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Multiple Coulomb Scattering a) Tandem Exit b) Booster Exit

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U.S. Department of Energy

USDOE Office of Science (SC)

DISCLAIMER

MULTIPLE COULOMB SCATTERING AND EMITTANCE GROWTH IN STRIPPER FOILS

a) Tandem Exit
b) Booster Exit

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February 23, 1984

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2/23/84

is small.

Rewriting the equation for $\langle \Theta'^2 \rangle$, we have

$$\sqrt{\langle \Theta'^2 \rangle} \equiv \theta^{rms}_{plane} = \sqrt{2\pi \frac{\rho N_A}{A} t Z_2^2 \ln(210 Z^{-1/3})} \; \frac{2 Z_1}{p} \frac{e^2}{\hbar c} \frac{\hbar c}{\beta c}$$

$(\beta = v/c)$

We can approximate $\frac{X_0^{-1}}{4 \alpha r_0^2 N_A} \sim \frac{Z^2 \ln(210 Z^{-1/3})}{A}$

and err by only 10% or less for our stripping foils, except for carbon where the neglect of the second coulomb log gives ~16% error for $(\theta^{rms}_{plane})^2$. This gives a convenient formula in terms of X_0, which is tabulated; the correction is included by multiplying by $(1+\delta)$ where δ is evaluated for each material. For carbon, $\delta = -0.079$

copper, $\delta = 0.00193$

tantalum, $\delta = 0.0478$

This then works out to

$$\boxed{\theta^{rms}_{plane} = (1+\delta) Z_1 \frac{14.98 \text{ MeV/c}}{p\beta} \sqrt{\frac{\Delta}{X_0}}}$$

where Δ is the scatterer thickness in g/cm² and X_0 is in g/cm². X_0 is tabulated in Tsai's article[*], or see the Particle Data Properties booklet. X_0 = 42.70 g/cm² carbon ; 6.82 g/cm² tantalum

12.86 g/cm² copper

2/23/84

In the following graphs, values of θ^{rms}_{plane} are given for C, S, Cu, I and Au ions for the following cases.

(a) Tandem exit — energy/nucleon expected for 2 stage operation; 5-20 $\mu g/cm^2$ carbon stripper

(b) booster exit — 300-500 MeV/nucleon i.e. matching to performance expected for gold; 10 - 100 mg/cm^2 copper stripper

Emittance growth can be found from

$$\varepsilon = \beta(\theta_0^2 + \theta^{rms\ 2}_{plane}) \quad , \quad \text{where } \varepsilon = \beta\theta_0^2 \text{ is the}$$

emittance prior to passing through the stripper. The last graphs then give values of the β function required to limit the emittance growth at the tandem exit to $0.5\ \pi$ mm mrad, and at the booster exit to $1.0\ \pi$ mm mrad. Carbon is fully stripped after the tandem exit stripper. The curves for ^{32}S and ^{63}Cu at the booster exit are included for completeness only. The stripper thicknesses listed are overkill by at least a factor of 100 or more.

As usual, gold yields the most stringent requirements. At the tandem exit, 20 $\mu g/cm^2$ carbon @ 1 keV/A requires $\beta^* = 14$ cm. At the booster exit, 367 MeV/A thru a 70 mg/cm^2 Copper stripper requires $\beta^* = 2.02$ m. This line can handle all lighter beams, as they use thinner stripping foils.

3/1/84

NOTE ADDED

Another formula for multiple scattering of heavy ions is given by Green (U. Washington, Seattle, Annual Report 1975). It gives somewhat more optimistic numbers than the one from Jackson.

The formula is

$$\theta_{1/2} = K \; Z \; \frac{Z_1 Z_2}{E_1} \; \tau^{n} \qquad \left(\text{Divide by } 1.175 \text{ to get } \sigma^{rms}_{plane}\right)$$

where $K = 1.92 \; 10^{-2}$ mrad, Z_1 = Z of ion, Z_2 = Z of foil, $Z = (Z_1^{2/3} + Z_2^{2/3})^{1/2}$ (this appears from calculating an effective screening radius), E_1 is the total kinetic energy of the ion, and τ and n are given by

$$\tau = \frac{41.5 \; \delta}{Z^2 \; A} \qquad \delta = \text{foil thickness in } \mu g/cm^2; \; A = \text{foil Atomic weight}$$

and $n = N^{-.115} - .115$ where $N = \ln(1.03 + \tau)$. Needless to say, this is a phenomenological formula.

PAST EXPERIENCE AT OAKRIDGE IS THAT THIS GIVES TOO SMALL VALUES BY ~ FACTOR OF TWO

To give some values to compare, consider ^{197}Au at the tandem and booster exits, passing thru 20 μg/cm² C and 70 mg/cm² Cu strippers, respectively. A comparison of the two formulae results is below.

Tandem	.5 MeV/A	1 MeV/A	1.5 MeV/A	.5 MeV/A	1 MeV/A	1.5 MeV/A
Jackson's	3.782	1.894	1.263	.035	0.14	0.31
Green's	.978	.489	.326	.52	2.09	4.70

Booster	300	350	400	450	300	350	400	450 MeV/A
Jackson's	.844	.733	.654	.590	1.40	1.86	2.34	2.87
Green's	.515	.441	.386	.343	3.77	5.14	6.71	8.50

Left columns: Θ^{rms}_{plane} (mrad); right columns: β^* value needed

CONCLUSION

(1) Some attention has to be paid to the transport between the tandem high energy end and the stripper foil. A quad doublet has to be used to focus x and y to a small spot. β^* values somewhere in the range of 0.14 m (Jackson formula) to 2m (Greene) formula are needed.

A measurement of emittance after the high energy stripper for the heavy beams is needed. We want a growth of less than 0.5 π mm mrad so the injection into the booster works.

(2) Stripping at the booster exit will require β^* of 2m or larger, but again in <u>both</u> x and y. Again a quad doublet before the stripper is needed.

(3) Better (smaller) values of scattering angles due to multiple Coulomb scattering could be had by using gold foils. Less thickness is needed to strip to the equilibrium charge state distribution due to the density effect in stripping.

For the tandem exit, this probably does not work as the thickness required (10 μg/cm²) is mechanically quite fragile.

For the booster exit, foils of 20-30 mg/cm² are needed, which are quite rugged (0.02 mm thick). As the β^* required varies as the thickness (inversely) the β^* needed after the booster could be relaxed by a factor of two in both planes.

θ^{rms}_{plane} Mult Scatt

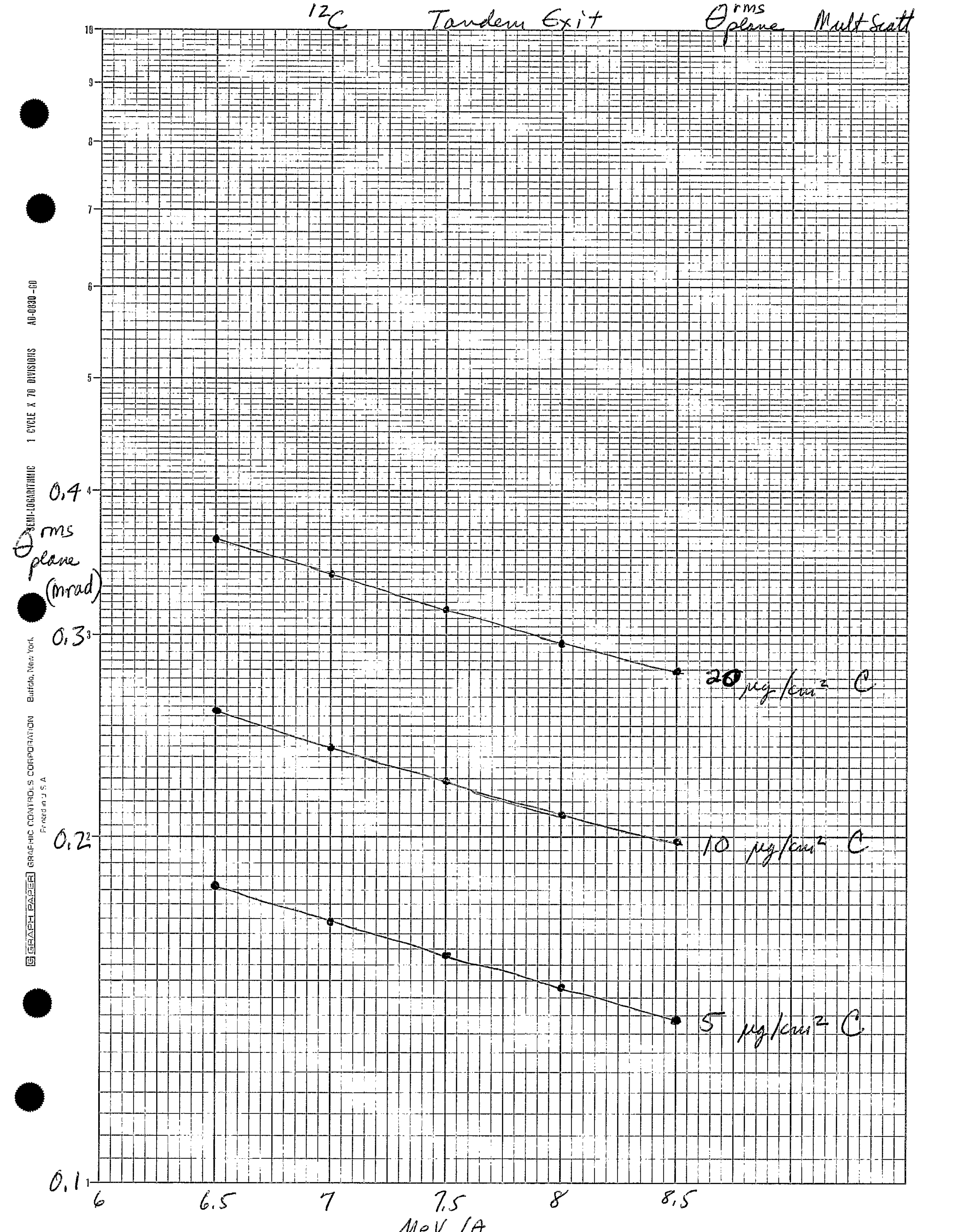
Tandem Exit — Carbon Stripper (5, 10, 20 μg/cm²)

