

## Crossing The Transition Energy At RHIC

J. M. Wang

February 1984

Collider Accelerator Department  
**Brookhaven National Laboratory**

**U.S. Department of Energy**

USDOE Office of Science (SC)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-76CH00016 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

## **DISCLAIMER**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

CROSSING THE TRANSITION ENERGY

AT RHIC

J. M. Wang

(BNL, February 7, 1984

①

9 shall discuss the following effects at or near transition energy

①  $\Delta P/p$

② Longitudinal phase space mismatch (Hereward - Sorensen effect)

③ RF phase jump mistiming (Umstätter effect)

④ Microwave instability (Courant Effect)

Only the last effect can be

Serious for RHIC.

(2)

1. Bunch area is an invariant (Poincare invariant), even though the Hamiltonian is time dependent & non-adiabatic near transition.

$$2. \quad \gamma = \gamma_{tr} + \dot{\gamma} t$$

$$\eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}$$

$$E = E_0 \gamma ; \quad \omega_0 = \beta \omega_{\infty} ; \quad \omega_{\infty} = c/R$$

$$\Omega_s \cdot T_s = 2\pi$$

$$\Omega_s^2 = - \frac{\omega_{\infty}^2 h}{E} \frac{eV}{2\pi} \eta \cos \varphi_s$$

$$\eta \cos \varphi_s < 0$$

3.  $T_{NA}$  : non-adiabatic time

$$\frac{1}{T_{NA}^3} \equiv \frac{\omega_\infty^2 \hbar}{E_0} \frac{2\dot{\gamma}}{\gamma_t^4} \frac{eV}{2\pi} |\cos \varphi_s|$$

If  $\gamma_{tr} \gg \dot{\gamma} t$ , then :

$$(a) \eta \approx 2\dot{\gamma} t / \gamma_t^3$$

$$(b) \Omega_s^2 \approx t / T_{NA}^3$$

$$(c) \left| \frac{dT_s}{dt} \right| \ll 1 \Rightarrow \frac{|t|}{T_{NA}} \gg 1$$

adiabatic condition



## ② Longitudinal space charge effect

$$\eta_{sc} \equiv \frac{e g_0 I_b h Z_0}{2\sqrt{2\pi} R \gamma^2 (\sigma_\varphi)^3} \left/ \left| \frac{eV}{2\pi} \cos \varphi_s \right| \right.$$

$$g_0 = \frac{1}{2} + 2 \log \frac{2b}{a_{xt+a}}$$

$$\frac{d}{dt} \Delta E = \frac{\varphi - \varphi_s}{T_{rev}} eV \cos \varphi_s \begin{cases} (1 - \eta_{sc}) & ; \gamma < \gamma_{tr} \\ (1 + \eta_{sc}) & ; \gamma > \gamma_{tr} \end{cases}$$

Space charge lengthen (shorten) the bunch  
before (after) the transition  $\Rightarrow$

filamentation after transition

$$\rho = \frac{\text{bunch area way after transition}}{\text{" before "}}$$

$$\rho = 1 + 1.4 \eta_{sc}$$

# Horizontal space charge effect (5)

Horizontal space charge  $\Rightarrow \Delta\gamma_{tr}$

$\Rightarrow$  mistiming of RF phase jump

for some particles  $\Rightarrow$  bunch blow up

$$\tau_{miss} \equiv \frac{\Delta\gamma_{tr}}{\dot{\gamma} T_{NA}} \simeq \frac{\Delta V_{H,sc}}{\dot{\gamma} T_{NA}}$$

$$\rho \simeq 1 + 1.7 \tau_{miss}$$



$S = 0.2 \text{ eV/Acc/AMU}$	15/120° Scheme A	12/90° Scheme B	9/120° ⑥ Scheme C
$\delta_T$	39.0	25.1	<del>25.9</del> 23.6
Accel. Period	30 sec	30 sec	5 min
Energy Gain / turn	38.1 keV/A	38.1	3.81
Peak Voltage for Acceleration	1.0 MV	1.0	0.5
$\phi_s$	5.45°	5.45°	1.09°
Bucket Area :			
inject.	0.66 eV/A-sec	0.72	0.52 0.5
top energy	5.7	3.5	2.7 2.3
Bucket Height:			
inject.	$\pm 0.24 \%$	0.26	0.19 0.1
top energy	0.24	0.15	0.11 0.1
Peak Voltage in Storage Mode at 100 GeV/A	1.0 MV	2.6	2.3 3.
$\nu_s$ @ 100 GeV/A	0.0005	0.0013	0.0013
$\bar{\eta}$	0.5	1.15	1.25

$E_N$

$10\pi \times 10^{-6} \text{ rad in.}$

The Bucket Area and Height given in this Table are for Stationary Bucket (no Acceleration)

