

Crossing The Transition Energy At RHIC

J. M. Wang

February 1984

Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-76CH00016 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

CROSSING THE TRANSITION ENERGY

AT RHIC

J. M. Wang

(BNL, February 7, 1984)

① shall discuss the following effects at or near transition energy

① $\Delta P/p$

② Longitudinal phase space mismatch (Hereward - Sorensen effect)

③ RF phase jump mistiming (Umstätter effect)

④ Microwave instability (Courant Effect)

Only the last effect can be

serious for RHIC.

1. Bunch area is an invariant (Poincare invariant), even though the Hamiltonian is time dependent & non-adiabatic near transition.

$$2. \quad \gamma = \gamma_{tr} + \dot{\gamma} t$$

$$\eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}$$

$$E = E_0 \gamma ; \quad \omega_0 = \beta \omega_{\infty} ; \quad \omega_{\infty} = \frac{c}{R}$$

$$\Omega_s \cdot T_s = 2\pi$$

$$\Omega_s^2 = - \frac{\omega_{\infty}^2 h}{E} \frac{eV}{2\pi} \eta \cos \varphi_s$$

$$\eta \cos \varphi_s < 0$$

3. T_{NA} : non-adiabatic time

3

$$\frac{1}{T_{NA}^3} \equiv \frac{\omega_{\infty}^2 \hbar}{E_0} \frac{2\dot{\gamma}}{\gamma_t^4} \frac{eV}{2\pi} |\cos \varphi_s|$$

If $\gamma_{tr} \gg \dot{\gamma} t$, then :

$$(a) \eta \approx 2\dot{\gamma} t / \gamma_t^3$$

$$(b) \Omega_s^2 \approx t / T_{NA}^3$$

$$(c) \left| \frac{dT_s}{dt} \right| \ll 1 \Rightarrow \frac{|t|}{T_{NA}} \gg 1$$

adiabatic condition



② Longitudinal space charge effect

$$\eta_{sc} \equiv \frac{e g_0 I_b h Z_0}{2\sqrt{2\pi} R \gamma^2 (\sigma_\varphi)^3} \left| \frac{eV}{2\pi} \cos \varphi_s \right|$$

$$g_0 = \frac{1}{2} + 2 \log \frac{z_b}{a_{rt+a_r}}$$

$$\frac{d}{dt} \Delta E = \frac{\varphi - \varphi_s}{T_{rev}} eV \cos \varphi_s \left. \begin{array}{l} (1 - \eta_{sc}) ; \quad \gamma < \gamma_{tr} \\ (1 + \eta_{sc}) ; \quad \gamma > \gamma_{tr} \end{array} \right\}$$

Space charge lengthen (shorten) the bunch before (after) the transition \implies

filamentation after transition

$$\rho = \frac{\text{bunch area way after transition}}{\text{" before "}}$$

$$\rho = 1 + 1.4 \eta_{sc}$$

Horizontal space charge effect (5)

Horizontal space charge $\Rightarrow \Delta\gamma_{tr}$

\Rightarrow mistiming of RF phase jump

for some particles \Rightarrow bunch blow up

$$\tau_{miss} \equiv \frac{\Delta\gamma_{tr}}{\dot{\gamma} T_{NA}} \approx \frac{\Delta V_{H,SC}}{\dot{\gamma} T_{NA}}$$

$$\rho \approx 1 + 1.7 \tau_{miss}$$

$S = 0.2 \text{ eV/Acc/AMU}$	15/120° Scheme A	12/90° Scheme B	9/120° (6) Scheme C
σ_T	39.0	25.1	25.9 23.6
Accel. Period	30 sec	30 sec	5 min
Energy Gain / turn	38.1 keV/A	38.1	3.81
Peak Voltage for Acceleration	1.0 MV	1.0	0.5
ρ_s	5.45°	5.45°	1.09°
Bucket Area :			
inject.	0.56 eV/A-sec	0.72	0.52 0.5
top energy	5.7	3.5	2.7 2.3
Bucket Height:			
inject.	$\pm 0.24 \%$	0.26	0.19 0.1
top energy	0.24	0.15	0.11 0.1
Peak Voltage in Storage Mode at 100 GeV/A	1.0 MV	2.6	2.3 3.
ν_s @ 100 GeV/A	0.0005	0.0013	0.0013
$\bar{\eta}$	0.5	1.15	1.25