Ion	E/A (MeV/A)	P/A	P	β	θ^{rms}_{plane} (mrad) 5 μg/cm²	10 μg/cm²	20 μg/cm²
^{12}C (Z=6)	6.5	110.2	1322.8	.1175	0.182	0.258	0.364
	7	114.4	1372.9	.1219	0.169	0.239	0.339
	7.5	118.4	1421.3	.1261	0.158	0.224	0.316
	8	122.3	1468.1	.1302	0.148	0.210	0.296
	8.5	126.1	1513.5	.1342	0.139	0.197	0.279
^{32}S (Z=16)	3.5	80.8	2586.4	.0864	0.338	0.478	0.676
	4	86.4	2765.4	.0924	0.296	0.418	0.591
	4.5	91.7	2933.5	.0979	0.263	0.372	0.526
	5	96.6	3092.6	.1032	0.237	0.335	0.473
	5.5	101.4	3244.0	.1082	0.215	0.304	0.430
^{63}Cu (Z=29)	2	61.1	3847.6	.0654	0.544	0.769	1.088
	2.5	68.3	4302.4	.0731	0.435	0.616	0.871
	3	74.8	4713.6	.0804	0.363	0.513	0.725
	3.5	80.8	5092.0	.0864	0.311	0.440	0.622
	4	86.4	5444.3	.0924	0.272	0.385	0.544
^{127}I (Z=53)	.5	30.5	3876.6	.0328	1.968	2.783	3.936
	1	43.2	5483.1	.0463	0.986	1.394	1.971
	1.5	52.9	6716.3	.0567	0.657	0.929	1.314
	2	61.1	7756.4	.0654	0.493	0.698	0.986
^{197}Au (Z=79)	.5	30.5	6013.3	.0328	1.891	2.674	3.782
	1	43.2	8505.3	.0463	0.947	1.339	1.894
	1.5	52.9	10418.2	.0567	0.631	0.893	1.263

θ^{rms}_{plane} Mult Scatt

Booster Exit — Copper Strippers (10, 40, 70, 100 mg/cm²)

Ion	E/A	p/A (MeV/cA)	p (GeV/c)	β	θ^{rms}_{plane} (mrad) 10 mg/cm²	40 mg/cm²	70 mg/cm²	100 mg/cm²
^{32}S	300	805.5	25.78	0.6541	0.397	0.794	1.051	1.255
Z=16	350	880.1	28.2	0.6868	0.346	0.692	0.915	1.094
	400	951.4	30.44	0.7145	0.308	0.616	0.815	0.974
	450	1020.2	32.65	0.7385	0.278	0.556	0.736	0.879
	500	1086.97	34.78	0.7593	0.254	0.508	0.672	0.803
^{63}Cu	300	805.5	50.75	0.6541	0.366	0.732	0.968	1.157
Z=29	350	880.1	55.4	0.6868	0.318	0.636	0.841	1.006
	400	951.4	59.94	0.7145	0.283	0.566	0.749	0.895
	450	1020.2	64.27	0.7385	0.256	0.502	0.677	0.810
	500	1087.0	68.48	0.7593	0.233	0.466	0.616	0.737
^{127}I	300	805.5	102.3	0.6541	0.331	0.662	0.876	1.047
Z=53	350	880.1	111.8	0.6868	0.289	0.578	0.765	0.914
	400	951.4	120.8	0.7145	0.257	0.514	0.680	0.813
	450	1020.2	129.6	0.7385	0.232	0.464	0.614	0.734
	500	1087.0	138.0	0.7593	0.212	0.424	0.561	0.670
^{197}Au	300	805.5	158.7	0.6541	0.319	0.638	0.844	1.009
Z=79	350	880.1	173.4	0.6868	0.277	0.554	0.733	0.876
	400	951.4	187.4	0.7145	0.247	0.494	0.654	0.781
	450	1020.2	200.9	0.7385	0.223	0.446	0.590	0.705
	500	1087.0	214.1	0.7593	0.203	0.406	0.537	0.642



12C Tandem Exit
θ rms plane Mult Scatt
θ rms plane (mrad)
0.4
0.3
0.2
0.1
20 μg/cm2 C
10 μg/cm2 C
5 μg/cm2 C
6
6.5
7
7.5
8
8.5
MeV/A
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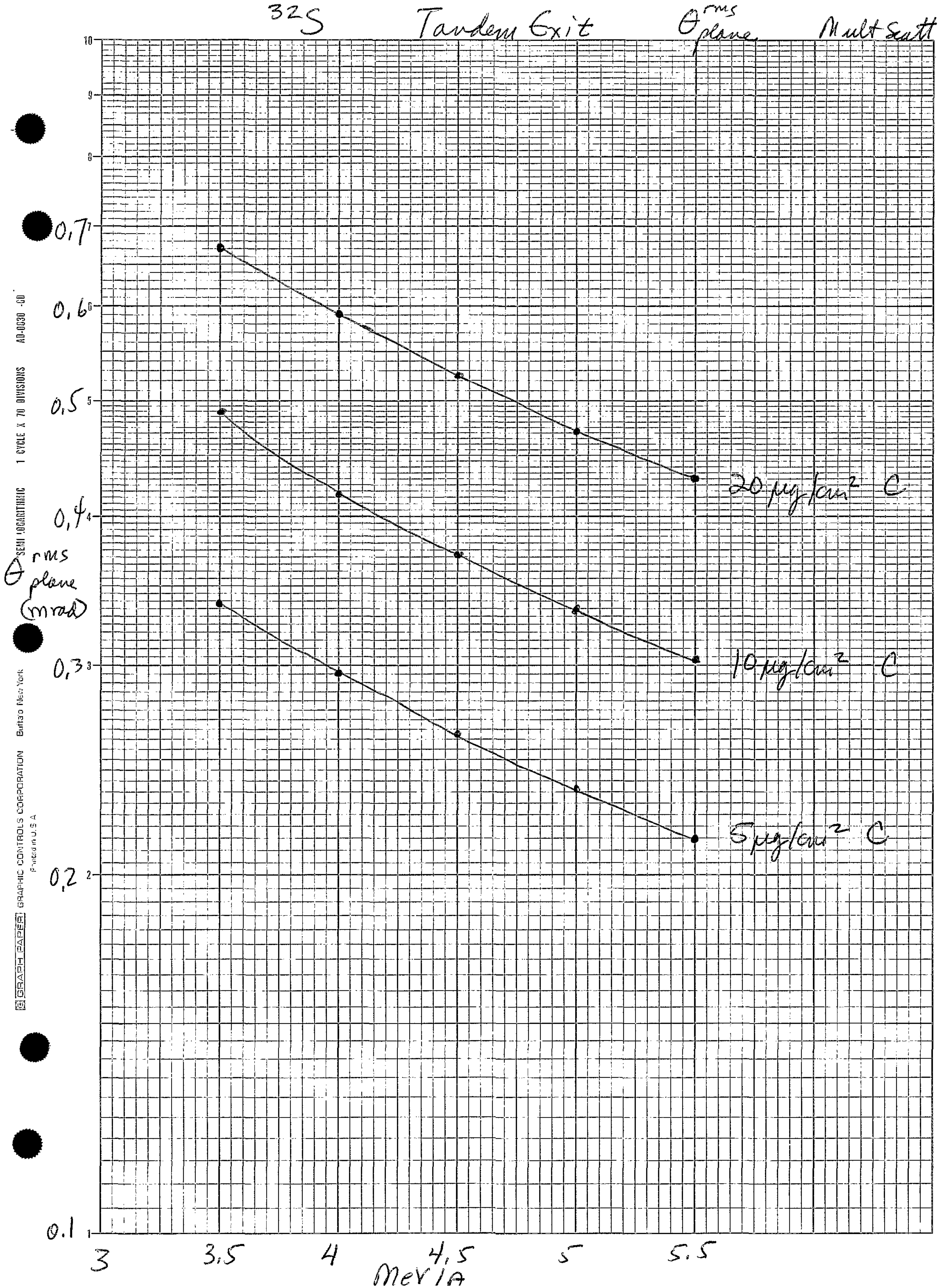
32S Tandem Exit θ rms plane Mult Scatt
θ rms plane (mrad)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
3
3.5
4
4.5
5
5.5
MeV/A
20 μg/cm² C
10 μg/cm² C
5 μg/cm² C
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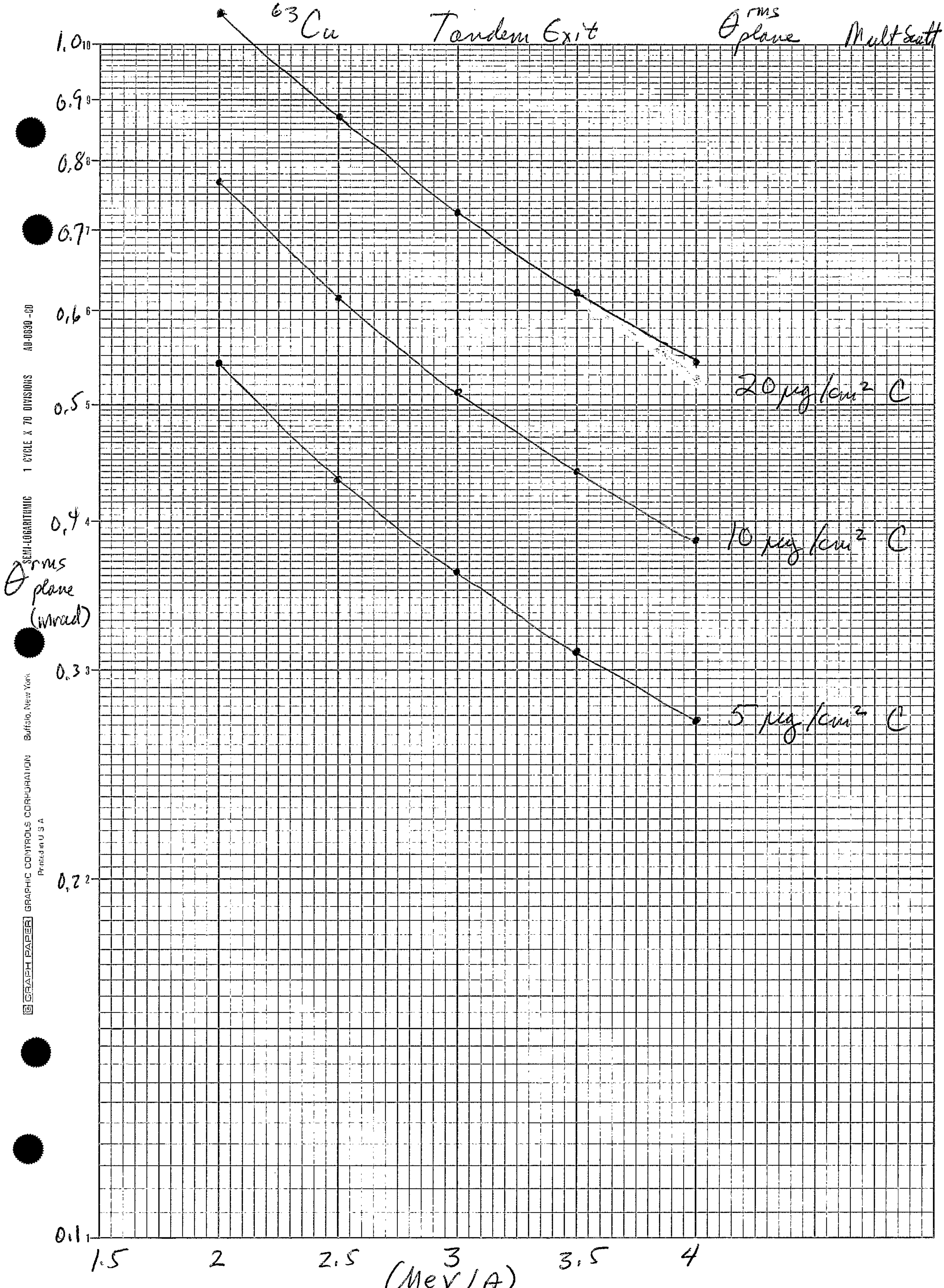

