult Scatt

10

5

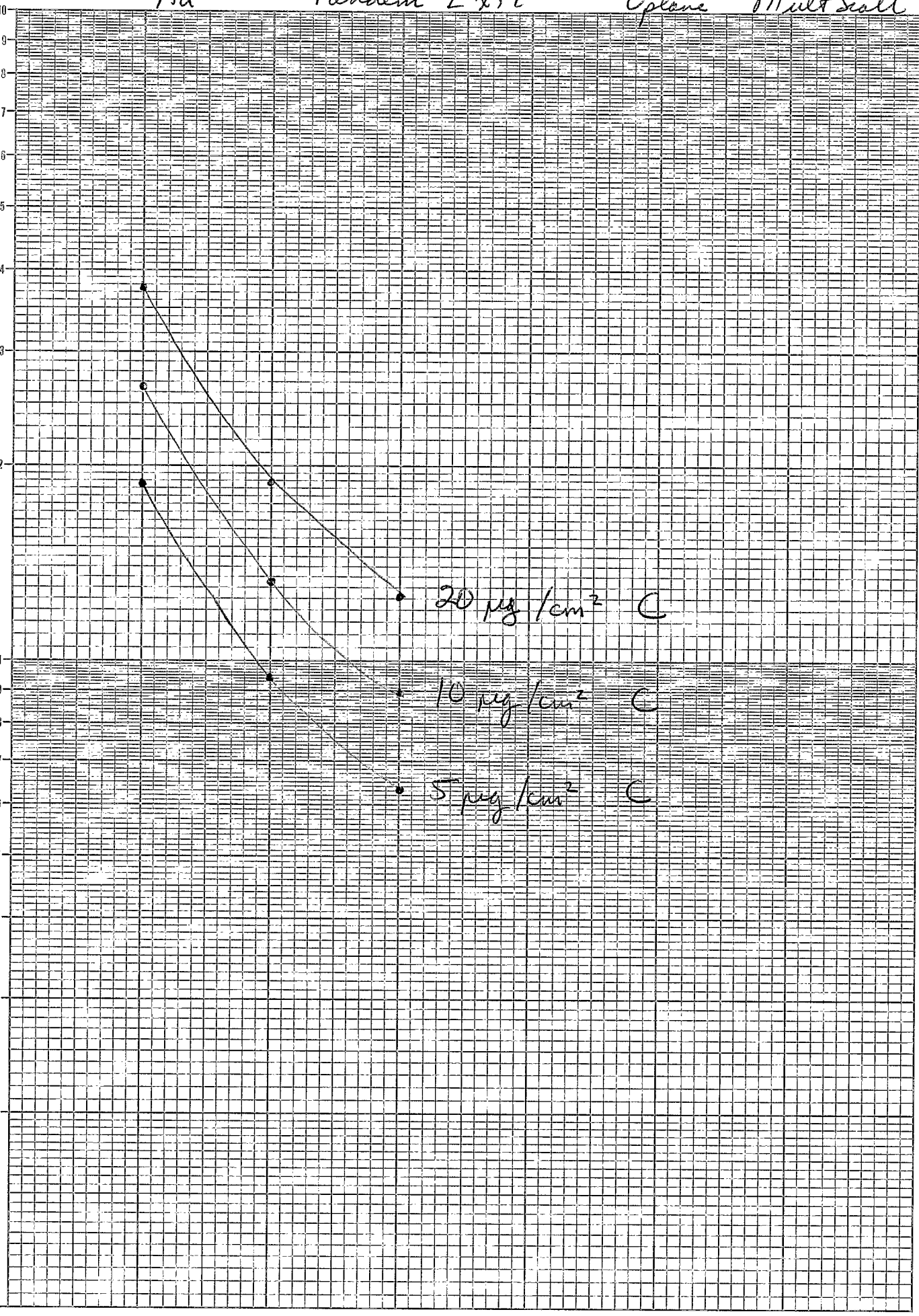
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0.5

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SEMI-LOGARITHMIC
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GRAPH PAPER GRAPHIC CONTROLS CORPORATION
Butte, Nev. U.S.A.
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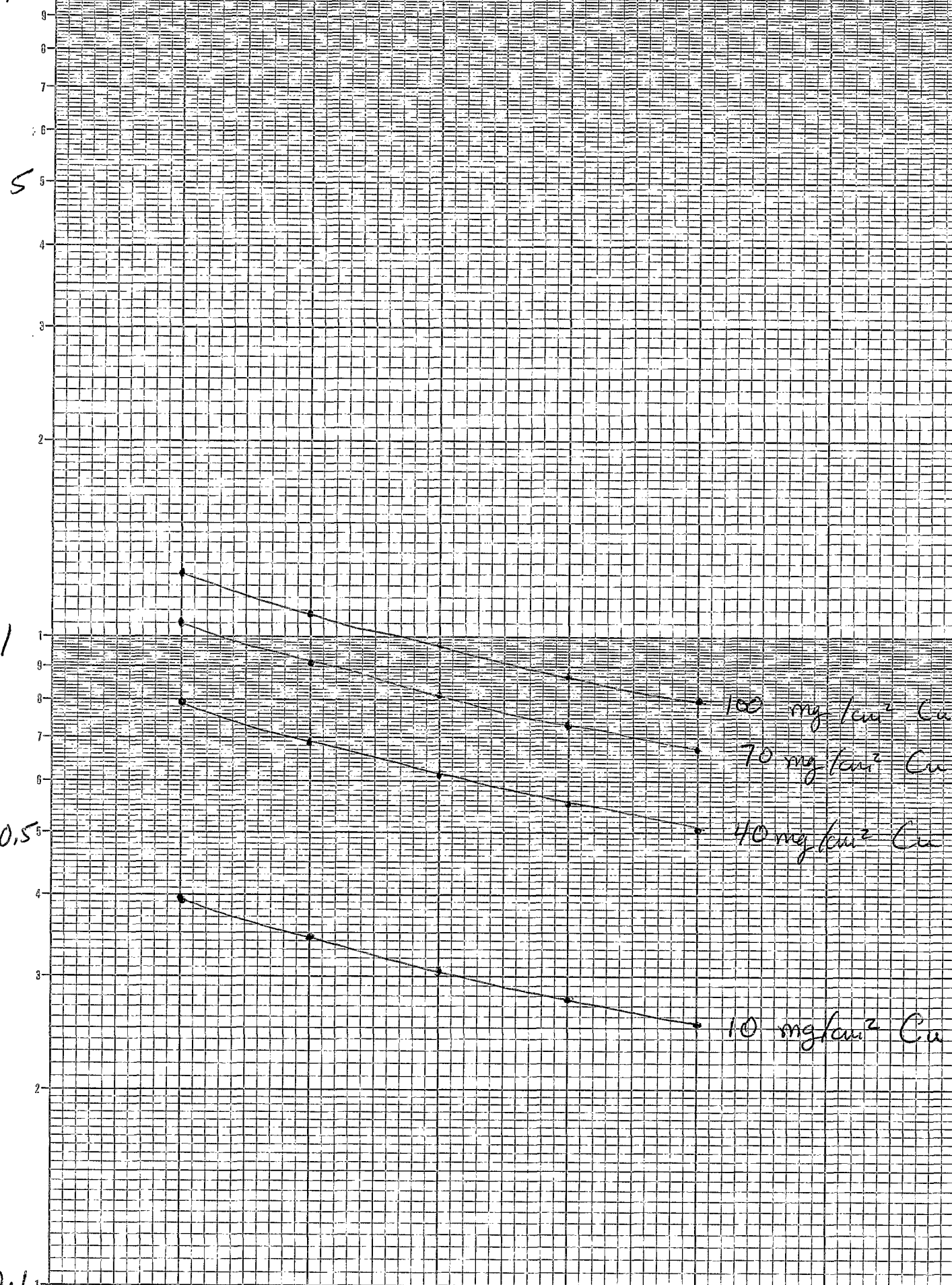
20 $\mu\text{g}/\text{cm}^2$ C