E_N

$10\pi \times 10^{-6}$ rad in (

The Bucket Area and Height given in this Table are for Stationary Bucket (no Acceleration)

At Transition

	Scheme A	B	C
$\Delta p/p$ (bottom to bottom)	$1. \times 10^{-2}$	1.1×10^{-2}	1.3×10^{-2}
η_{sc}	1.7×10^{-5}	1.4×10^{-5}	4×10^{-5}
T_{miss}	3.6×10^{-3}	5.6×10^{-3}	2×10^{-2}
T_{NA} (sec)	3.3×10^{-2}	1.8×10^{-2}	4.6×10^{-2}

Crossing the Transition Energy

	A.	D.	E.
$S = 0.2 \text{ eV/A-sec}$	$15/120^\circ$	$12/90^\circ$	$9/120^\circ$
γ_T	39.0	25.1	26.9 23.6
f_{RF}	53.487 MHz	26.743	26.743
V	1.0 MV	0.2	0.2
ϕ_s	5.45°	6.82	6.82
charact. time T_{NA}	31.8 msec	60.4	66.3 55.7
bunch length	$\pm 0.40 \text{ nsec}$	0.92	0.88 0.96
bunch spread	$\pm 0.51 \%$	0.34	0.33 0.35
Betatron Emittance			
$Q_{\gamma = \gamma_T}$	0.39	0.97	0.85 0.97
95% of beam	$\pi \text{ mm-mrad}$		
η_{sc}		6×10^{-5}	6×10^{-5}
T_{miss}		7×10^{-3}	8×10^{-3}

Microwave Instability

?

$$\xi = \Delta E \quad ; \quad \eta = \varphi - \varphi_s \quad ; \quad \varphi: \text{RF phase}$$

Assume that the threshold happens at

$$t = \pm T_{th} \quad ;$$

$$\dot{\gamma} T_{th} \ll \gamma_{tr}, \quad \frac{T_{th}}{T_{NA}} \gg 1; \quad \eta_{th} = \eta(T_{th}) = \frac{2\dot{\gamma} T_{th}}{\gamma_{tr}^3}$$

(has to be checked a posteriori)

$$\left\{ \begin{array}{l} \eta = \eta_{max} \cos\left(\gamma - \frac{7}{12}\pi - \psi\right) \\ \xi = -\xi_{max} \sin\left(\gamma - \frac{7}{12}\pi - \psi\right) \end{array} \right.$$

$$\xi_{max} = \eta_{max} \sqrt{\frac{E}{h|\eta|} \frac{eV}{2\pi} |\cos\varphi_s|}$$

$$A = \pi \xi_{max} \eta_{max} \frac{1}{\omega_0 h}$$

$$\eta_{max} \propto t^{-1/4} \quad ; \quad \xi_{max} \propto t^{1/4}$$

$$\gamma = \int_0^t dt' \Omega_s(t') = \frac{2}{3} \left| \frac{t}{T_{NA}} \right|^{3/2}$$

$$\xi_{max} = 2.5 \sigma_\xi, \quad \eta_{max} = 2.5 \sigma_\eta$$

Threshold Condition

(10)

$$\frac{1}{|\eta(t)|} \left| \frac{z_n}{n} \right| \frac{1}{\sqrt{2\pi}} e^{I_{AV}^B E_0 \gamma_{tr}} \frac{h}{\sigma_{\theta}(t)} \frac{1}{\sigma_{\xi}(t)^2} = 1$$

Solution: $t = \pm \tau_{th}$

Unstable at $t=0$, since $\eta(t=0) = 0$.

$$\eta(t) = \frac{2\dot{\gamma}t}{\gamma_{tr}^3}; \quad \dot{\eta} = \frac{2\ddot{\gamma}}{\gamma_{tr}^3}$$

If $\dot{\eta} \ll 1$, then there is a large time interval around $t=0$ when $\eta(t)$ is very small, and the beam unstable, (τ_{th} large).