^{63}Cu
Tandem Exit
θ^{rms}_{plane}
Mult Scatt
θ^{rms}_{plane} (mrad)
20 μg/cm² C
10 μg/cm² C
5 μg/cm² C
(MeV/A)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.5
2
2.5
3
3.5
4
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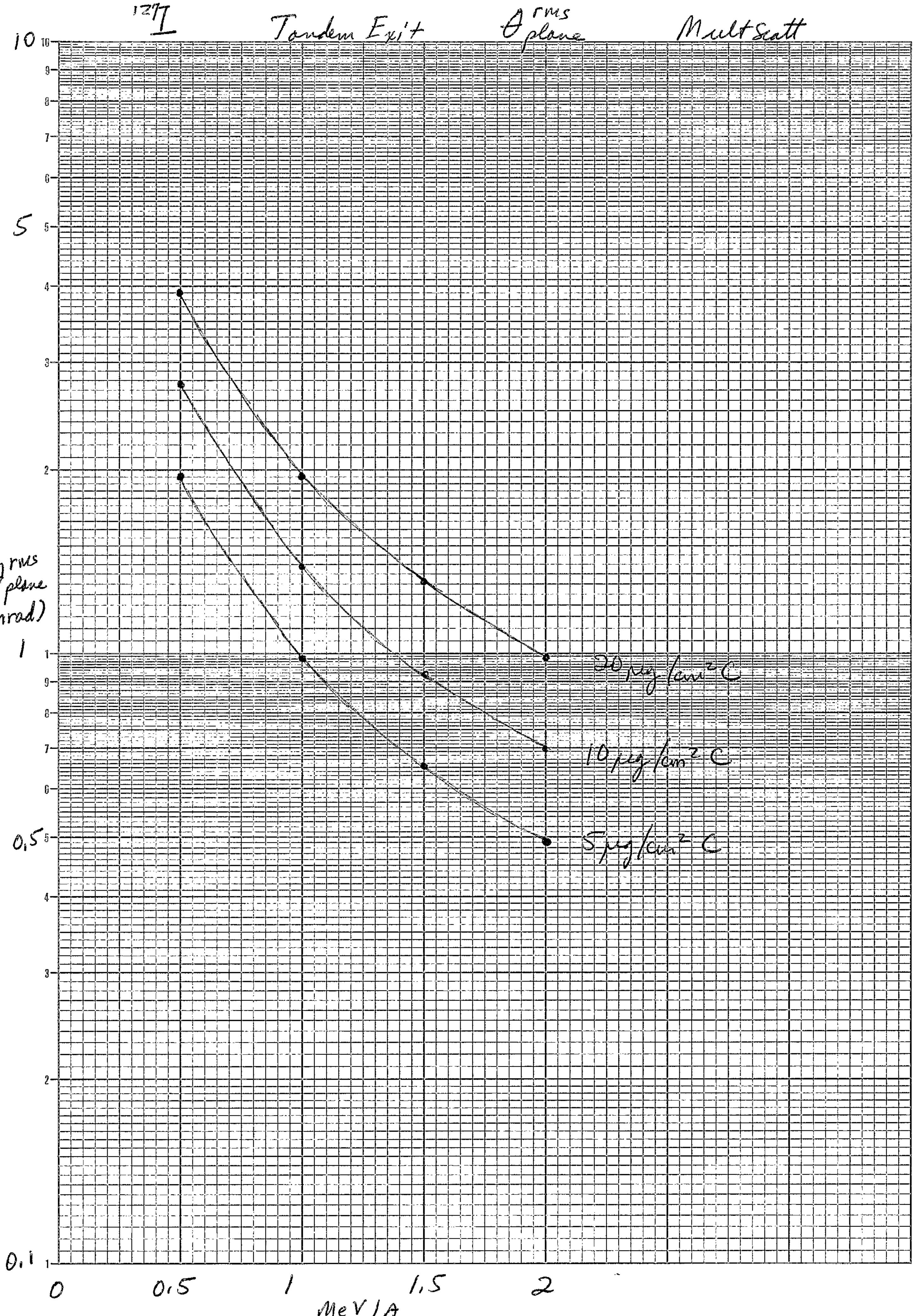
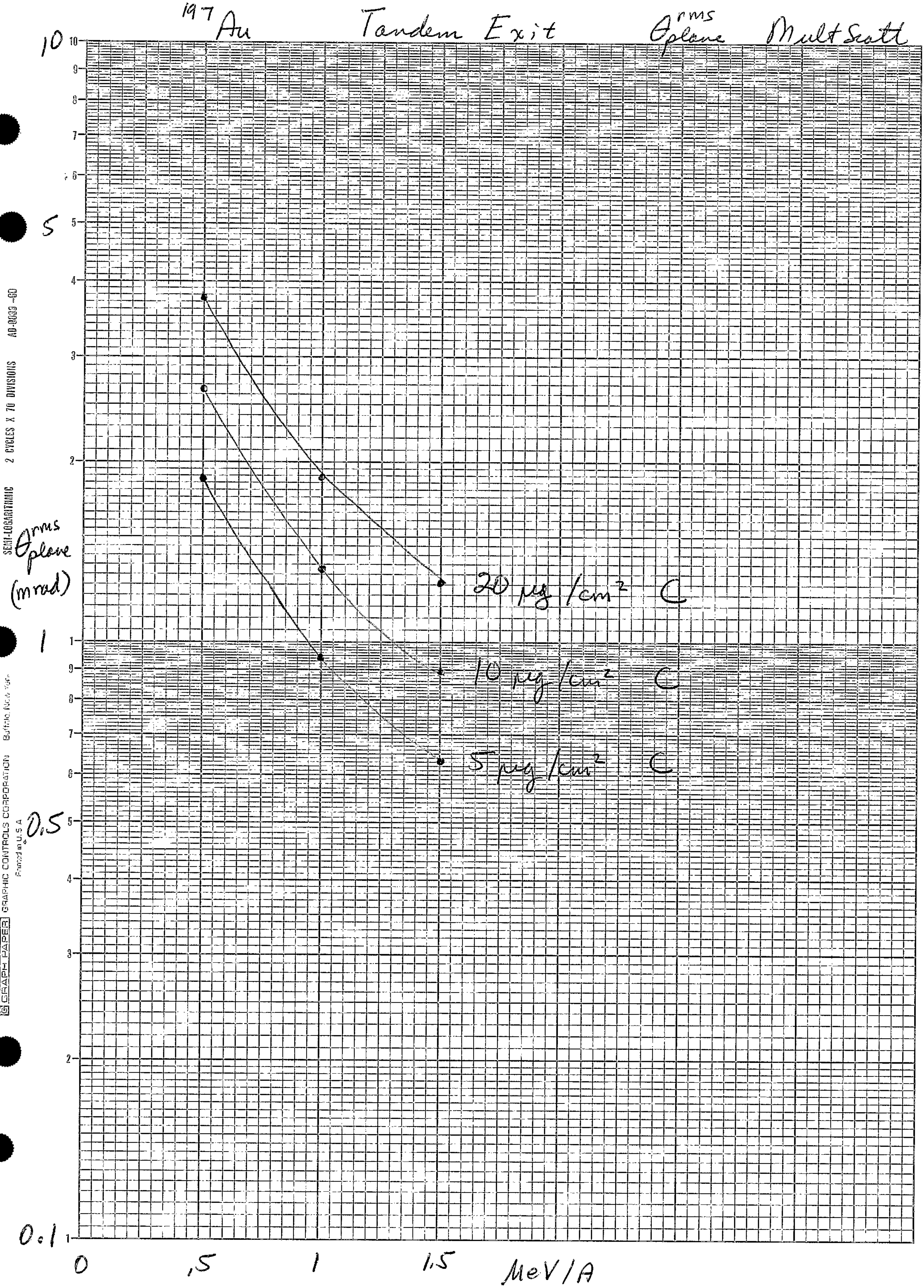


127 I
Tandem Exit
θ rms plane
MultScatt
θ rms plane (mrad)
20 μg/cm² C
10 μg/cm² C
5 μg/cm² C
MeV/A
10
5
1
0.5
0.1
0
0.5
1
1.5
2
SEMI-LOGARITHMIC 2 CYCLES X 70 DIVISIONS AO-0833-60
GRAPH PAPER GRAPHIC CONTROLS CORPORATION Buffalo New York