10 $\mu\text{g}/\text{cm}^2$ C

5 $\mu\text{g}/\text{cm}^2$ C

MeV/A

10 32S Booster Exit θ_{plane}^{rms} Mult Scatt

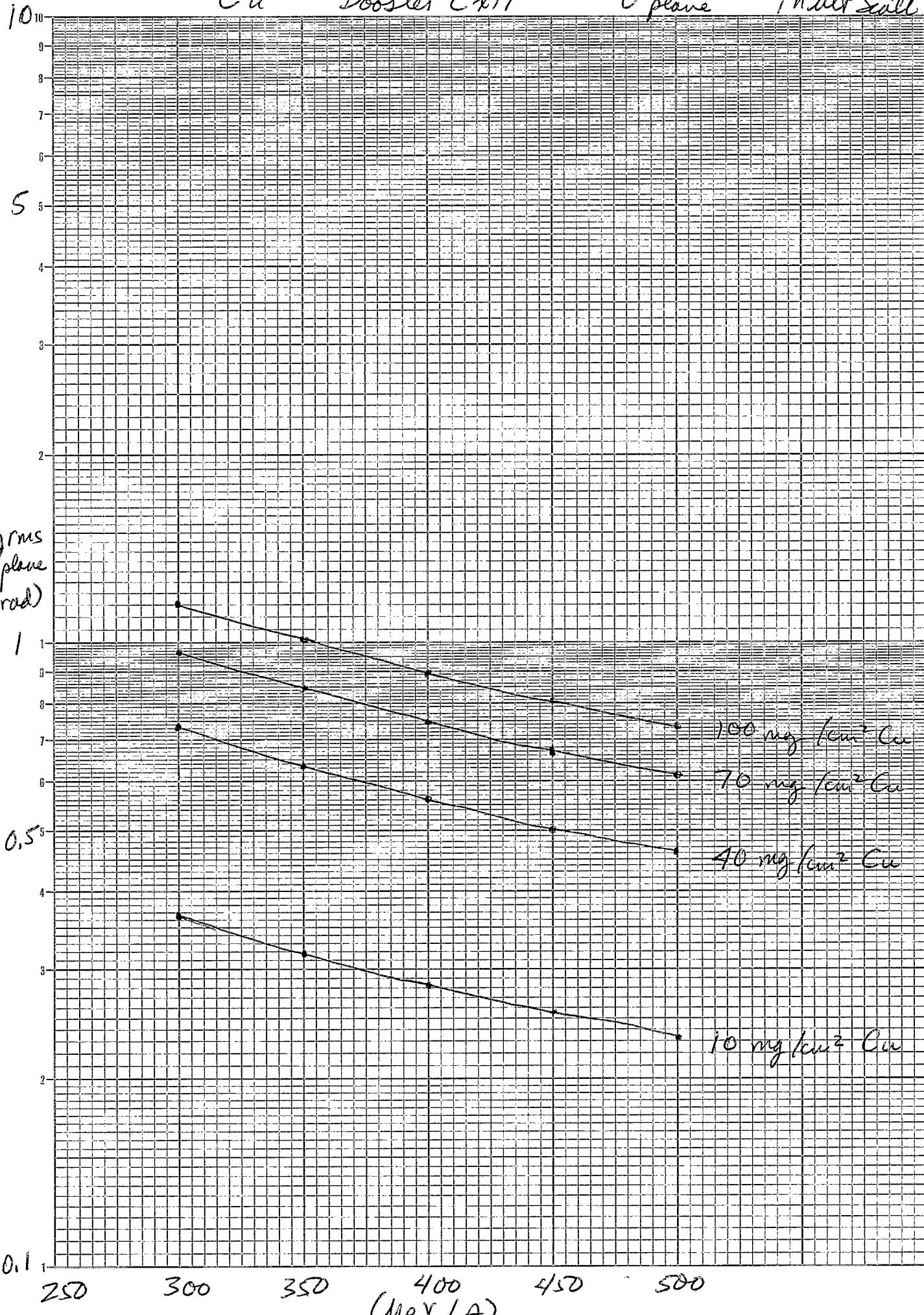


SEMI-LOGARITHMIC 2 CYCLES X 70 DIVISIONS AB-0033-60

GRAPHIC CONTROLS CORPORATION B. 1100 N. 17th Ave. Phoenix, U.S.A.

250 300 350 400 450 500 (MeV/A)

^{63}Cu Booster Exit $\theta_{\text{plane}}^{\text{rms}}$ Mult Scatt



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PR 4025-111 1 A

SEMI-LOGARITHMIC

2 CYCLES X 70 DIVISIONS

AD-0820-80

127 I

Booster Exit

θ_{rms}
plane

Mult Scatt

10

5

40-0830-60

2 CYCLES X 70 DIVISIONS

SEMI-LOGARITHMIC

θ_{rms}
plane
(mrad)

1

GRAPHIC CONTROLS CORPORATION

Billings, MT, U.S.A.