τ_{th} large \Rightarrow enough time for the beam to blow up appreciably, before the threshold is reached at $t = \tau_{th}$

$$2 \dot{\gamma} \tau_{th} = 113 \left(e I_{Av}^B \left| \frac{Z_n}{h} \right| \right)^{4/3} \frac{E_0 \gamma_{tr}^4}{A^2 \omega_0^2 h^{1/3} \left[\frac{eV}{2\pi} |\cos \psi_s| \right]^{1/3}}$$

$$\delta \gamma_{tr} \equiv 2 \dot{\gamma} \tau_{th}$$

$$\delta \gamma_{tr} \propto \frac{\gamma_{tr}^4}{A^2}$$

Growth rate (No frequency spread)

Imaginary part of

$$G(t) = \Delta\Omega(t) = \sqrt{i \frac{e I_{\text{peak}} \omega_0^2 \eta(t)}{2\pi E} n Z_n}$$

$$I_{\text{peak}} = \frac{\sqrt{2\pi} I_{\text{AV}}^B}{\sigma_{\omega}} \cdot h ; \text{ I take } n \sim R/b$$

The growth rate vanishes at $t=0$,

($\because \eta=0$) and at $t = \pm T_{th}$. (Landau Damping)

A good measure of the extent of the beam blow up is the imaginary part of:

$$\chi \equiv G\left(\frac{1}{2} T_{th}\right) \cdot T_{th}$$

(13)

If $Z_n = |Z_n| e^{i\alpha}$, then

$$G\left(\frac{1}{2} \tau_{th}\right) = 6 \sqrt{i e^{i\alpha}} e^{I_{AV}^B} \cdot |Z_n| / A$$

Independent of γ_t , $\dot{\gamma}$, V & ψ_s !!

