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Crossing The Transition Energy At RHIC

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RHIC-PG-32

CROSSING THE TRANSITION ENERGY

AT RHIC

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I shall discuss the following effects at or near transition energy.

1. $\Delta p/p$
2. Longitudinal phase space mismatch (Hereward - Sorensen effect)
3. RF phase jump mistiming (Umstätter effect)
4. Microwave instability (Courant Effect)

Only the last effect can be serious for RHIC.

1. Bunch area is an invariant (Poincare invariant), even though the Hamiltonian is time dependent & non-adiabatic near transition.

2. $\gamma = \gamma_{tr} + \dot{\gamma} t$

$\eta = 1/\gamma_{tr}^2 - 1/\gamma^2$

$E = E_0 \gamma$; $\omega_0 = \beta \omega_\infty$; $\omega_\infty = c/R$

$\Omega_s \cdot T_s = 2\pi$

$$\Omega_s^2 = -\frac{\omega_\infty^2 h}{E} \frac{eV}{2\pi} \eta \cos \varphi_s$$

$$\eta \cos \varphi_s < 0$$

3. T_{NA} : non-adiabatic time

$$\frac{1}{T_{NA}^3} \equiv \frac{\omega_\infty^2 h}{E_0} \frac{2\dot{\gamma}}{\gamma_t^4} \frac{eV}{2\pi} |\cos \varphi_s|$$

If $\gamma_{tr} >> \dot{\gamma} t$, then :

(a) $\eta \simeq 2\dot{\gamma} t / \gamma_t^3$

(b) $\Omega_s^2 \simeq t / T_{NA}^3$

(c) $\left| \frac{d T_s}{dt} \right| << 1 \Rightarrow \frac{|t|}{T_{NA}} >> 1$

adiabatic condition

2 Longitudinal space charge effect

$$\eta_{sc} \equiv \frac{e\, g_0\, I_B\, h\, Z_0}{2\sqrt{2\pi}\, R\, \gamma^2 (\;\sigma_\varphi)^3} \Big/ \left| \frac{eV}{2\pi} \cos\varphi_s \right|$$

$$g_0 = \frac{1}{2} + 2 \log \frac{2b}{a_x + a_y}$$

$$\frac{d}{dt}\Delta E = \frac{\varphi - \varphi_s}{T_{rev}} eV \cos\varphi_s \begin{cases} (1 - \eta_{sc}) \;; & \gamma < \gamma_{tr} \\ (1 + \eta_{sc}) \;; & \gamma > \gamma_{tr} \end{cases}$$

Space charge lengthen (shorten) the bunch before (after) the transition $\Longrightarrow$

filamentation after transition

$$\rho = \frac{\text{bunch area way after transition}}{\text{"} \quad \text{before} \quad \text{"}}$$

$$\boxed{\rho \simeq 1 + 1.4\, \eta_{sc}}$$

Horizotal space Charge effect

Horizontal space charge $\Rightarrow$ $\Delta\gamma_{tr}$

$\Rightarrow$ mistiming of RF phase jump

for some particles $\Rightarrow$ bunch blow up

$$\tau_{miss} \equiv \frac{\Delta\gamma_{tr}}{\dot{\gamma}\, T_{NA}} \simeq \frac{\Delta\nu_{H,sc}}{\dot{\gamma}\, T_{NA}}$$

$$\boxed{\rho \simeq 1 + 1.7\, \tau_{miss}}$$

$S = 0.2$ eV-sec/AMU

	15/120° Scheme A	12/90° Scheme B	9/120° Schem C	
γ_T	39.0	25.1	~~25.9~~	23.6
Acceler. Period	30 sec	30 sec	5 min	
Energy Gain/turn	38.1 KeV/A	38.1	3.81	
Peak Voltage for Acceleration	1.0 MV	1.0	0.5	
φ_s	5.45°	5.45°	1.09°	
Bucket Area: inject.	0.56 eV/A-sec	0.72	0.52	0.5
top energy	5.7	3.5	2.7	2.3
Bucket Height: inject.	± 0.24 %	0.26	0.19	0.1
top energy	0.24	0.15	0.11	0.1
Peak Voltage in Storage Mode at 100 GeV/A	1.0 MV	2.6	2.3	3.
ν_s @ 100 GeV/A	0.0005	0.0013	0.0013	
$\bar{\eta}$	0.5	1.15	1.25	