0.5

GRAPH PAPER

0.1

250

300

350

400

450

500

(ANAL)

100 mg/cm² Cu

70 mg/cm² Cu

40 mg/cm² Cu

10 mg/cm² Cu



197Au

Booster Exit

θ_{plane}^{rms}

Mult Scatt

10

5

00-0033-80

2 CYCLES X 70 DIVISIONS

SEMI LOGARITHMIC

θ_{plane}^{rms}

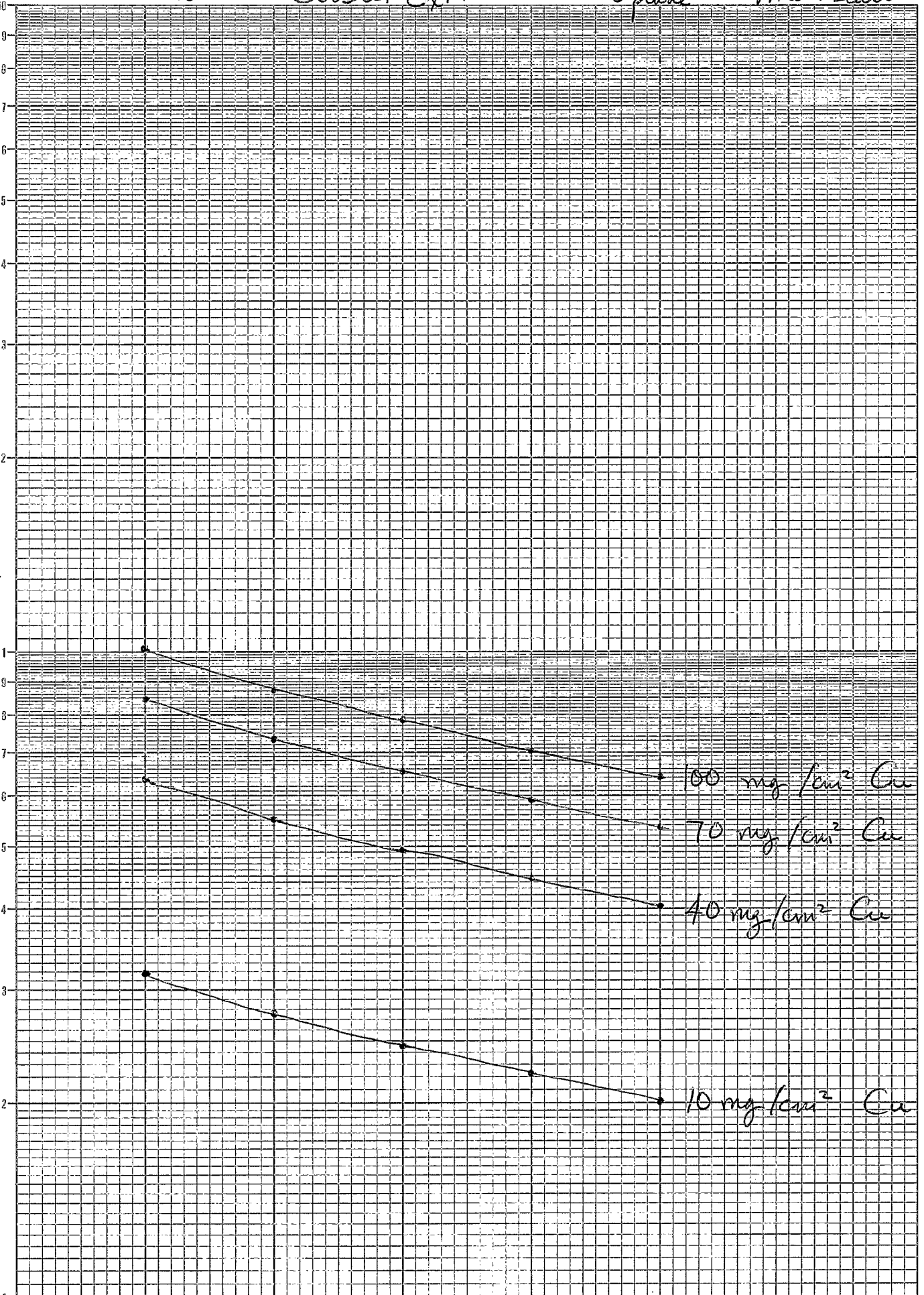
(mrad)

1

Buffalo, N.Y. 14203

GRAPHIC CONTROLS CORPORATION
P.O. Box 1154
Buffalo, N.Y. 14203

0.5



0.1

250 300 350 400 450 500
(MeV/A)

Tandem $241T$, Carbon, β_{max}^* values for $\Delta E = 0.577 \text{ mm mrad}$

Ion	MeV/A	$5 \mu\text{g/cm}^2$	$10 \mu\text{g/cm}^2$	$20 \mu\text{g/cm}^2$
^{12}C	6.5	15.1	7.51	3.77
	7	17.5	8.75	4.35
	7.5	20.0	10.0	5.01
	8	22.8	11.4	5.71
	8.5	25.9	12.5	6.42
^{32}S	3.5	4.38	2.19	1.09
	4	5.71	2.85	1.42
	4.5	7.23	3.61	1.81
	5	8.90	4.45	2.23
	5.5	10.8	5.41	2.71
^{63}Cu	2	1.69	0.85	0.42
	2.5	2.64	1.32	0.66
	3	3.79	1.89	0.95
	3.5	5.17	2.59	1.29
	4	6.76	3.38	1.69
^{127}I	.5	0.13	0.065	0.032
	1	0.51	0.257	0.128
	1.5	1.16	0.579	0.29
	2	2.06	1.03	0.51
^{197}Au	.5	0.14	0.070	0.035
	1	0.56	0.28	0.14
	1.5	1.26	0.63	0.31

Booster Exit, Copper, β^+ max values for $\Delta E = 1 \pi$ mm mrad

Ion	MeV/A	10 mg/cm ²	40 mg/cm ²	70 mg/cm ²	100 mg/cm ²
³² S	300	6.34	1.59	0.91	0.63
	350	8.35	2.09	1.19	0.84
	400	10.5	2.63	1.5	1.05
	450	12.9	3.23	1.84	1.29
	500	15.5	3.88	2.21	1.55

⁶³ Cu	300	7.47	1.87	1.07	0.75
	350	9.89	2.47	1.41	0.99
	400	12.5	3.12	1.79	1.25
	450	15.3	3.83	2.19	1.53
	500	18.4	4.6	2.63	1.84

¹²⁷ I	300	9.13	2.28	1.30	0.91
	350	11.97	2.99	1.71	1.20
	400	15.1	3.78	2.16	1.51
	450	18.6	4.65	2.66	1.86
	500	22.2	5.55	3.17	2.22

¹⁹⁷ Au	300	9.83	2.46	1.40	0.98
	350	13.0	3.25	1.86	1.30
	400	16.4	4.1	2.34	1.64
	450	20.1	5.03	2.87	2.01
	500	24.3	6.08	3.47	2.43

Tandem Exit

^{12}C

β_{mix}^{40} values (m)

MuHScott

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SEMI-LOGARITHMIC 2 CYCLES X 70 DIVISIONS AD-0003-80

β_{mix}^{40} (m)