197 Au
Tandem Exit
θ rms plane
Mult Scatt
10
5
1
0.5
0.1
θ rms plane (mrad)
20 μg/cm² C
10 μg/cm² C
5 μg/cm² C
0
.5
1
1.5
MeV/A
SEMI-LOGARITHMIC 2 CYCLES X 70 DIVISIONS AD-0633-60
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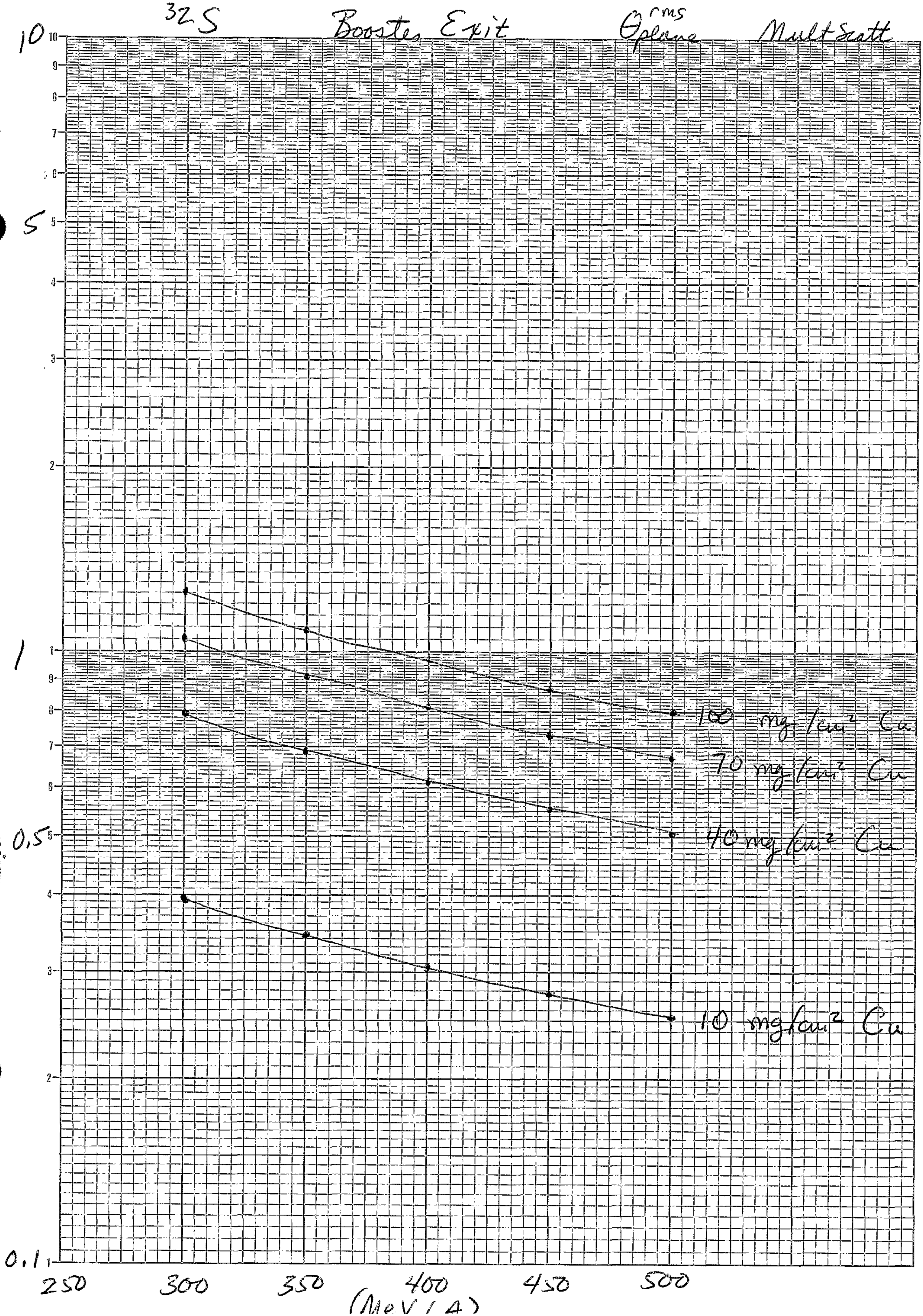
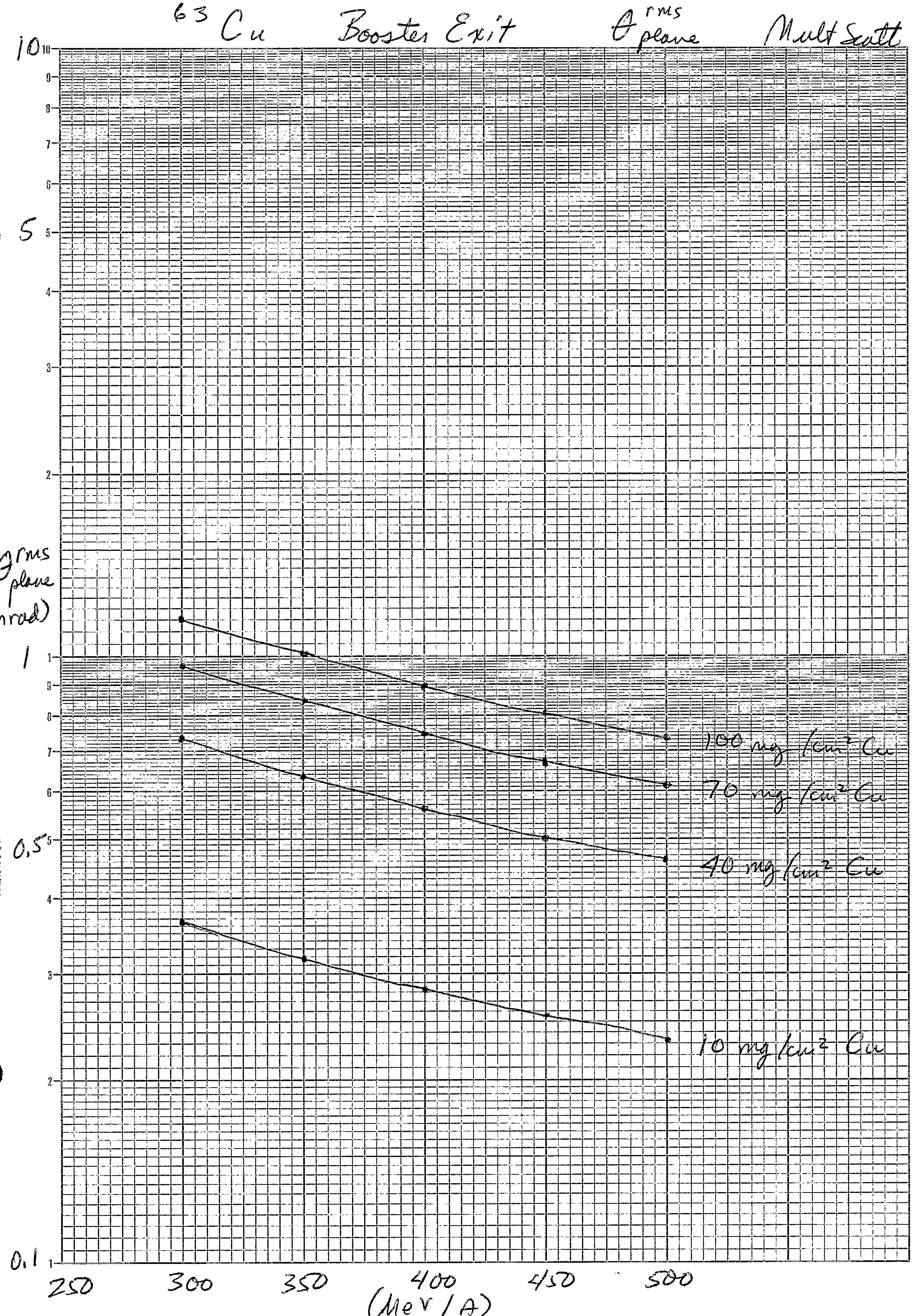
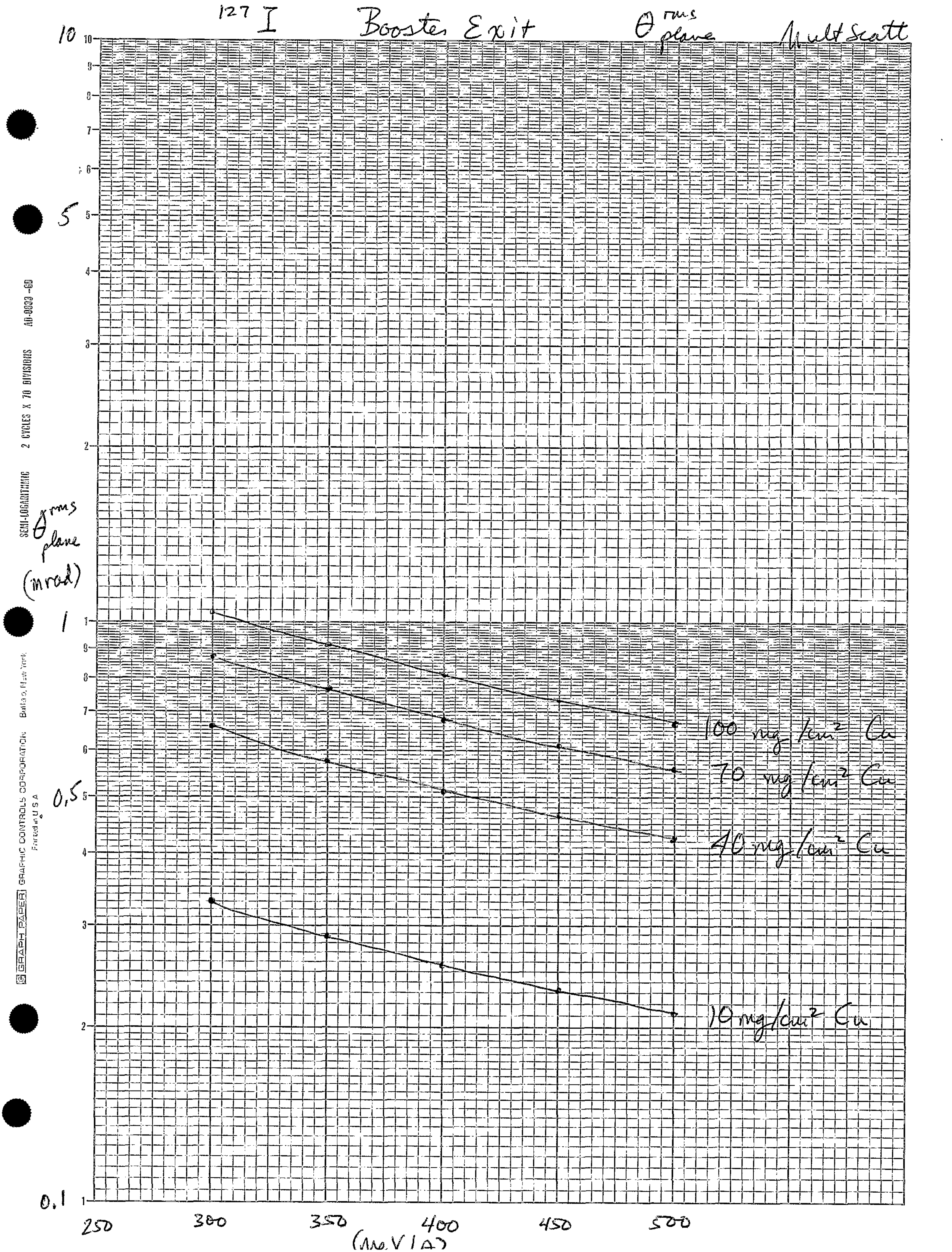


32S
Booster Exit
θ rms plane
Mult Scatt
100 mg/cm² Cu
70 mg/cm² Cu
40 mg/cm² Cu
10 mg/cm² Cu
10
5
1
0.5
0.1
250
300
350
400
450
500
(MeV/A)



63 Cu
Booster Exit
θ^{rms}_{plane}
Mult Scatt
θ^{rms}_{plane} (mrad)
100 mg/cm² Cu
70 mg/cm² Cu
40 mg/cm² Cu
10 mg/cm² Cu
(MeV/A)
SEMI-LOGARITHMIC 2 CYCLES X 70 DIVISIONS
GRAPHIC CONTROLS CORPORATION Buffalo, New York


