

Crossing The Transition Energy At RHIC

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CROSSING THE TRANSITION ENERGY

AT RHIC

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9 shall discuss the following effects at or near transition energy

① $\Delta P/p$

② Longitudinal phase space mismatch (Hereward - Sorensen effect)

③ RF phase jump mistiming (Umstätter effect)

④ Microwave instability (Courant Effect)

Only the last effect can be

Serious for RHIC.

1. Bunch area is an invariant (Poincare invariant), even though the Hamiltonian is time dependent & non-adiabatic near transition.

$$2. \quad \gamma = \gamma_{tr} + \dot{\gamma} t$$

$$\eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}$$

$$E = E_0 \gamma ; \quad \omega_0 = \beta \omega_{\infty} ; \quad \omega_{\infty} = \frac{c}{R}$$

$$\Omega_s \cdot T_s = 2\pi$$

$$\Omega_s^2 = - \frac{\omega_{\infty}^2 h}{E} \frac{eV}{2\pi} \eta \cos \varphi_s$$

$$\eta \cos \varphi_s < 0$$

3. T_{NA} : non-adiabatic time

3

$$\frac{1}{T_{NA}^3} \equiv \frac{\omega_{\infty}^2 \hbar}{E_0} \frac{2\dot{\gamma}}{\gamma_t^4} \frac{eV}{2\pi} |\cos \varphi_s|$$

If $\gamma_{tr} \gg \dot{\gamma} t$, then :

(a) $\eta \approx 2\dot{\gamma} t / \gamma_t^3$

(b) $\Omega_s^2 \approx t / T_{NA}^3$

(c) $\left| \frac{dT_s}{dt} \right| \ll 1 \Rightarrow \frac{|t|}{T_{NA}} \gg 1$

adiabatic condition



② Longitudinal space charge effect

$$\eta_{sc} \equiv \frac{e g_0 I_b h Z_0}{2\sqrt{2\pi} R \gamma^2 (\sigma_\varphi)^3} \left| \frac{eV}{2\pi} \cos \varphi_s \right|$$

$$g_0 = \frac{1}{2} + 2 \log \frac{z_b}{a_{rt+a}}$$

$$\frac{d}{dt} \Delta E = \frac{\varphi - \varphi_s}{T_{rev}} eV \cos \varphi_s \left. \begin{array}{l} (1 - \eta_{sc}) ; \quad \gamma < \gamma_{tr} \\ (1 + \eta_{sc}) ; \quad \gamma > \gamma_{tr} \end{array} \right\}$$

Space charge lengthen (shorten) the bunch before (after) the transition \implies

filamentation after transition

$$\rho = \frac{\text{bunch area way after transition}}{\text{" before "}}$$

$$\rho = 1 + 1.4 \eta_{sc}$$

Horizontal space charge effect (5)

Horizontal space charge $\Rightarrow \Delta\gamma_{tr}$

\Rightarrow mistiming of RF phase jump

for some particles \Rightarrow bunch blow up

$$\tau_{miss} \equiv \frac{\Delta\gamma_{tr}}{\dot{\gamma} T_{NA}} \approx \frac{\Delta V_{H,SC}}{\dot{\gamma} T_{NA}}$$

$$\rho \approx 1 + 1.7 \tau_{miss}$$

$S = 0.2 \text{ eV/Acc/AMU}$	15/120° Scheme A	12/90° Scheme B	9/120° (6) Scheme C
σ_T	39.0	25.1	25.9 23.6
Accel. Period	30 sec	30 sec	5 min
Energy Gain / turn	38.1 keV/A	38.1	3.81
Peak Voltage for Acceleration	1.0 MV	1.0	0.5
ρ_s	5.45°	5.45°	1.09°
Bucket Area :			
inject.	0.66 eV/A-sec	0.72	0.52 0.5
top energy	5.7	3.5	2.7 2.3
Bucket Height:			
inject.	± 0.24 %	0.26	0.19 0.1
top energy	0.24	0.15	0.11 0.1
Peak Voltage in Storage Mode at 100 GeV/A	1.0 MV	2.6	2.3 3.
ν_s @ 100 GeV/A	0.0005	0.0013	0.0013
$\bar{\eta}$	0.5	1.15	1.25