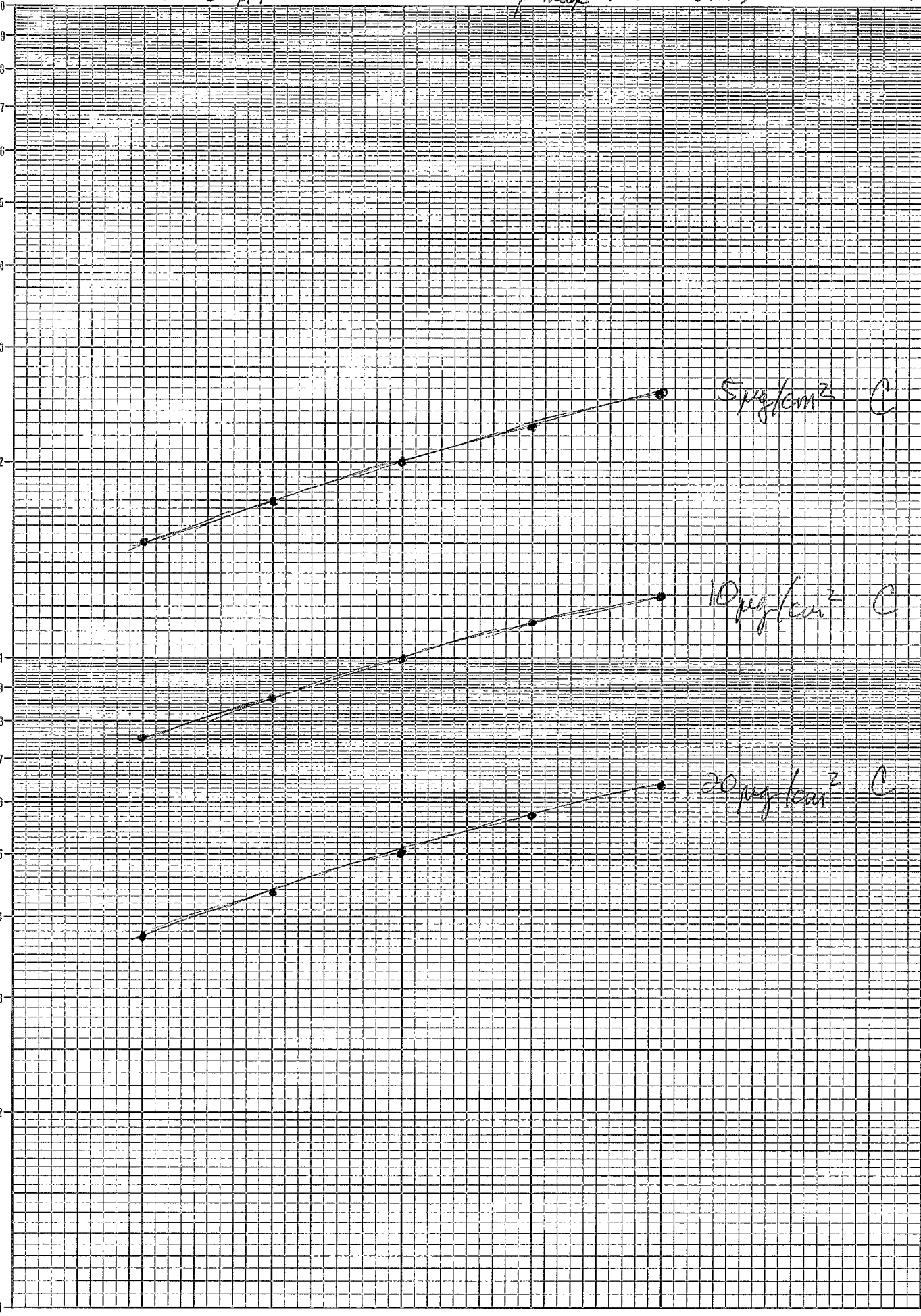
20

10

5

2

1



Tandem Exit

³²S

β^+ max values (m)

Mult Scatt

AG-0085-30
2 CYCLES X 70 DIVISIONS
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20²

β^+ (m)