127 I Booster Exit θ rms plane Mult Scatt
θ rms plane (mrad)
10
5
1
0,5
0.1
250
300
350
400
450
500
(MeV/A)
100 mg/cm² Cu
70 mg/cm² Cu
40 mg/cm² Cu
10 mg/cm² Cu
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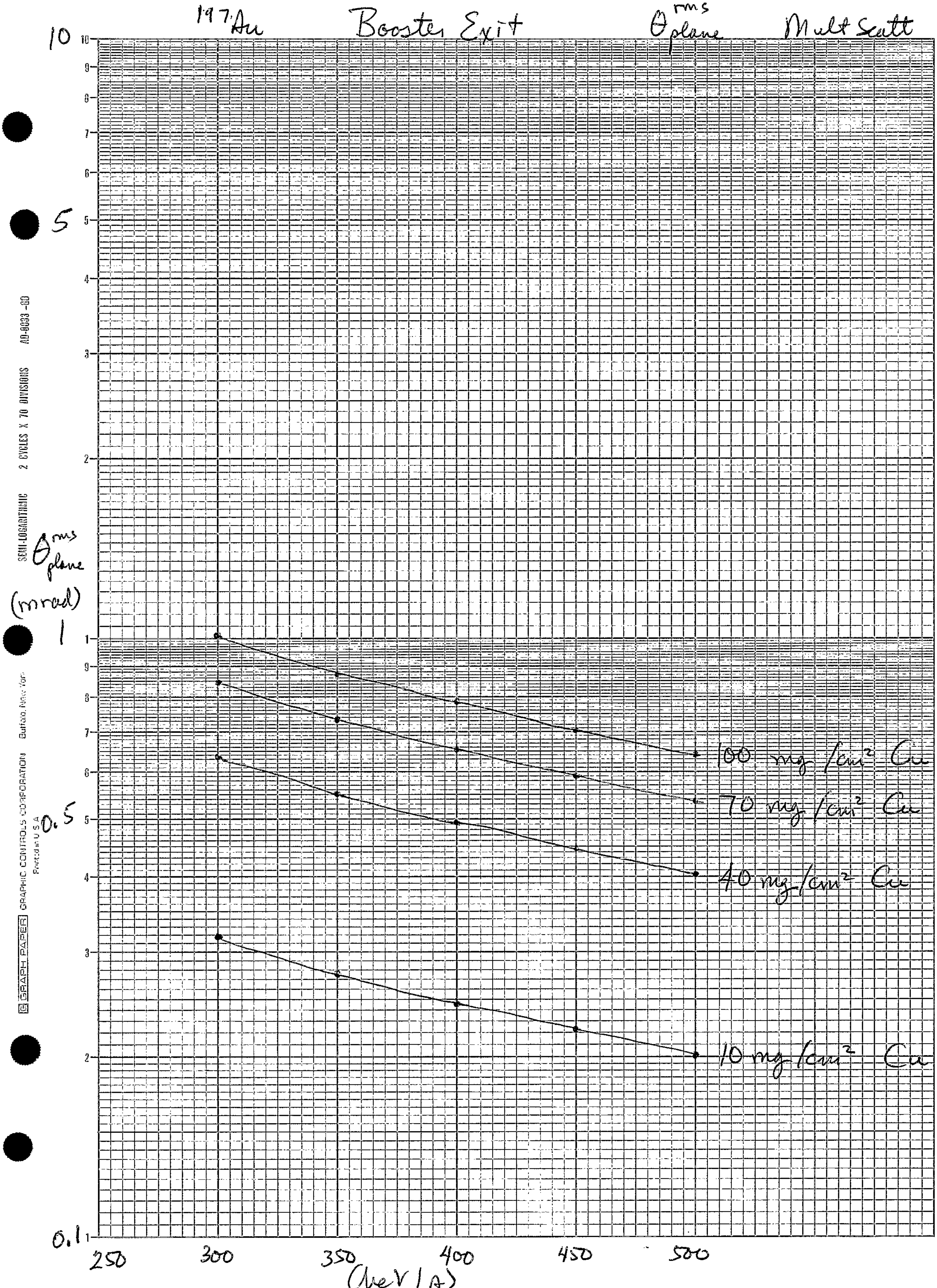

197 Au
Booster Exit
θ_{plane}^{rms}
Mult Scatt
10
5
1
0.5
0.1
θ_{plane}^{rms} (mrad)
100 mg/cm² Cu
70 mg/cm² Cu
40 mg/cm² Cu
10 mg/cm² Cu
250
300
350
400
450
500
(MeV/A)
SEMI-LOGARITHMIC 2 CYCLES X 70 DIVISIONS AD-8633-GO
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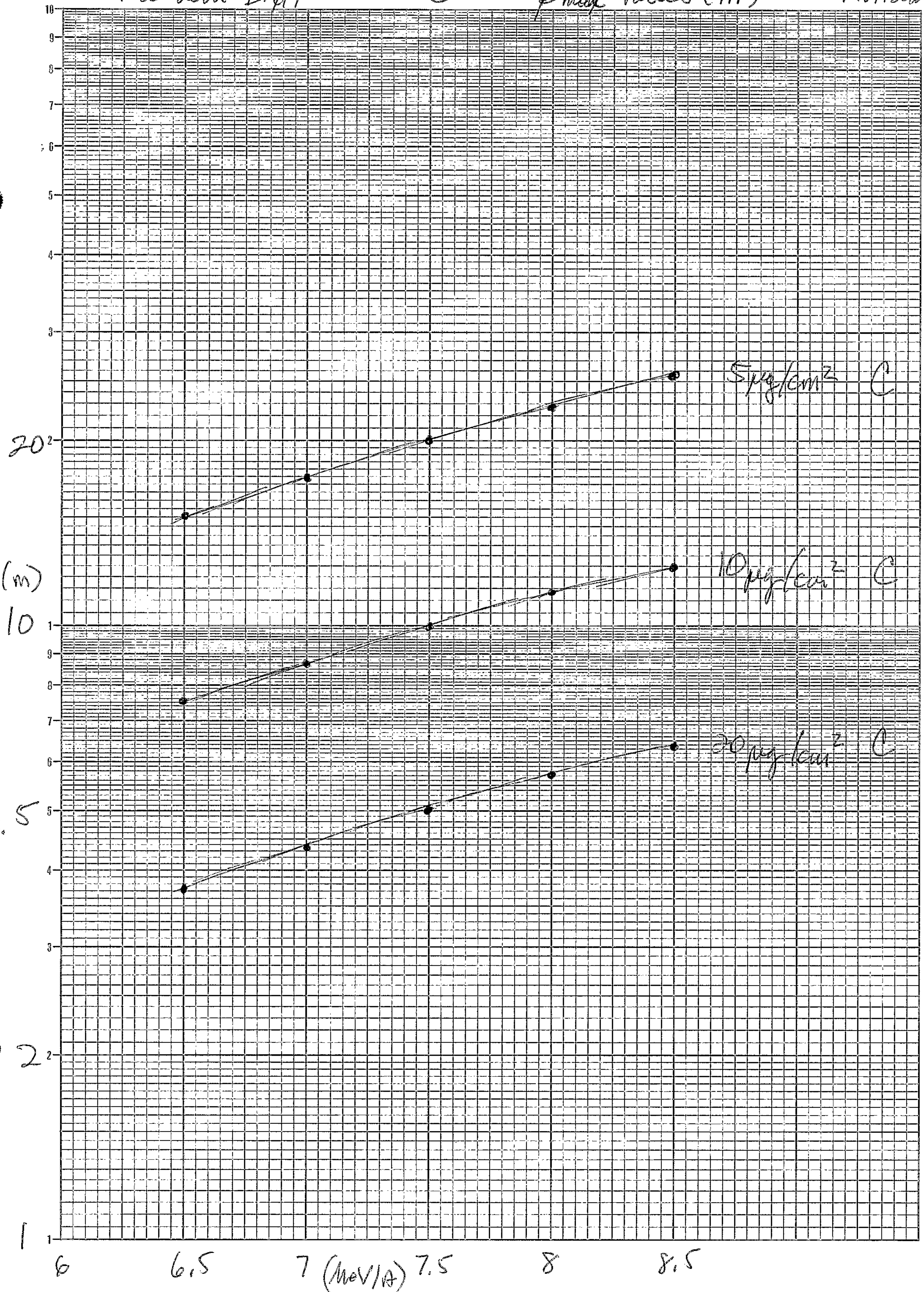
Tandem Exit, Carbon, β^*_{max} values for $\Delta\varepsilon = 0.5\pi$ mm mrad

Ion	MeV/A	5 μg/cm²	10 μg/cm²	20 μg/cm²
^{12}C	6.5	15.1	7.51	3.77
	7	17.5	8.75	4.35
	7.5	20.0	10.0	5.01
	8	22.8	11.4	5.71
	8.5	25.9	12.5	6.42
^{32}S	3.5	4.38	2.19	1.09
	4	5.71	2.85	1.42
	4.5	7.23	3.61	1.81
	5	8.90	4.45	2.23
	5.5	10.8	5.41	2.71
^{63}Cu	2	1.69	0.85	0.42
	2.5	2.64	1.32	0.66
	3	3.79	1.89	0.95
	3.5	5.17	2.59	1.29
	4	6.76	3.38	1.69
^{127}I	.5	0.13	0.065	0.032
	1	0.51	0.257	0.128
	1.5	1.16	0.579	0.29
	2	2.06	1.03	0.51
^{197}Au	.5	0.14	0.070	0.035
	1	0.56	0.28	0.14
	1.5	1.26	0.63	0.31