E_N

$10\pi \times 10^{-6}$ rad in (

The Bucket Area and Height given in this Table are for Stationary Bucket (no Acceleration)

At Transition

	Scheme A	B	C
$\Delta p/p$ (bottom to bottom)	$1. \times 10^{-2}$	1.1×10^{-2}	1.3×10^{-2}
η_{sc}	1.7×10^{-5}	1.4×10^{-5}	4×10^{-5}
T_{miss}	3.6×10^{-3}	5.6×10^{-3}	2×10^{-2}
T_{NA} (sec)	3.3×10^{-2}	1.8×10^{-2}	4.6×10^{-2}

Crossing the Transition Energy

	A.	D.	E.
$S = 0.2 \text{ eV/A-sec}$	$15/120^\circ$	$12/90^\circ$	$9/120^\circ$
γ_T	39.0	25.1	26.9 23.6
f_{RF}	53.487 MHz	26.743	26.743
V	1.0 MV	0.2	0.2
ϕ_s	5.45°	6.82	6.82
charact. time T_{NA}	31.8 msec	60.4	66.3 55.7
bunch length	$\pm 0.40 \text{ nsec}$	0.92	0.88 0.96
bunch spread	$\pm 0.51 \%$	0.34	0.33 0.35
Betatron Emittance			
$Q \text{ } \delta = \gamma_T$	0.39	0.97	0.85 0.97
95% of beam	$\pi \text{ mm-mrad}$		
η_{sc}		6×10^{-5}	6×10^{-5}
L_{miss}		7×10^{-3}	8×10^{-3}

Microwave Instability

?

$$\xi = \Delta E \quad ; \quad \eta = \varphi - \varphi_s \quad ; \quad \varphi: \text{RF phase}$$

Assume that the threshold happens at

$$t = \pm T_{th} \quad ;$$

$$\dot{\gamma} T_{th} \ll \gamma_{tr}, \quad \frac{T_{th}}{T_{NA}} \gg 1; \quad \eta_{th} = \eta(T_{th}) = \frac{2\dot{\gamma} T_{th}}{\gamma_{tr}^3}$$

(has to be checked a posteriori)

$$\left\{ \begin{array}{l} \eta = \eta_{max} \cos\left(\gamma - \frac{7}{12}\pi - \psi\right) \\ \xi = -\xi_{max} \sin\left(\gamma - \frac{7}{12}\pi - \psi\right) \end{array} \right.$$

$$\xi_{max} = \eta_{max} \sqrt{\frac{E}{h|\eta|} \frac{eV}{2\pi} |\cos\varphi_s|}$$

$$A = \pi \xi_{max} \eta_{max} \frac{1}{\omega_0 h}$$

$$\eta_{max} \propto t^{-1/4} \quad ; \quad \xi_{max} \propto t^{1/4}$$

$$\gamma = \int_0^t dt' \Omega_s(t') = \frac{2}{3} \left| \frac{t}{T_{NA}} \right|^{3/2}$$

$$\xi_{max} = 2.5 \sigma_\xi, \quad \eta_{max} = 2.5 \sigma_\eta$$

Threshold Condition

(10)

$$\frac{1}{|\eta(t)|} \left| \frac{z_n}{n} \right| \frac{1}{\sqrt{2\pi}} e^{I_{AV}^B E_0 \gamma_{tr}} \frac{h}{\sigma_{\theta}(t)} \frac{1}{\sigma_{\xi}(t)^2} = 1$$

Solution: $t = \pm \tau_{th}$

Unstable at $t=0$, since $\eta(t=0) = 0$.

$$\eta(t) = \frac{2\dot{\gamma}t}{\gamma_{tr}^3}; \quad \dot{\eta} = \frac{2\ddot{\gamma}}{\gamma_{tr}^3}$$

If $\dot{\eta} \ll 1$, then there is a large time interval around $t=0$ when $\eta(t)$ is very small, and the beam unstable, (τ_{th} large).