10

5

2

1

3

3.5

4

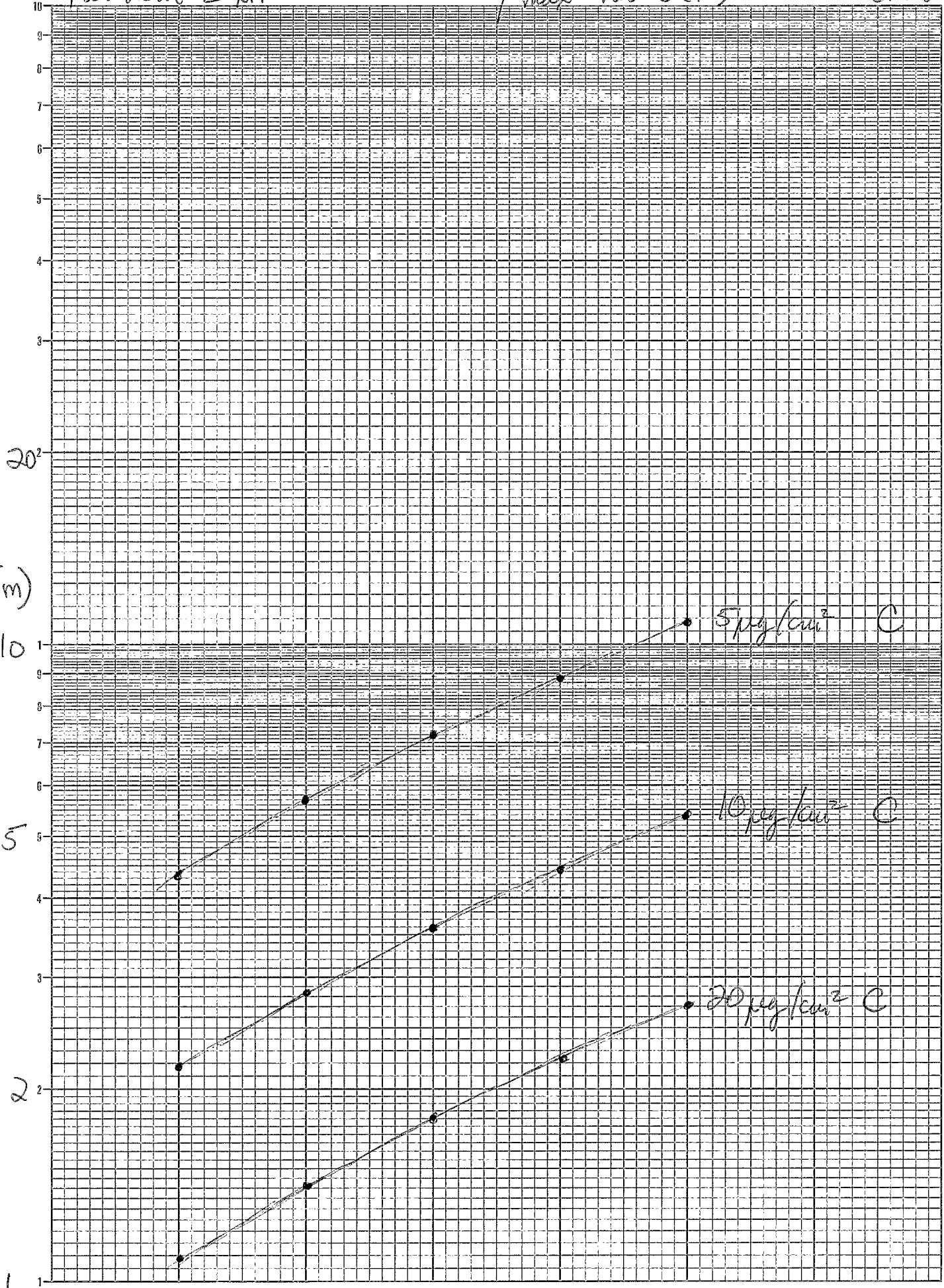
4.5 (MeV/A)⁵

5.5

5 $\mu\text{g}/\text{cm}^2$ C

10 $\mu\text{g}/\text{cm}^2$ C

20 $\mu\text{g}/\text{cm}^2$ C



Tandem Exit

^{63}Cu

β^* max values (m)

Mult Scatt

10

5

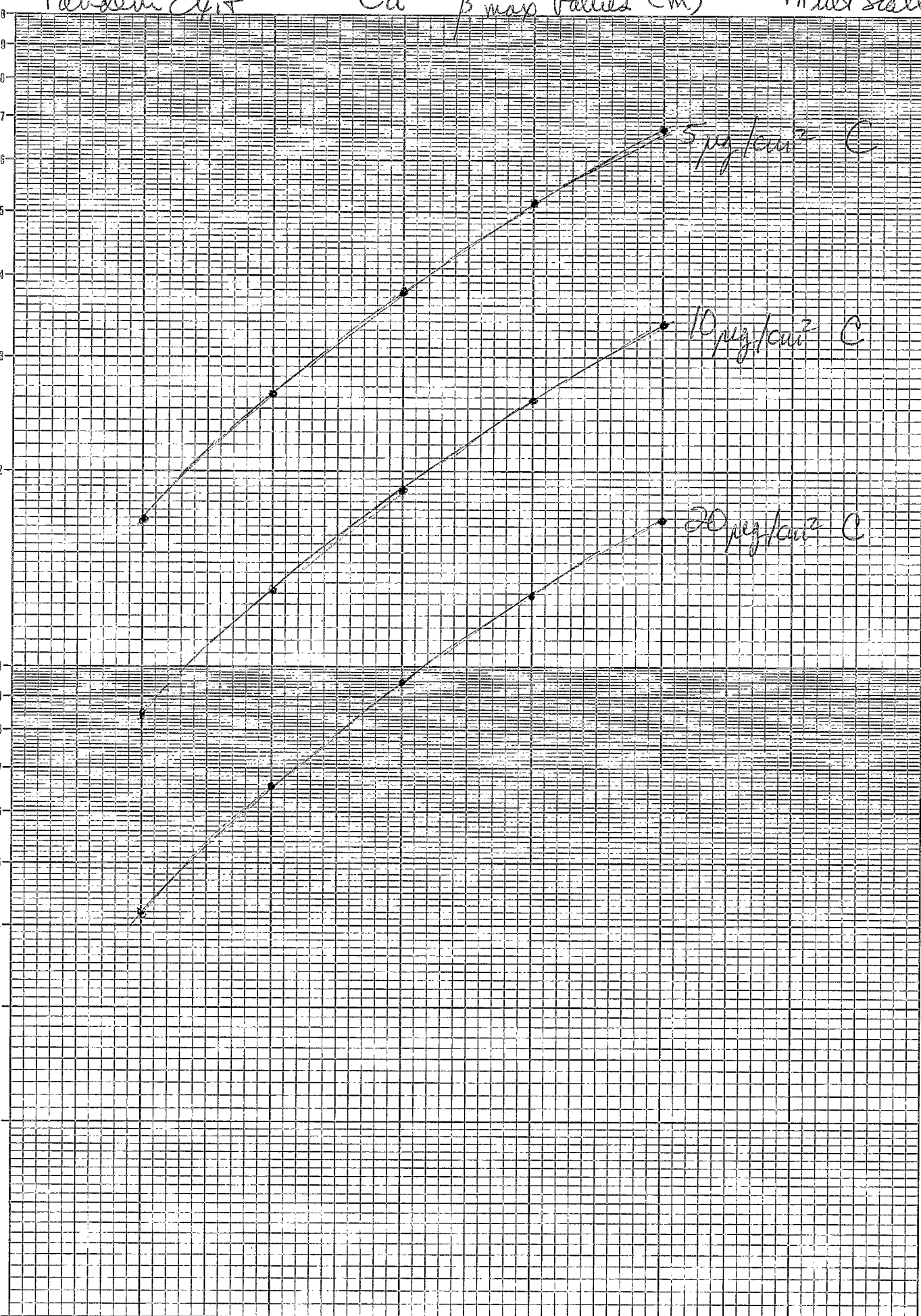
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1