## At Transition

	Scheme A	B	C
$\Delta p/p$ (bottom to bottom)	$1. \times 10^{-2}$	$1.1 \times 10^{-2}$	$1.3 \times 10^{-2}$
$\eta_{sc}$	$1.7 \times 10^{-5}$	$1.4 \times 10^{-5}$	$4 \times 10^{-5}$
$T_{miss}$	$3.6 \times 10^{-3}$	$5.6 \times 10^{-3}$	$2 \times 10^{-2}$
$T_{NA} \text{ (sec)}$	$3.3 \times 10^{-2}$	$1.8 \times 10^{-2}$	$4.6 \times 10^{-2}$

# Crossing the Transition Energy

(8)

	A.	D	E	
$S = 0.2 \text{ eV/A-sec}$	$15/120^\circ$	$12/90^\circ$	$9/120^\circ$	
$\gamma_T$	39.0	25.1	<del>26.9</del>	23.6
$f_{RF}$	53.487 MHz	26.743	26.743	
$V$	1.0 MV	0.2	0.2	
$\phi_s$	$5.45^\circ$	6.82	6.82	
charact. time $T_{NA}$	31.8 msec	60.4	66.3	55.7
bunch length	$\pm 0.40 \text{ nsec}$	0.92	0.88	0.96
bunch spread	$\pm 0.51 \%$	0.34	0.33	0.35
Betatron Emittance				
@ $\gamma = \gamma_T$	0.39	0.97	0.85	0.97
95% of beam	$\pi \text{ mm-mrad}$			
$\eta_{sc}$		$6 \times 10^{-5}$	$6 \times 10^{-5}$	
$T_{miss}$		$7 \times 10^{-3}$	$8 \times 10^{-3}$	

# Microwave Instability

9

$$\xi = \Delta E \quad ; \quad \eta = \varphi - \varphi_s \quad ; \quad \varphi: \text{RF phase}$$

Assume that the threshold happens at

$$t = \pm T_{th} \quad ;$$

$$\gamma T_{th} \ll \gamma_{tr}, \quad \frac{T_{th}}{T_{NA}} \gg 1, \quad \eta_{th} = \eta(T_{th}) = \frac{2\gamma T_{th}}{\gamma_{tr}^3}$$

(has to be checked a posteriori)

$$\begin{cases} \eta = \eta_{max} \cos(\gamma - \frac{7}{12}\pi - \psi) \\ \xi = -\xi_{max} \sin(\gamma - \frac{7}{12}\pi - \psi) \end{cases}$$

$$\xi_{max} = \eta_{max} \sqrt{\frac{E}{h|\eta|} \frac{eV}{2\pi} |\cos \varphi_s|}$$

$$A = \pi \xi_{max} \eta_{max} \frac{1}{\omega_0 h}$$

$$\eta_{max} \propto t^{-1/4} \quad ; \quad \xi_{max} \propto t^{1/4}$$

$$\gamma = \int_0^t dt' \Omega_s(t') = \frac{2}{3} \left| \frac{t}{T_{NA}} \right|^{3/2}$$

$$\xi_{max} = 2.5 \sigma_\xi, \quad \eta_{max} = 2.5 \sigma_\eta$$

# Threshold Condition

(10)

$$\frac{1}{|\eta(t)|} \left| \frac{z_n}{n} \right| \frac{1}{\sqrt{2\pi}} e I_{AV}^B E_0 \gamma_{tr} \frac{h}{\sigma_y(t)} \frac{1}{\sigma_z(t)^2} = 1$$

Solution:  $t = \pm \tau_{th}$

Unstable at  $t=0$ , since  $\eta(t=0)=0$ .

$$\eta(t) = \frac{2\dot{\gamma}t}{\gamma_{tr}^3} ; \quad \dot{\eta} = 2\dot{\gamma}/\gamma_{tr}^3$$

If  $\dot{\eta} \ll 1$ , then there is a large time interval around  $t=0$  when  $\eta(t)$  is very small, and the beam unstable, ( $\tau_{th}$  large).

$\tau_{th}$  large  $\Rightarrow$  enough time for the beam to blow up appreciably, before the threshold is reached at  $t=\tau_{th}$

$$2 \dot{\gamma} \tau_{th} = 113 \left( e I_{Av}^B \left| \frac{Z_n}{h} \right| \right)^{4/3} \frac{E_0 \gamma_{tr}^4}{A^2 \omega_0^2 h^{1/3} \left[ \frac{eV}{2\pi} |\cos \varphi_s| \right]^{1/3}}$$

$$\delta \gamma_{tr} \equiv 2 \dot{\gamma} \tau_{th}$$

$$\delta \gamma_{tr} \propto \frac{\gamma_{tr}^4}{A^2}$$

## Growth rate (No frequency spread)

Imaginary part of

$$G(t) = \Delta\Omega(t) = \sqrt{i \frac{e I_{\text{peak}} \omega_0^2 \eta(t)}{2\pi E} n Z_n}$$

$$I_{\text{peak}} = \frac{\sqrt{2\pi} I_{\text{Av}}^B}{\sigma_\omega} \cdot h ; \text{ I take } n \sim R/b$$

The growth rate vanishes at  $t=0$ ,

( $\because \eta=0$ ) and at  $t=\pm T_{th}$ . (Landau Damping)

A good measure of the extent of the beam blow up is the imaginary part of:

$$\chi \equiv G\left(\frac{1}{2} T_{th}\right) \cdot T_{th}$$

If  $Z_n = |Z_n| e^{i\alpha}$ , then

$$G\left(\frac{1}{2} \tau_{th}\right) = 6 \sqrt{i e^{i\alpha}} e^{I_{AV}^B} \cdot |Z_n| / A$$

Independent of  $\gamma_t$ ,  $\dot{\gamma}$ ,  $V$  &  $\psi_s$  !!