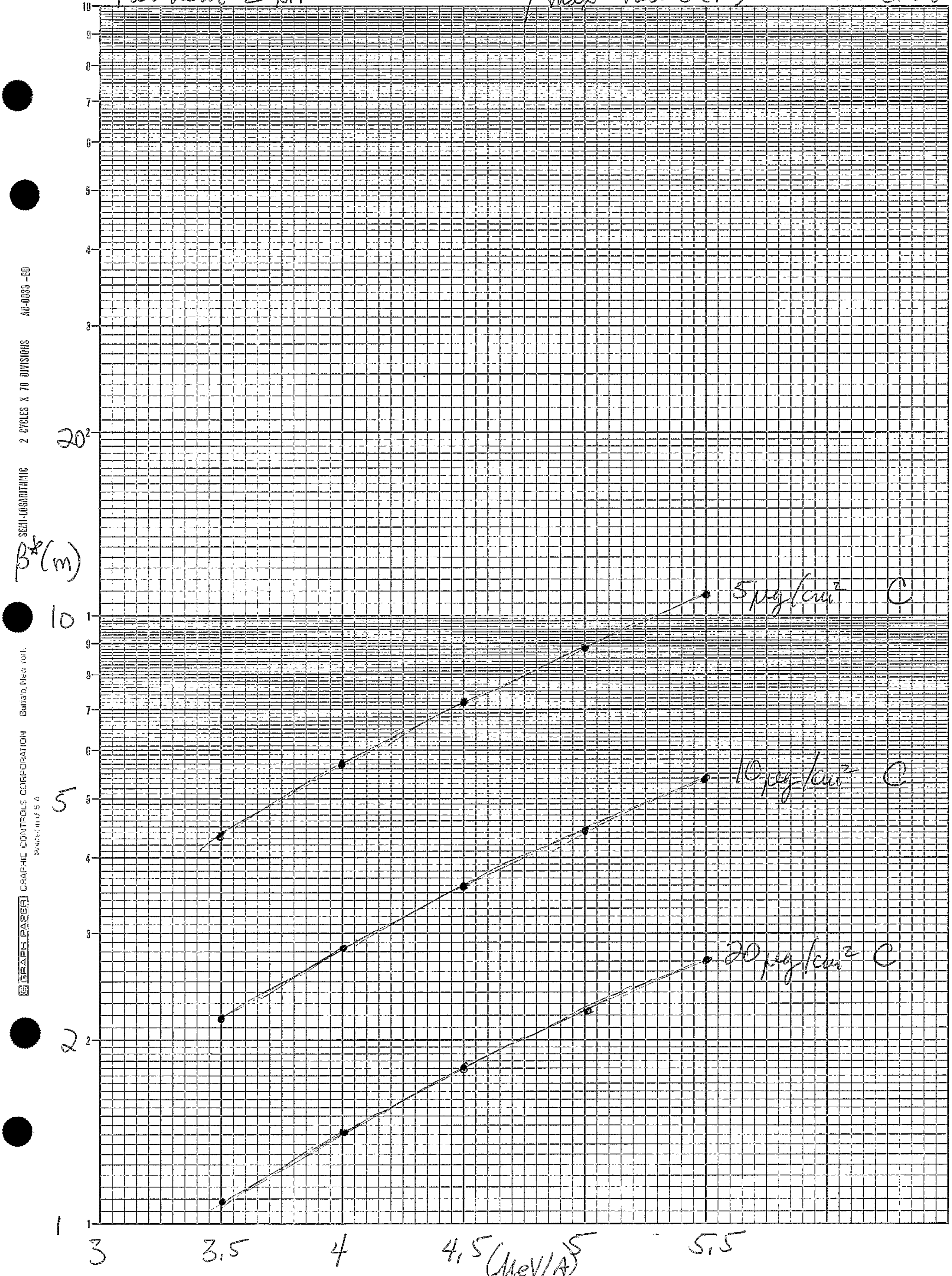
Booster Exit, Copper, <u>β^*_{max} values</u> for $\Delta\varepsilon = 1\pi$ mm mrad

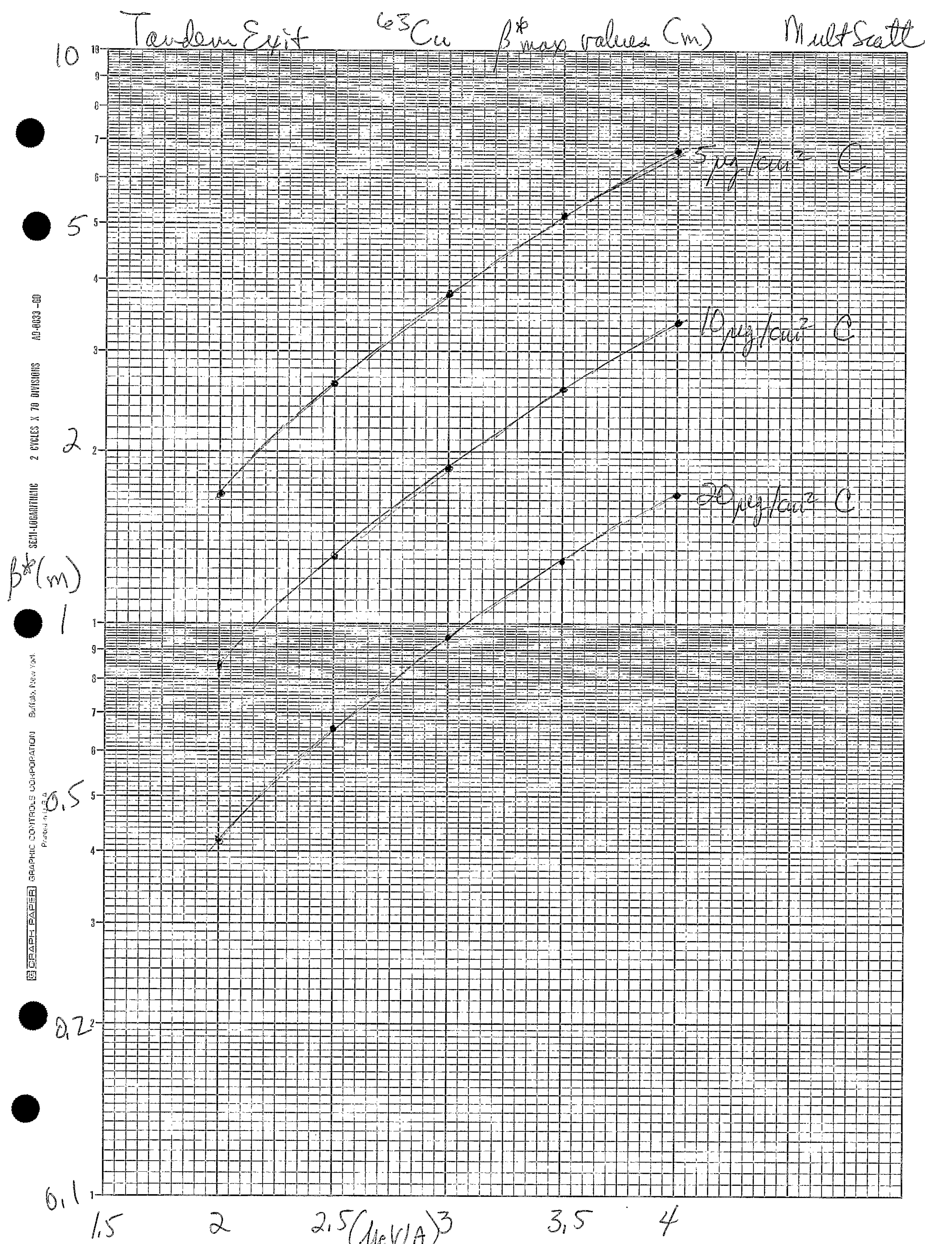
Ion	MeV/A	10 mg/cm²	40 mg/cm²	70 mg/cm²	100 mg/cm²
^{32}S	300	6.34	1.59	0.91	0.63
	350	8.35	2.09	1.19	0.84
	400	10.5	2.63	1.5	1.05
	450	12.9	3.23	1.84	1.29
	500	15.5	3.88	2.21	1.55
^{63}Cu	300	7.47	1.87	1.07	0.75
	350	9.89	2.47	1.41	0.99
	400	12.5	3.12	1.79	1.25
	450	15.3	3.83	2.19	1.53
	500	18.4	4.6	2.63	1.84
^{127}I	300	9.13	2.28	1.30	0.91
	350	11.97	2.99	1.71	1.20
	400	15.1	3.78	2.16	1.51
	450	18.6	4.65	2.66	1.86
	500	22.2	5.55	3.17	2.22
^{197}Au	300	9.83	2.46	1.40	0.98
	350	13.0	3.25	1.86	1.30
	400	16.4	4.1	2.34	1.64
	450	20.1	5.03	2.87	2.01
	500	24.3	6.08	3.47	2.43

Tandem Exit ^{12}C β^*_{max} values (m) McHScott



Tandem Exit ^{32}S β^*_{max} values (m) Mult Scatt





Tandem Exit ^{63}Cu β^*_{max} values (m) MultScatt
5 μg/cm² C
10 μg/cm² C
20 μg/cm² C
β*(m)
10
5
2
1
0.5
0.2
0.1
1.5
2
2.5 (MeV/A) 3
3.5
4
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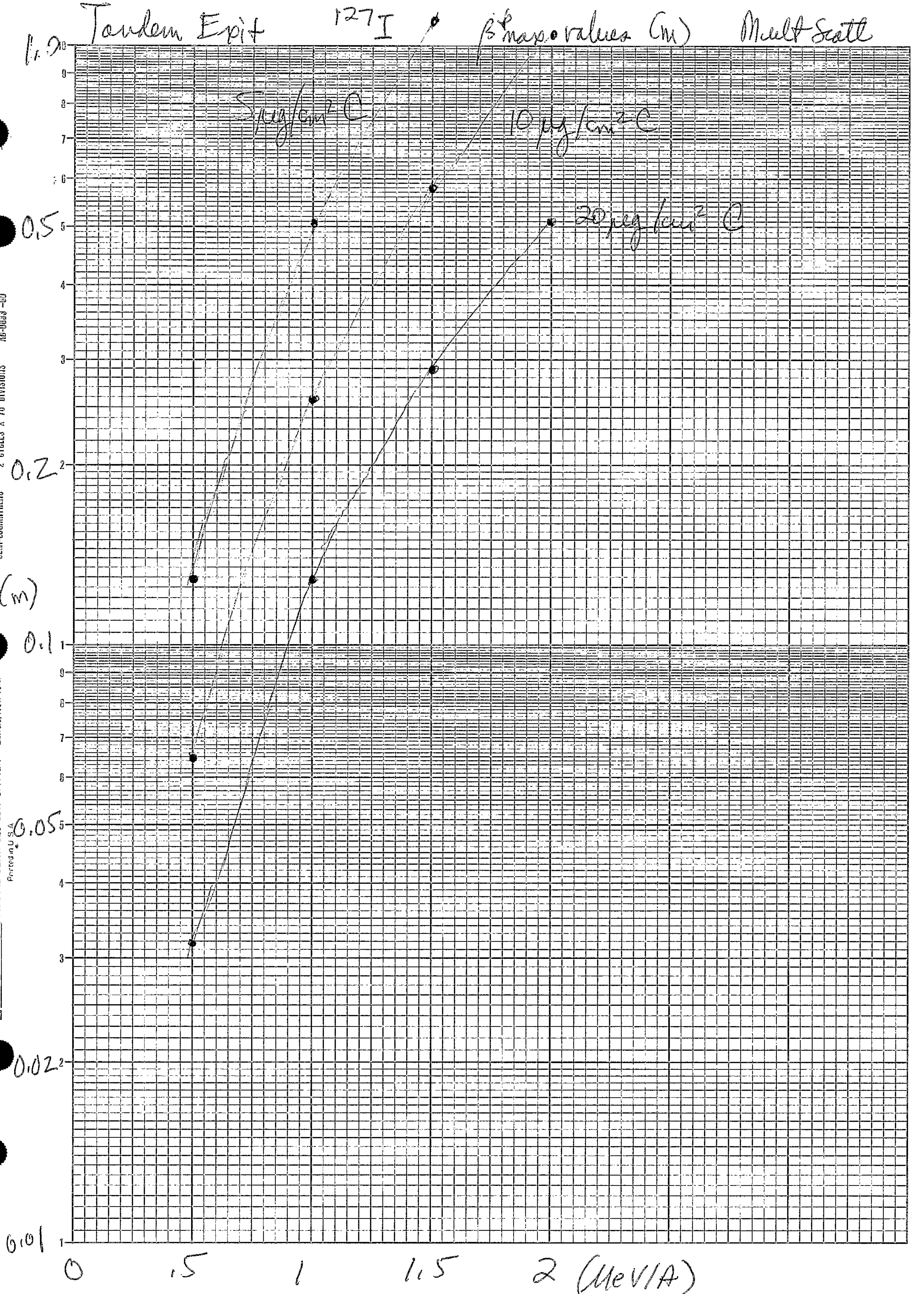