0.5

0.2²

0.1



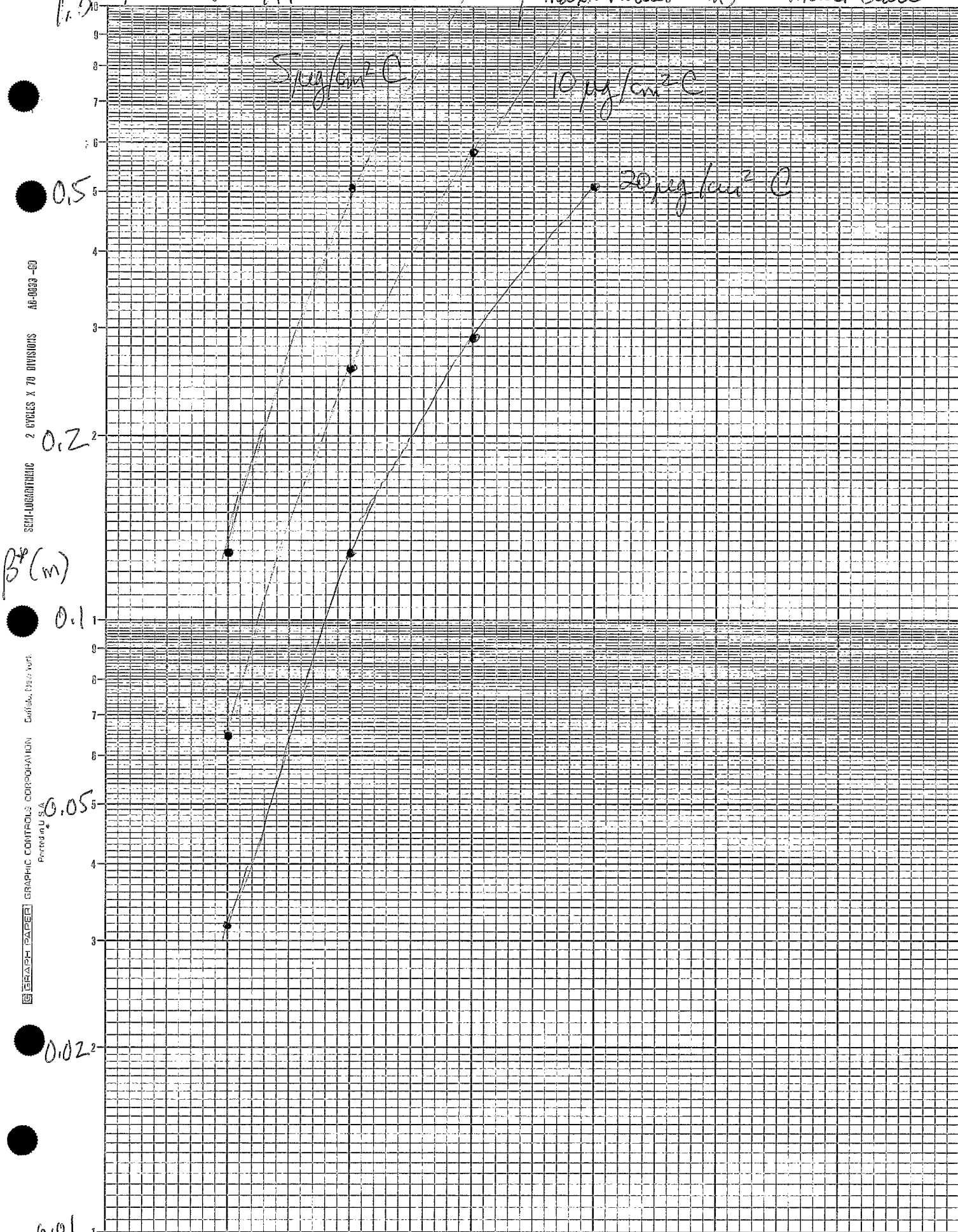
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 SEMI-LOGARITHMIC
 2 CYCLES X 70 DIVISIONS AD-6033-60

Tandem Epit 127I β^+ max. values (m) Mult Scatt

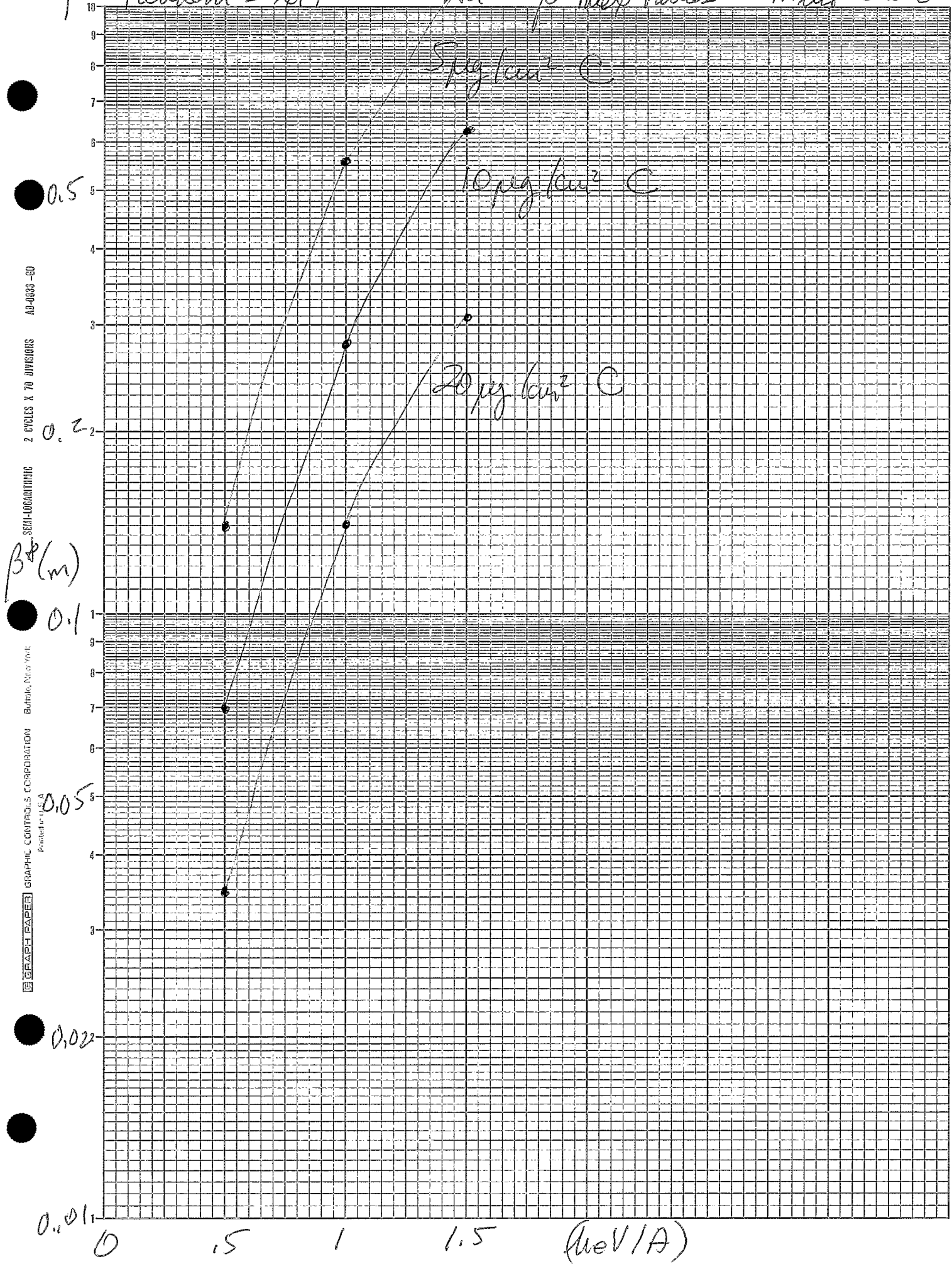
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β^+ (m)
 0.1
 0.05
 0.02
 0.01