E_N $10\pi \times 10^{-6}$ rad m (

The Bucket Area and Height given in this Table are for Stationary Bucket (no Acceleration

At Transition

	Scheme A	B	C
$\Delta p/p$ (bottom to bottom)	$1. \times 10^{-2}$	1.1×10^{-2}	1.3×10^{-2}
η_{sc}	1.7×10^{-5}	1.4×10^{-5}	4×10^{-5}
τ_{miss}	3.6×10^{-3}	5.6×10^{-3}	2×10^{-2}
T_{NA} (sec)	3.3×10^{-2}	1.8×10^{-2}	4.6×10^{-2}

Crossing the Transition Energy

	A	D	E	
$S = 0.2$ eV/A-sec	15/120°	12/90°	9/120°	
γ_T	39.0	25.1	~~26.9~~	23.6
f_{RF}	53.487 MHz	26.743	26.743	
V	1.0 MV	0.2	0.2	
φ_s	5.45°	6.82	6.82	
charact. time T_{NA}	31.8 msec	60.4	66.3	55.7
bunch length	± 0.40 nsec	0.92	0.88	0.96
bunch spread	± 0.51 %	0.34	0.33	0.35
Betatron Emittance				
@ $\gamma = \gamma_T$	0.39	0.97	0.85	0.97
95% of beam	π mm-mrad			
η_{sc}		6×10^{-5}	6×10^{-5}	
T_{miss}		7×10^{-3}	8×10^{-3}	

Microwave Instability

$\varepsilon = \Delta E \quad ; \quad \vartheta = \varphi - \varphi_s \quad ; \quad \varphi : \text{RF phase}$

Assume that the threshold happens at

$t = \pm \tau_{th}$;

$$\dot{\gamma}\, \tau_{th} \ll \gamma_{tr}, \quad \frac{\tau_{th}}{T_{NA}} \gg 1 ; \quad \eta_{th} = \eta(\tau_{th}) = \frac{2\dot{\gamma}\,\tau_{th}}{\gamma_{tr}^3}$$

(has to be checked a posteriori)

$$\begin{cases} \vartheta = \vartheta_{max} \cos\left(Y - \frac{7}{12}\pi - \psi\right) \\ \varepsilon = -\varepsilon_{max} \sin\left(Y - \frac{7}{12}\pi - \psi\right) \end{cases}$$

$$\varepsilon_{max} = \vartheta_{max} \sqrt{\frac{E}{h|\eta|} \frac{eV}{2\pi} |\cos\varphi_s|}$$

$$A = \pi\, \varepsilon_{max}\, \vartheta_{max} \frac{1}{\omega_0 h}$$

$$\vartheta_{max} \propto t^{-1/4} \quad ; \quad \varepsilon_{max} \propto t^{1/4}$$

$$Y = \int_0^t dt'\, \Omega_s(t') = \frac{2}{3} \left| t / T_{NA} \right|^{3/2}$$

$$\varepsilon_{max} = 2.5\, \sigma_\varepsilon , \quad \vartheta_{max} = 2.5\, \sigma_\vartheta$$

Threshold Condition

$$\frac{1}{|\eta(t)|}\left|\frac{Z_n}{n}\right|\frac{1}{\sqrt{2\pi}}\, e\, I_{AV}^{B}\, E_0\, \gamma_{tr}\, \frac{h}{\sigma_\vartheta(t)}\, \frac{1}{\sigma_\varepsilon(t)^2} = 1$$

Solution: $t = \pm \tau_{th}$

unstable at $t=0$, since $\eta(t=0)=0$.

$$\eta(t) = \frac{2\dot{\gamma} t}{\gamma_{tr}^3} \quad ; \quad \dot{\eta} = 2\dot{\gamma}/\gamma_{tr}^3$$

If $\dot{\eta} \ll 1$, then there is a large time interval around $t=0$ when $\eta(t)$ is very small, and the beam unstable, (τ_{th} large).