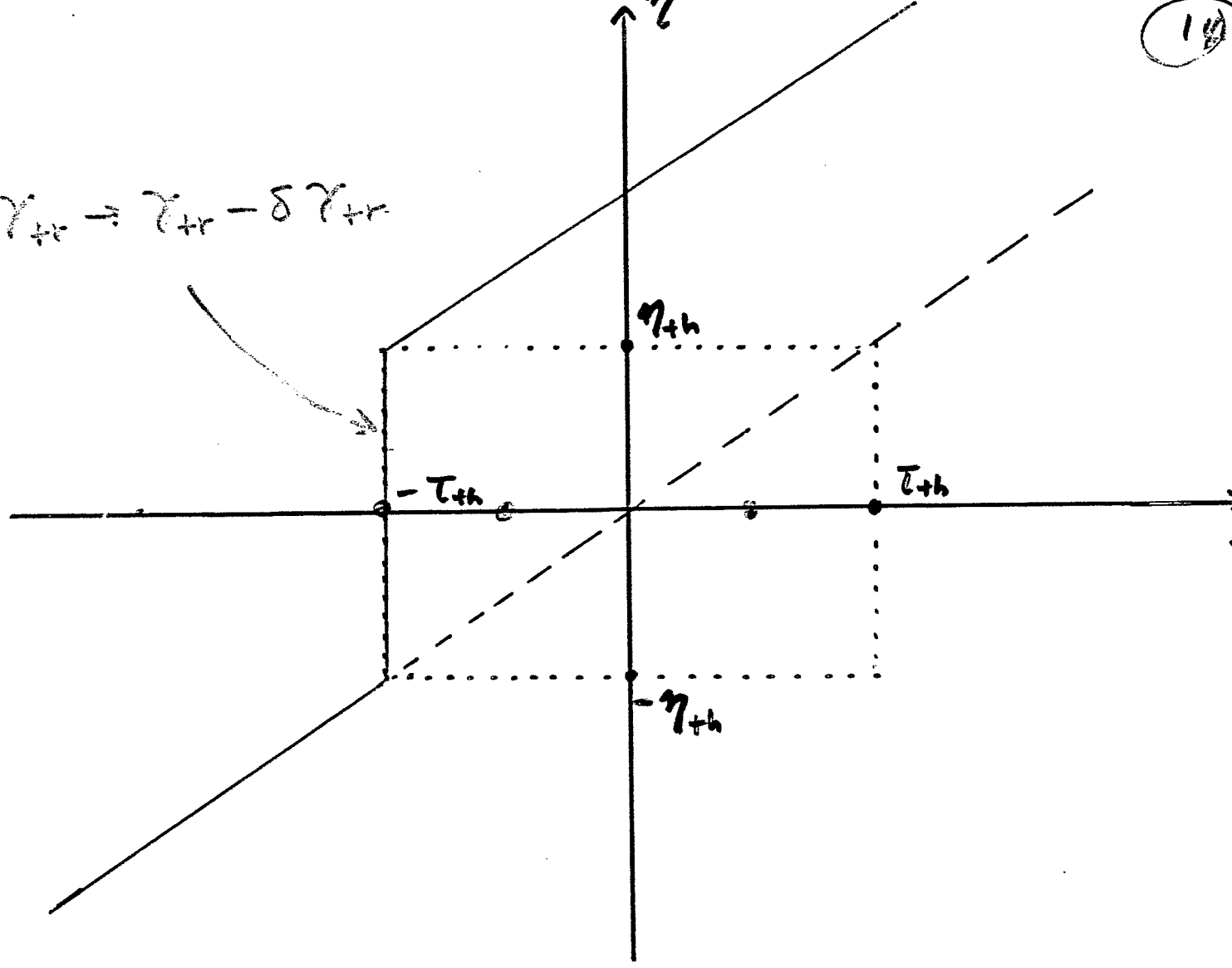
$$\chi \propto \tau_{th} / A$$

τ_{th} is large for RHIC.

$\delta\gamma_{tr} \equiv 2 \dot{\gamma} \tau_{th}$: The amount of

γ_{tr} jump needed.

$$\gamma_{tr} \Rightarrow \tau_{tr} - \delta \gamma_{tr}$$



$$\delta \gamma_{tr} \equiv z \dot{\gamma} \tau_{th}$$

If the growth time at $t = \pm \frac{1}{2} \tau_{th}$ is much shorter than τ_{th} , then γ_{tr} has to jump.

$$Z_n/n = 5 + i Z_{sc}/n$$

Scheme	A	B	C
$\delta\gamma_{tr}$	24	4.3	4.3
T_{th}	4 (sec)	0.7	7.2
$G(\frac{1}{2}T_{th})$	812 (sec ⁻¹)	904	929
Z_{sc}/n at $t = T_{th}$	0.8 (ohm)	1.7	1.9

$$Z_n/n = 2 + i Z_{sc}/n$$

Scheme	A	B	C
$\delta\gamma_{tr}$	7.6	1.7	1.8
T_{th}	1.3 (sec)	0.3	3
$G(\frac{1}{2}T_{th})$	377 (sec ⁻¹)	496	532
Z_{sc}/n at $t = T_{th}$	0.8	1.7	1.9

$$Z_n/n = 0 + i Z_{sc}/n$$

Scheme	A	B	C
$\delta\gamma_{tr}$	2/2 *	0.9/2 *	1/2 *
T_{th}	0.35 (sec)	0.15	1.8
$G(\frac{1}{2}T_{th})$	171 (sec ⁻¹)	345	393
Z_{sc}/n at $t = T_{th}$	0.8	1.7	1.9

* Divided by 2 since instability start at $t=0$ instead of $t = -T_{th}$

Crossing the Transition Energy

	D	D	E
$S = 0.2 \text{ eV/A-sec}$	$15/120^\circ$	$12/90^\circ$	$9/120^\circ$
γ_T	39.0	25.1	26.9 23.6
f_{RF}	53.487 MHz	26.743	26.743
V	1.0 MV	0.2	0.2
ϕ_s	5.45°	6.82	6.82
charact. time	31.8 msec	60.4	66.3 55.7
bunch length	$\pm 0.40 \text{ nsec}$	0.92	0.88 0.96
bunch spread	$\pm 0.51 \%$	0.34	0.33 0.35
Betatron Emittance			
$\epsilon \gamma = \gamma_T$	0.39	0.97	0.85 0.97
95% of beam	$\pi \text{ mm-mrad}$		
	$Z_n/n = 2 +$	Z_{sc}/n	
$\delta \gamma_{tr}$		4	3
τ_{th}		2.8 (sec)	2
$G(\frac{1}{2} \tau_{th})$		483 (sec ⁻¹)	544
$Z_{sc}/n \text{ at } t = \frac{1}{2} \tau_{th}$		1.6 (ohm)	2

Recommendations

1. Lower γ_{tr}
2. Study how to jump γ_{tr}
3. More detailed analysis.