5 $\mu\text{g}/\text{cm}^2 \text{ C}$
 10 $\mu\text{g}/\text{cm}^2 \text{ C}$
 20 $\mu\text{g}/\text{cm}^2 \text{ C}$



1 Tandem Exit ^{197}Au β^* max values Mult Scatt



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SEMI-LOGARITHMIC

2 CYCLES X 70 DIVISIONS

AB-0033-80

0.5

0.2

0.1

0.05

0.02

0.01

10 Booster Exit ^{32}S β^+ max values Mult Scatt

5

2

AD-6893-60

2 CYCLES X 70 DIVISIONS

β^+ SEMI-LOGARITHMIC

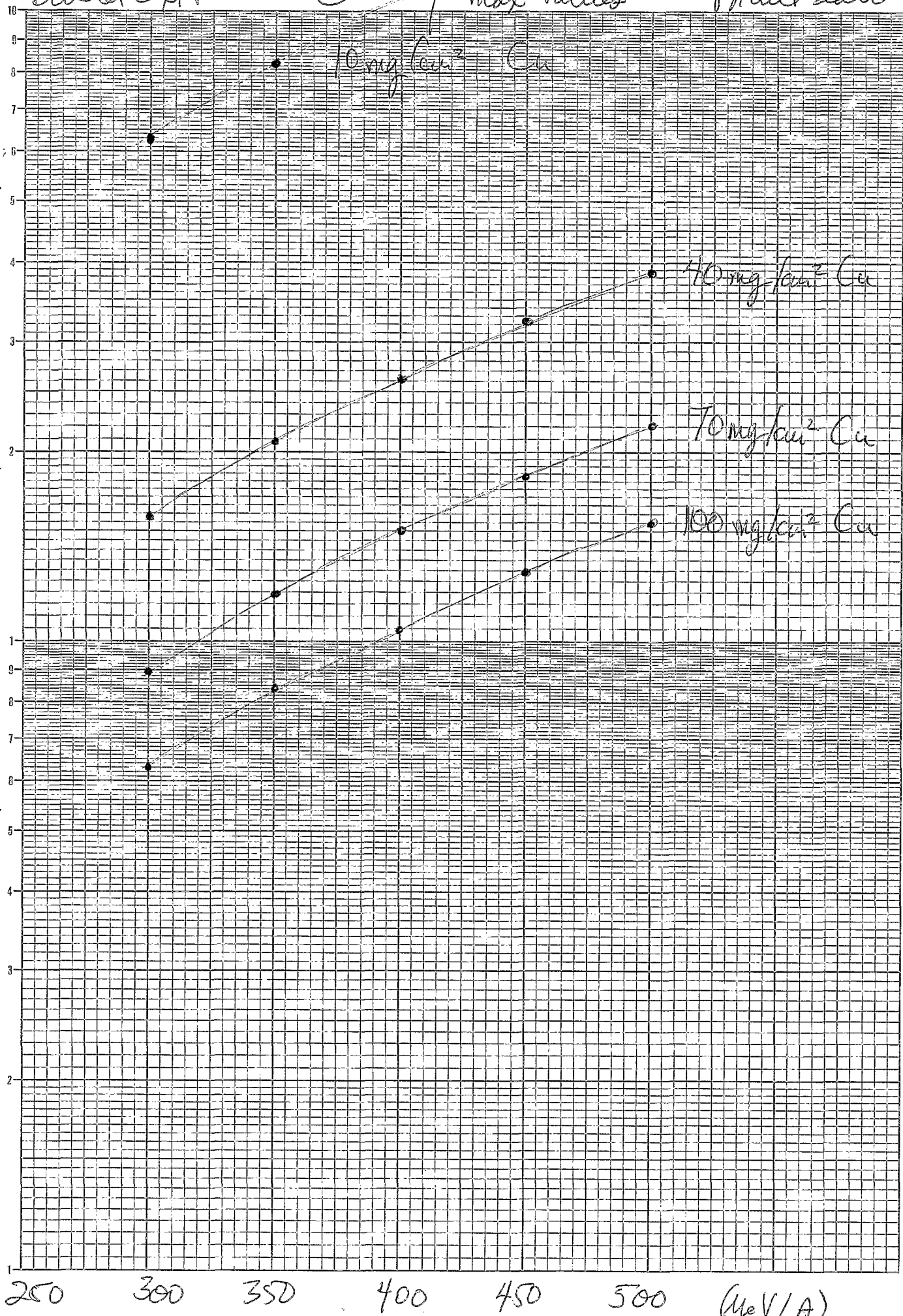
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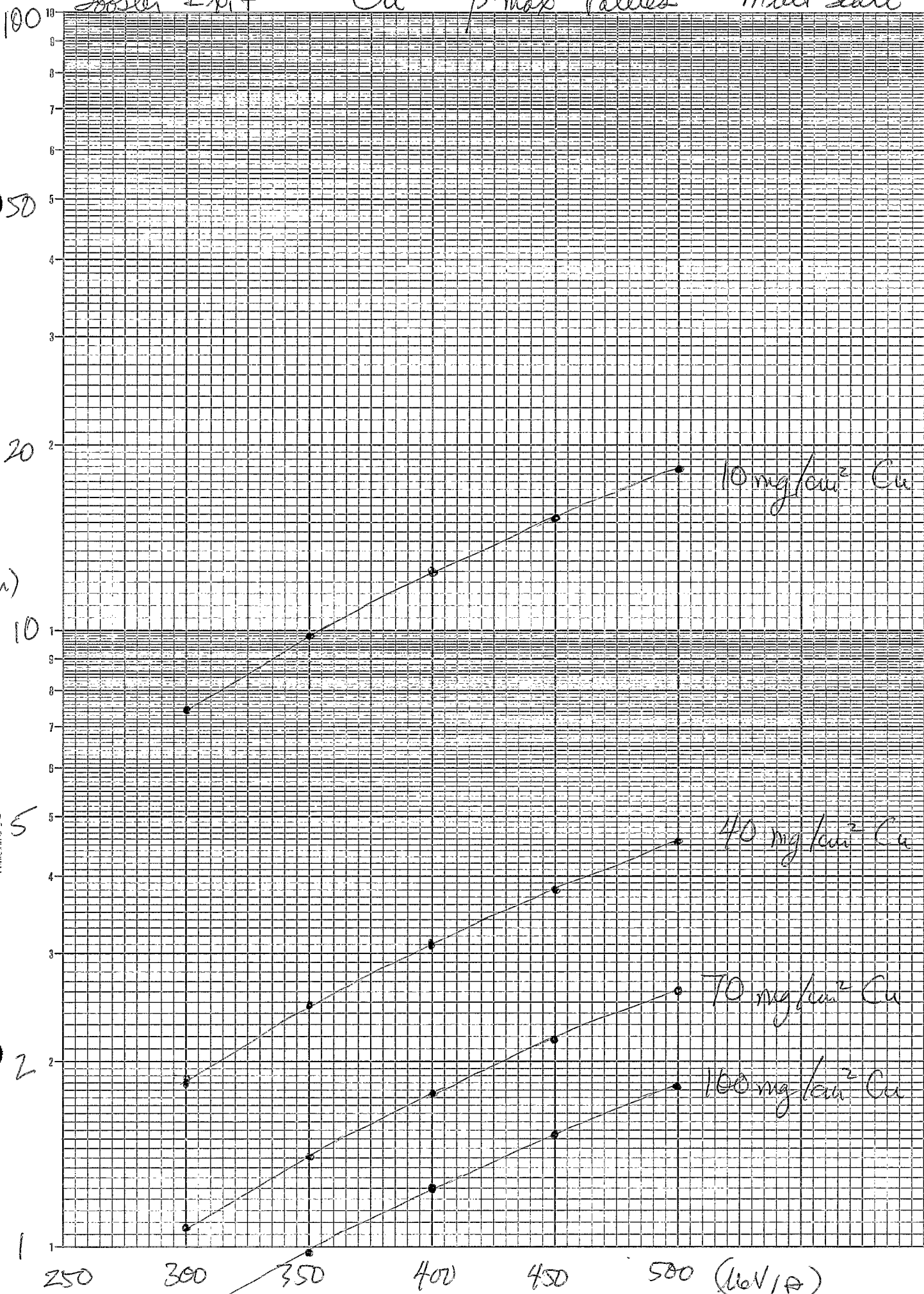
0.5