τ_{th} large \Rightarrow enough time for the beam to blow up appreciably, before the threshold is reached at $t = \tau_{th}$

$$2 \dot{\gamma} \tau_{th} = 113 \left(e I_{Av}^B \left| \frac{Z_n}{h} \right| \right)^{4/3} \frac{E_0 \gamma_{tr}^4}{A^2 \omega_0^2 h^{1/3} \left[\frac{eV}{2\pi} |\cos \psi_s| \right]^{1/3}}$$

$$\delta \gamma_{tr} \equiv 2 \dot{\gamma} \tau_{th}$$

$$\delta \gamma_{tr} \propto \frac{\gamma_{tr}^4}{A^2}$$

Growth rate (No frequency spread)

Imaginary part of

$$G(t) = \Delta\Omega(t) = \sqrt{i \frac{e I_{\text{peak}} \omega_0^2 \eta(t)}{2\pi E} n Z_n}$$

$$I_{\text{peak}} = \frac{\sqrt{2\pi} I_{\text{AV}}^B}{\sigma_{\omega}} \cdot h ; \text{ I take } n \sim R/b$$

The growth rate vanishes at $t=0$,

($\because \eta=0$) and at $t = \pm T_{th}$. (Landau Damping)

A good measure of the extent of the beam blow up is the imaginary part of:

$$\chi \equiv G\left(\frac{1}{2} T_{th}\right) \cdot T_{th}$$

(13)

If $Z_n = |Z_n| e^{i\alpha}$, then

$$G\left(\frac{1}{2} \tau_{th}\right) = 6 \sqrt{i e^{i\alpha}} e^{I_{AV}^B} \cdot |Z_n| / A$$

Independent of γ_t , $\dot{\gamma}$, V & ψ_s !!