Tandem Exit
^{127}I
β* max. values (m)
Mult Scatt
5 μg/cm² C
10 μg/cm² C
20 μg/cm² C
β* (m)
1.0
0.5
0.2
0.1
0.05
0.02
0.01
0
.5
1
1.5
2 (MeV/A)
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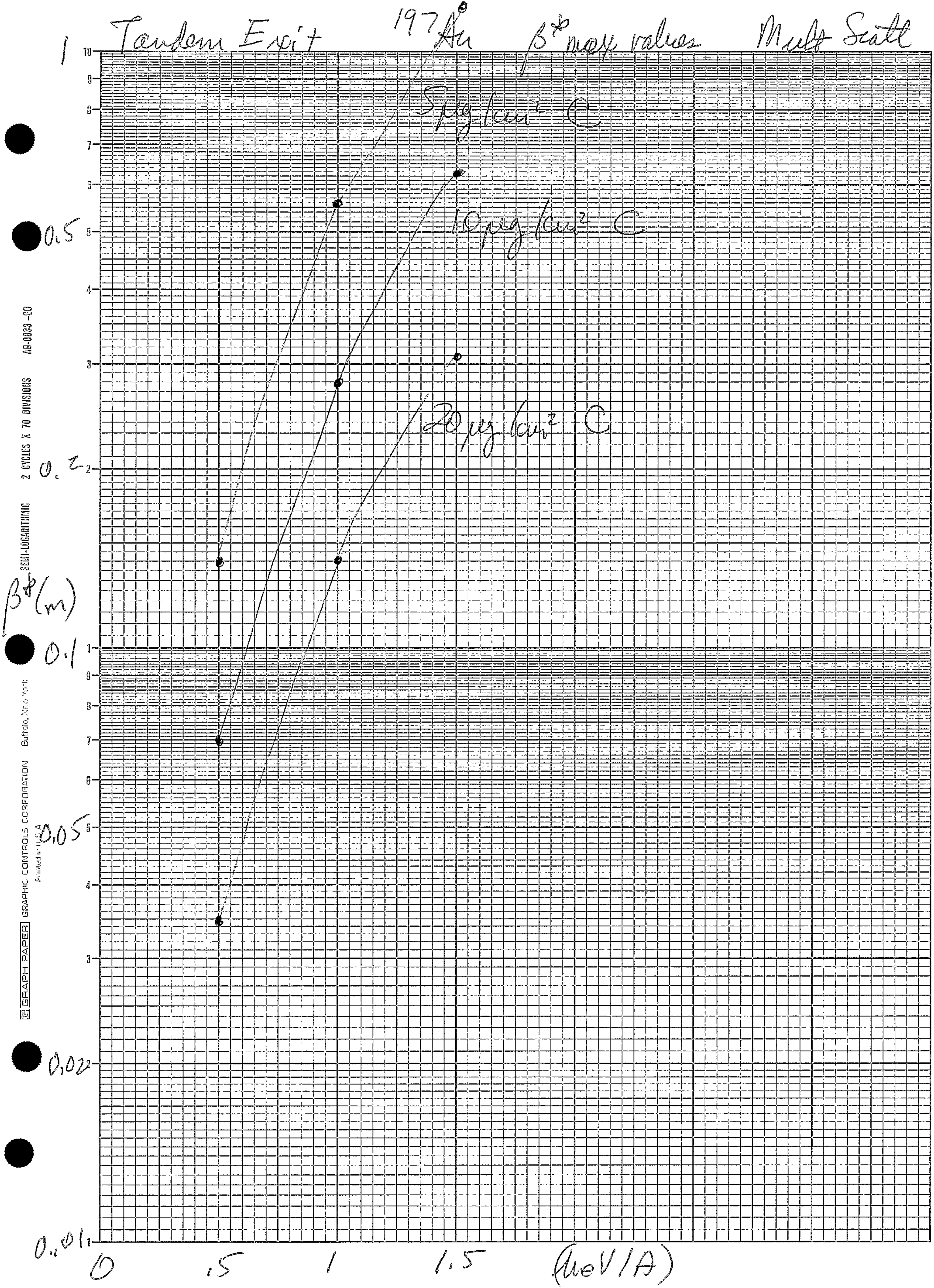

1
Tandem Exit
197Au
β* max values
Mult Scatt
5μg/cm² C
10μg/cm² C
20μg/cm² C
β*(m)
0.5
0.2
0.1
0.05
0.02
0.01
0
.5
1
1.5
(MeV/A)
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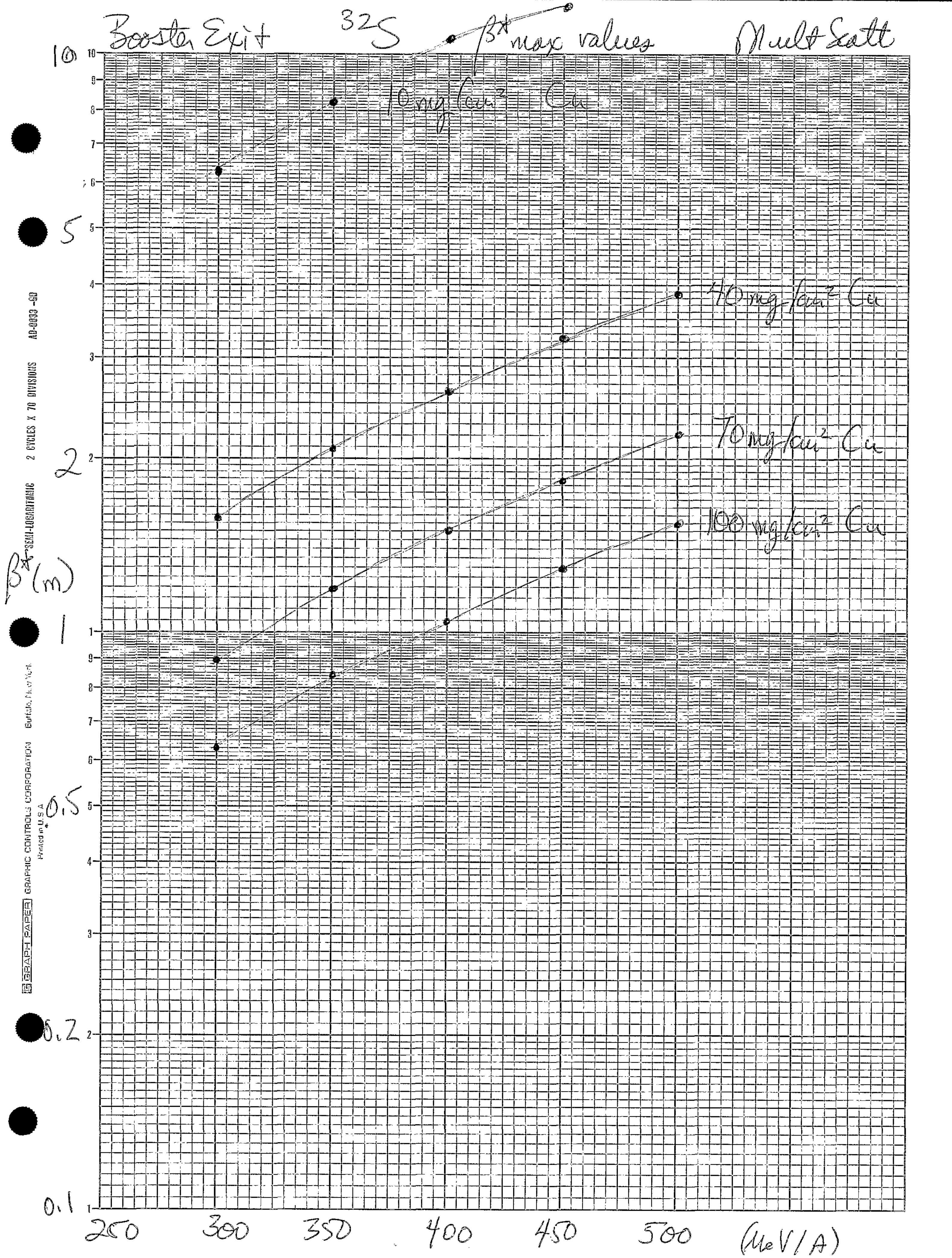
SEMI-LOGARITHMIC 2 CYCLES X 70 DIVISIONS