0.2

0.1



Booster Epi+ ⁶³Cu β* max values Mult Seatt



GRAPH PAPER GRAPHIC CONTROLS CORPORATION Buffalo, New York
 2 CYCLES X 70 DIVISIONS
 SEMI-LOGARITHMIC
 70-0033-68
 50
 20
 10
 5
 2
 1

Booster Exit 127 I β^* max values Mult Scatt

SEMI-LOGARITHMIC 2 CYCLES X 70 DIVISIONS AD-8033-60 GRAPHIC CONTROLS CORPORATION Buffalo, New York Printed in U.S.A.

β_{rel}^* (m)

50

10

2

1

100
9
8
7
6
5
4
3
2
1

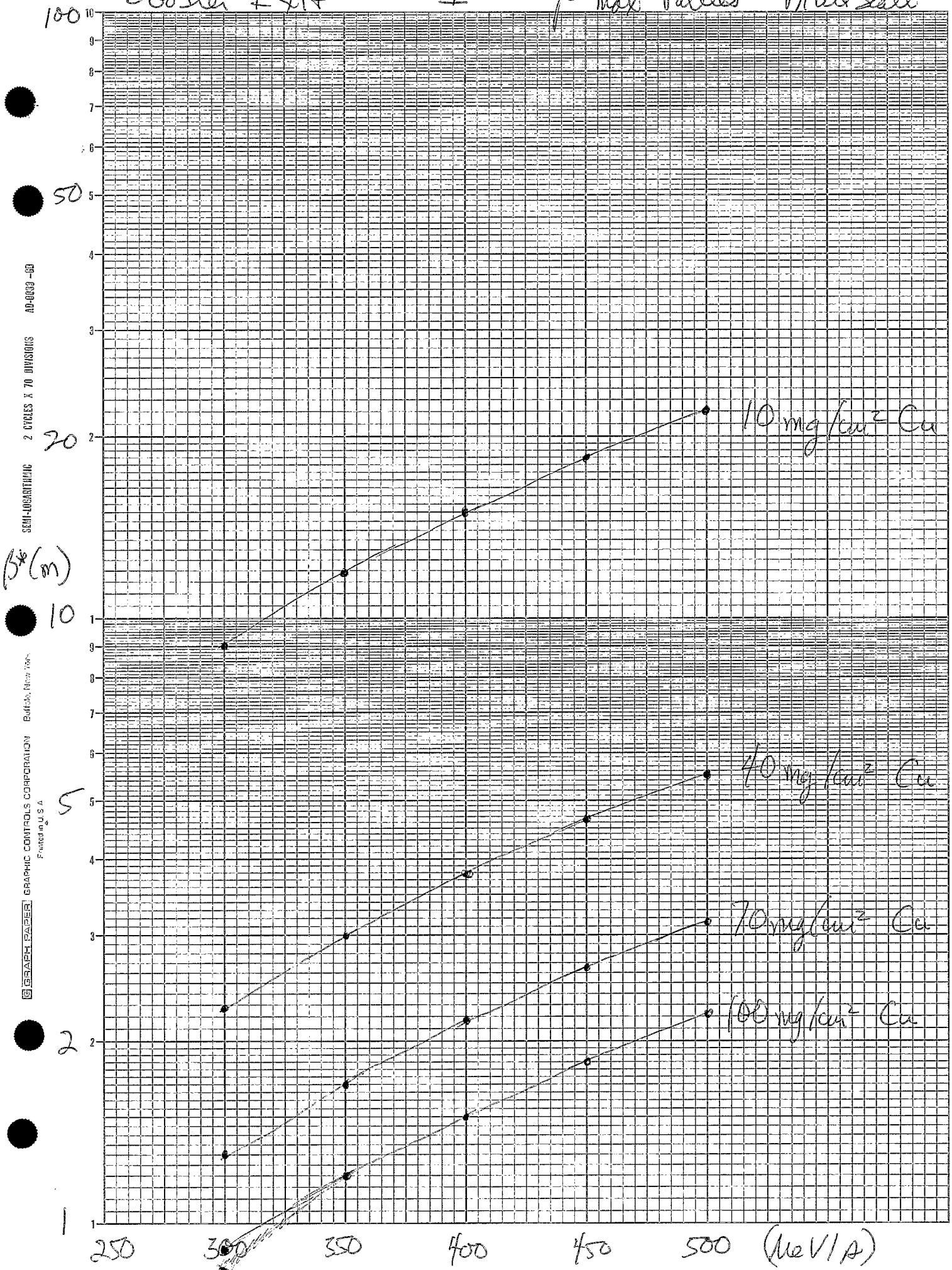
250 300 350 400 450 500 (MeV/A)

10 mg/cm² Cu

40 mg/cm² Cu

70 mg/cm² Cu

100 mg/cm² Cu



100 Booster Exit ^{197}Au β^* max values Mult Scatt

GRAPH PAPER GRAPHIC CONTROLS CORPORATION Buffalo, New York U.S.A.
 SEMI-LOGARITHMIC 2 CYCLES X 70 DIVISIONS AU-6533-80

β^* (m)