τ_{th} large $\Rightarrow$ enough time for the beam to blow up appreciably, before the threshold is reached at $t=\tau_{th}$

$$\boxed{2\dot{\gamma}\,\tau_{th} = 113\left(e\,I_{AV}^{B}\left|\frac{Z_n}{n}\right|\right)^{4/3}\frac{E_0\,\gamma_{tr}^{4}}{A^2\,\omega_0^{2}\,h^{1/3}\left[\frac{eV}{2\pi}\left|\cos\varphi_s\right|\right]^{1/3}}}$$

$$\delta\gamma_{tr} \equiv 2\dot{\gamma}\,\tau_{th}$$

$$\delta\gamma_{tr} \propto \frac{\gamma_{tr}^{4}}{A^{2}}$$

Growth rate (No frequency spread)

Imaginary part of

$$G(t) = \Delta\Omega(t) = \sqrt{i\,\frac{e\, I_{peak}\, \omega_0^2\, \eta(t)}{2\pi E}\; n\, Z_n}$$

$$I_{peak} = \frac{\sqrt{2\pi}\; I_{AV}^{B}}{\sigma_\vartheta}\cdot h \quad ; \quad \text{I take } n \sim R/b$$

The growth rate vanishes at $t=0$, ($\because \eta = 0$) and at $t = \pm T_{th}$. (Landau Damping)

A good measure of the extent of the beam blow up is the imaginary part of:

$$\chi \equiv G\left(\tfrac{1}{2} T_{th}\right)\cdot T_{th}$$

If $Z_n = |Z_n| e^{i\alpha}$, then

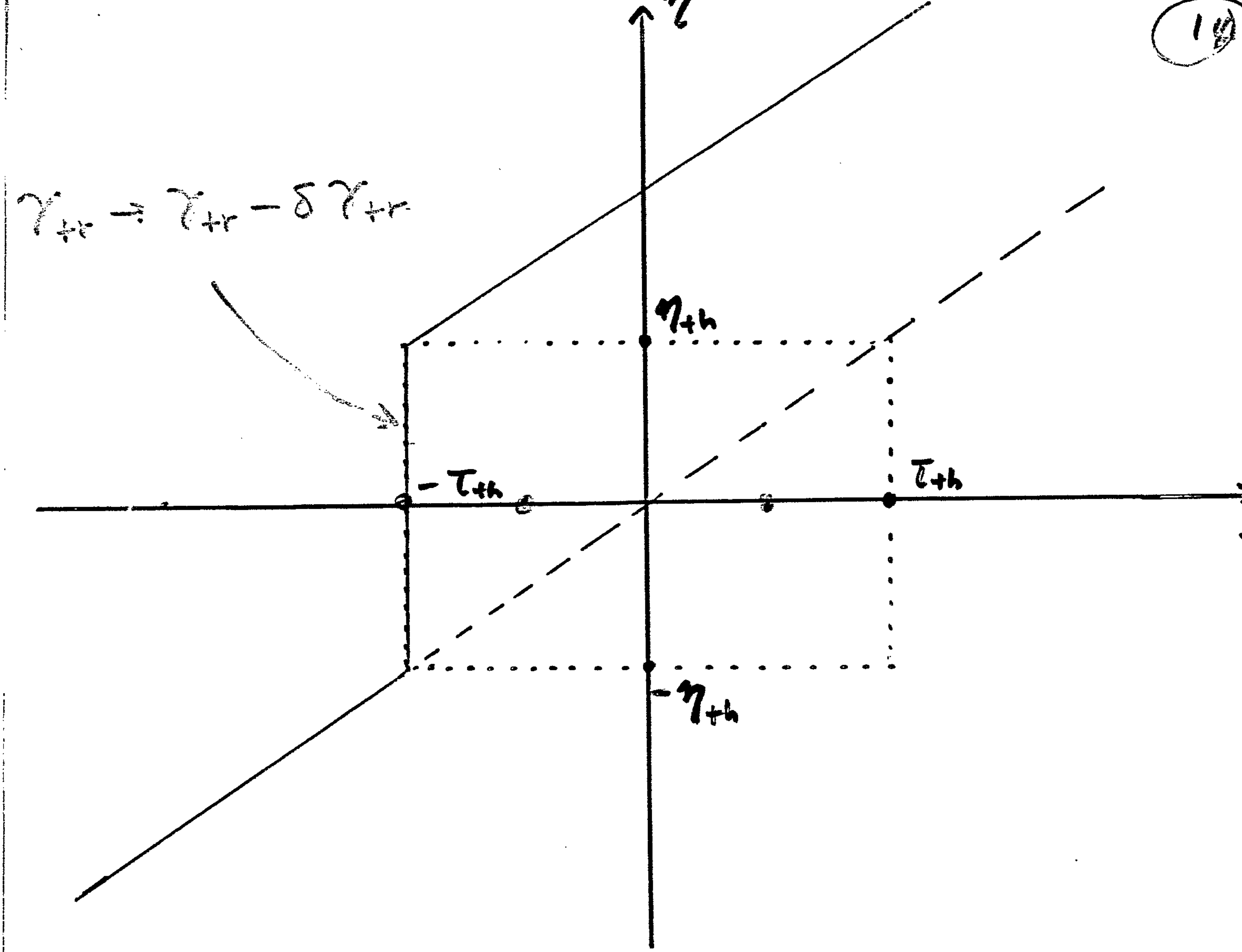
$$\boxed{G(\tfrac{1}{2}\tau_{th}) = 6\sqrt{i e^{i\alpha}}\; e I_{AV}^{B} \cdot |Z_n| / A}$$