Booster Exit
32S
β* max values
Mult Scatt
10 mg/cm² Cu
40 mg/cm² Cu
70 mg/cm² Cu
100 mg/cm² Cu
β*(m)
10
5
2
1
0.5
0.2
0.1
250
300
350
400
450
500
(MeV/A)
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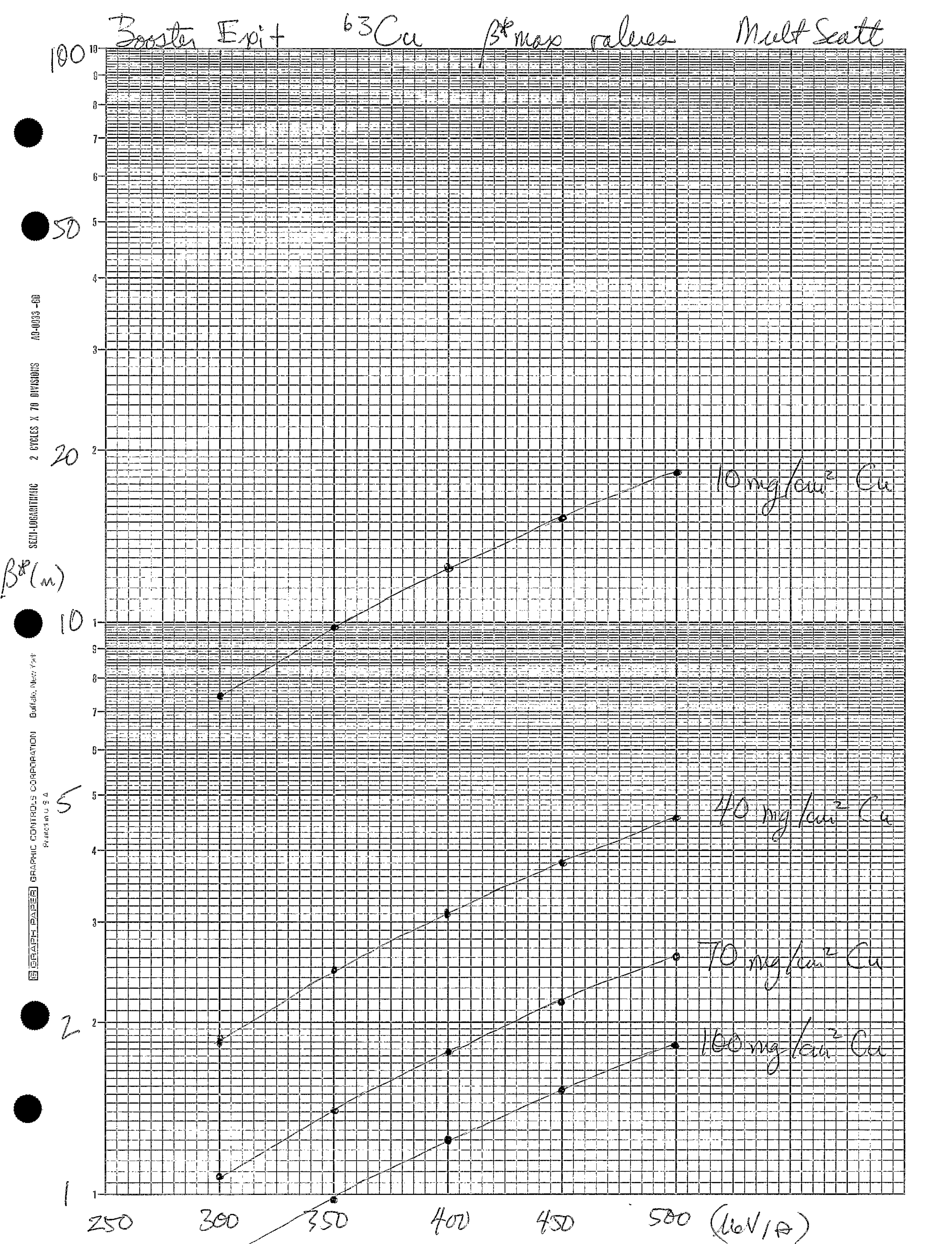
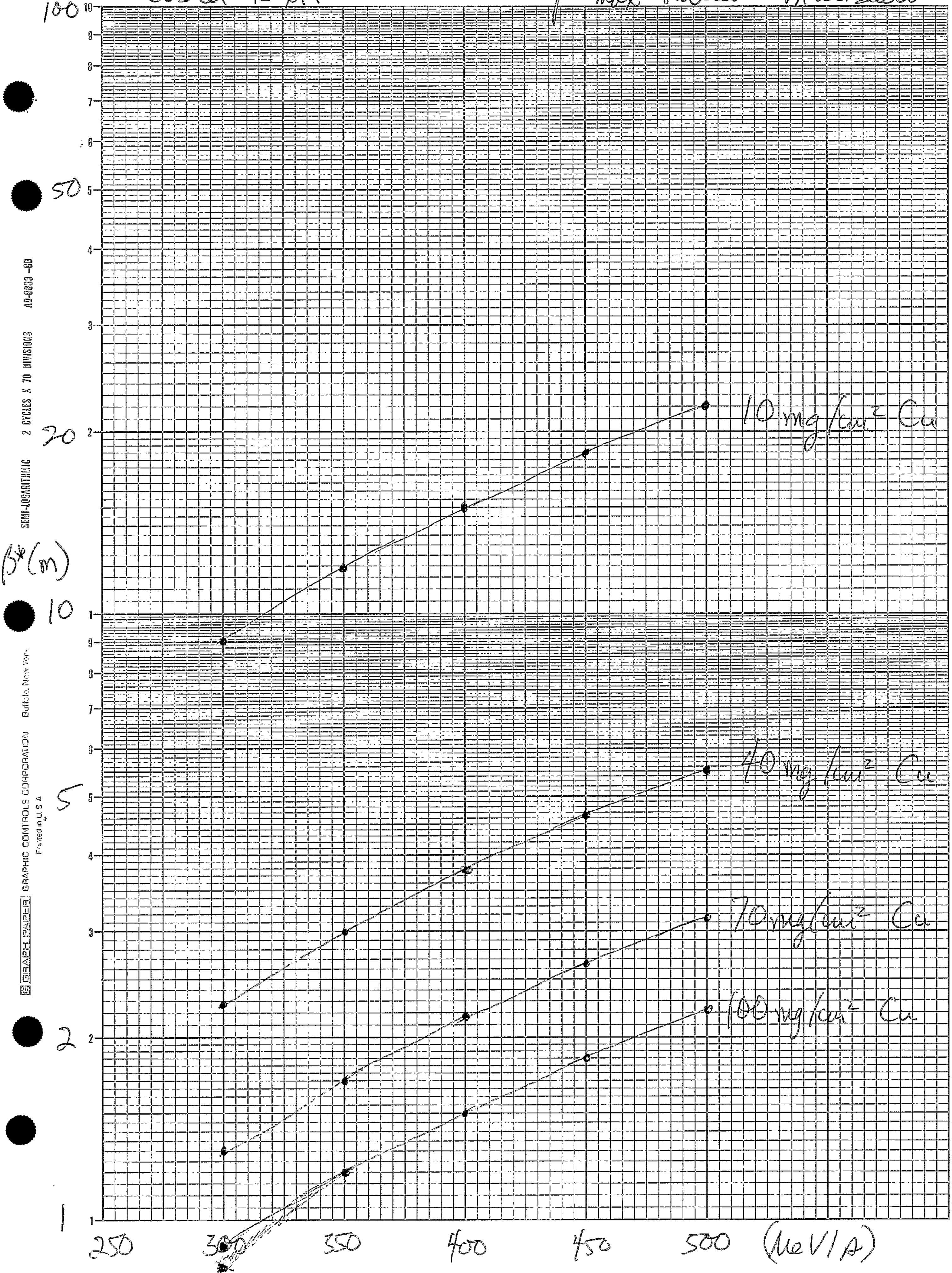
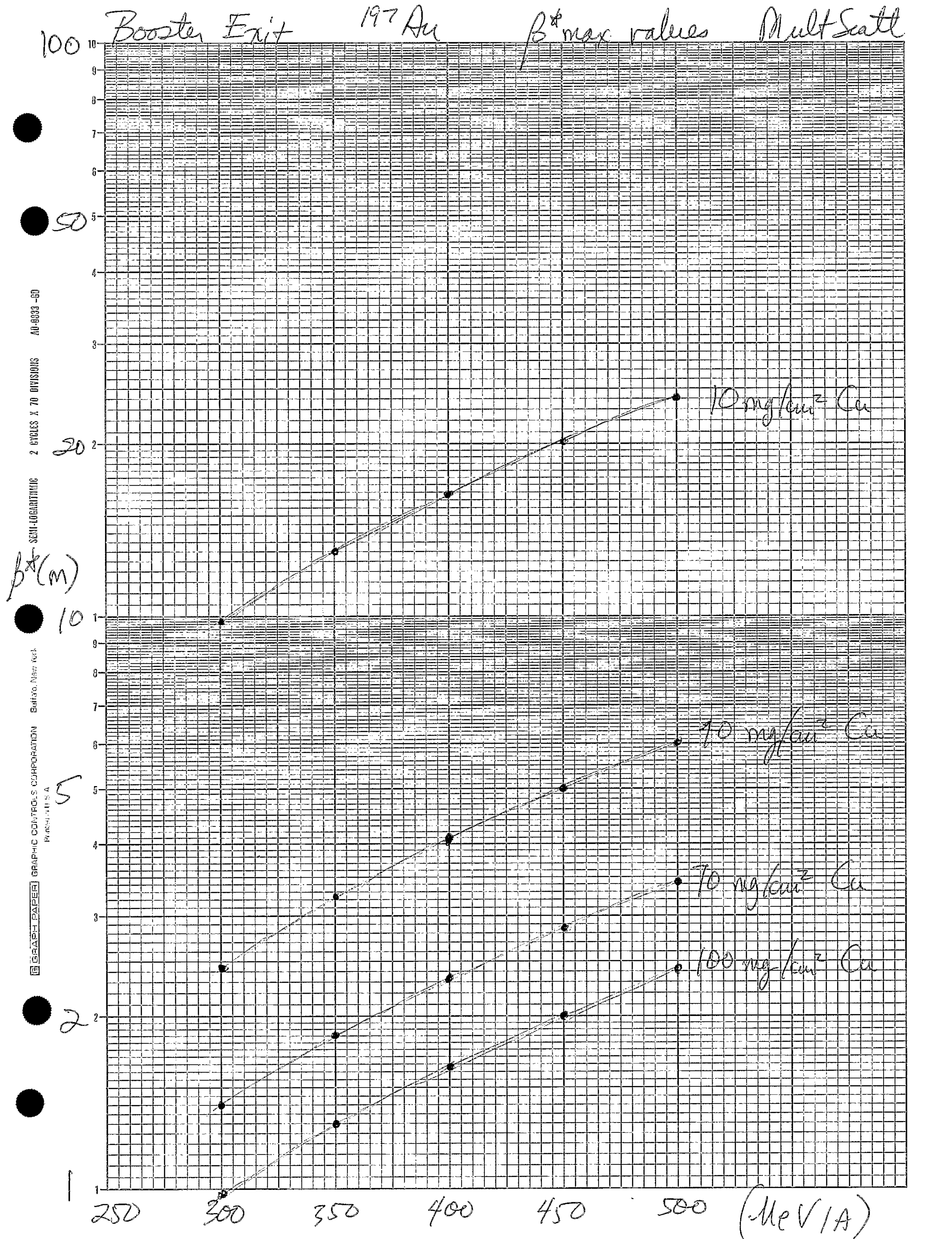


Booster Exit ^{63}Cu β* max values Mult Scatt
100
50
20
β*(m)
10
5
2
1
10 mg/cm² Cu
40 mg/cm² Cu
70 mg/cm² Cu
100 mg/cm² Cu
250
300
350
400
450
500 (MeV/A)
SEMI-LOGARITHMIC 2 CYCLES X 70 DIVISIONS
GRAPHIC CONTROLS CORPORATION Buffalo, New York

Booster Exit ^{127}I β^* max values MultScatt





Booster Exit
197 Au
β* max values
Mult Scatt
100
50
20
10
5
2
1
β*(m)
10 mg/cm² Cu
40 mg/cm² Cu
70 mg/cm² Cu
100 mg/cm² Cu
250
300
350
400
450
500
(MeV/A)
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