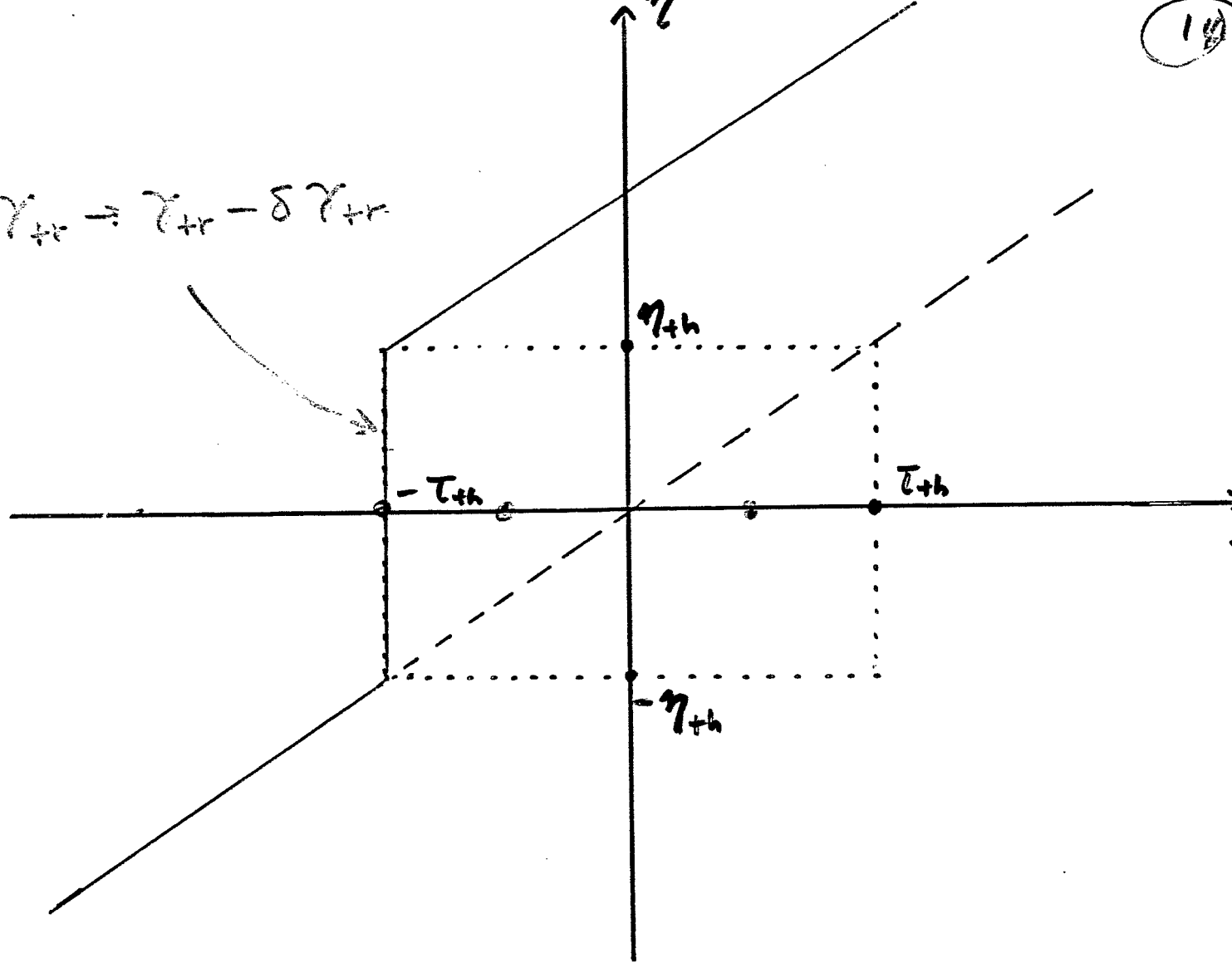
$$\chi \propto \tau_{th} / A$$

τ_{th} is large for RHIC.

$\delta\gamma_{tr} \equiv 2 \dot{\gamma} \tau_{th}$: The amount of

γ_{tr} jump needed.

$$\gamma_{tr} \Rightarrow \tau_{tr} - \delta \gamma_{tr}$$



$$\delta \gamma_{tr} \equiv z \dot{\gamma} \tau_{th}$$

If the growth time at $t = \pm \frac{1}{2} \tau_{th}$ is much shorter than τ_{th} , then γ_{tr} has to jump.

$$Z_n/n = 5 + i Z_{sc}/n$$

Scheme	A	B	C
$\delta\gamma_{tr}$	24	4.3	4.3
T_{th}	4 (sec)	0.7	7.2
$G(\frac{1}{2}T_{th})$	812 (sec ⁻¹)	904	929
Z_{sc}/n at $t = T_{th}$	0.8 (ohm)	1.7	1.9

$$Z_n/n = 2 + i Z_{sc}/n$$

Scheme	A	B	C
$\delta\gamma_{tr}$	7.6	1.7	1.8
T_{th}	1.3 (sec)	0.3	3
$G(\frac{1}{2}T_{th})$	377 (sec ⁻¹)	496	532
Z_{sc}/n at $t = T_{th}$	0.8	1.7	1.9

$$Z_n/n = 0 + i Z_{sc}/n$$

Scheme	A	B	C
$\delta\gamma_{tr}$	2/2 *	0.9/2 *	1/2 *
T_{th}	0.35 (sec)	0.15	1.8
$G(\frac{1}{2}T_{th})$	171 (sec ⁻¹)	345	393
Z_{sc}/n at $t = T_{th}$	0.8	1.7	1.9

* Divided by 2 since instability start at $t=0$ instead of $t = -T_{th}$

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Betatron Emittance			
@ $\gamma = \gamma_T$	0.39	0.97	0.85 0.97
95% of beams	$\pi \text{ mm-mrad}$		
	$Z_n/n = 2 +$	Z_{sc}/n	
$\delta \gamma_{tr}$		4	3
τ_{th}		2.8 (sec)	2
$G(\frac{1}{2} \tau_{th})$		483 (sec ⁻¹)	544
$Z_{sc}/n \text{ at } t = \frac{1}{2} \tau_{th}$		1.6 (ohm)	2

Recommendations

1. Lower γ_{tr}
2. Study how to jump γ_{tr}
3. More detailed analysis.