Independent of γ_t, $\dot{\gamma}$, V & φ_s !!

$$\chi \propto \tau_{th} / A$$

τ_{th} is large for RHIC.

$\boxed{\delta\gamma_{tr} \equiv 2\dot{\gamma}\tau_{th}}$: The amount of γ_{tr} jump needed.



$$\delta\gamma_{tr} \equiv 2\dot{\gamma}\tau_{th}$$

If the growth time at $t = \pm\frac{1}{2}\tau_{th}$ is much shorter than τ_{th}, then γ_{tr} has to jump.

$Z_n/n = 5 + i\, Z_{sc}/n$

Scheme	A	B	C
$\delta\gamma_{tr}$	24	4.3	4.3
τ_{th}	4 (sec)	0.7	7.2
$G(\frac{1}{2}\tau_{th})$	812 (sec^{-1})	904	929
Z_{sc}/n at $t=\tau_{th}$	0.8 (ohm)	1.7	1.9

$Z_n/n = 2 + i\, Z_{sc}/n$

Scheme	A	B	C
$\delta\gamma_{tr}$	7.6	1.7	1.8
τ_{th}	1.3 (sec)	0.3	3
$G(\frac{1}{2}\tau_{th})$	377 (sec^{-1})	496	532
Z_{sc}/n at $t=\tau_{th}$	0.8	1.7	1.9

$Z_n/n = 0 + i\, Z_{sc}/n$

Scheme	A	B	C
$\delta\gamma_{tr}$	2/2 *	0.9/2 *	1/2 *
τ_{th}	0.35 (sec)	0.15	1.8
$G(\frac{1}{2}\tau_{th})$	171 (sec^{-1})	345	393
Z_{sc}/n at $t=\tau_{th}$	0.8	1.7	1.9

* Divided by 2 since instability start at $t=0$ instead of $t=-\tau_{th}$

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95% of beam	π mm-mrad			
	$Z_n/n = 2 +$	Z_{sc}/n		
$8\,\tau_{tr}$		4	3	
τ_{th}		2.8 (sec)	2	
$G(\frac{1}{2}\tau_{th})$		483 (sec^{-1})	544	
Z_{sc}/n at $t = \frac{1}{2}\tau_{th}$		1.6 (ohm)	2.	

Recommendations

1. Lower γ_{tr}
2. Study how to jump γ_{tr}
3. More detailed analysis.