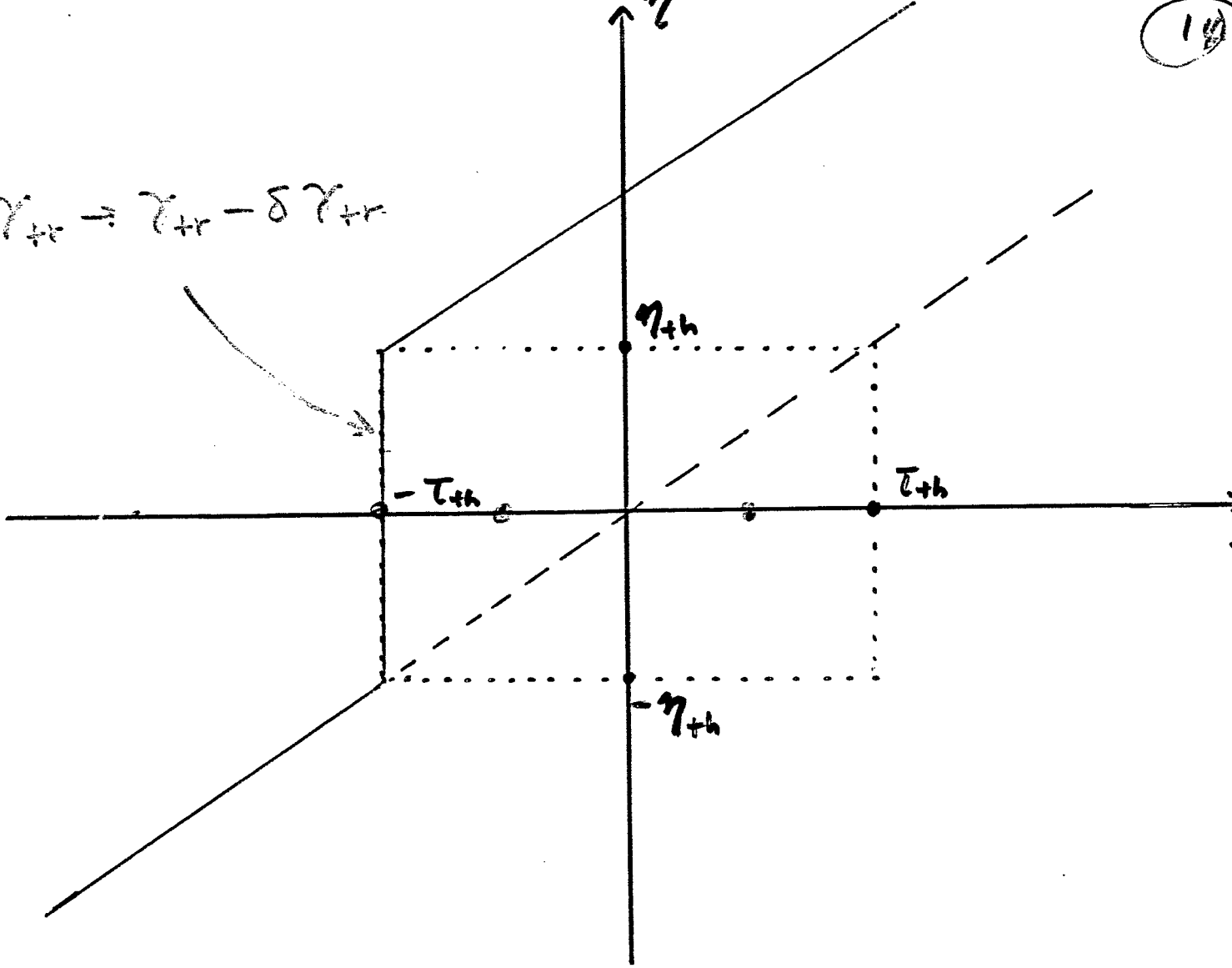
$$\chi \propto \tau_{th} / A$$

$\tau_{th}$  is large for RHIC.

$\boxed{\delta \gamma_{tr} \equiv 2 \dot{\gamma} \tau_{th}}$  : The amount of  $\gamma_{tr}$  jump needed.



$$\gamma_{tr} \Rightarrow \gamma_{tr} - \delta \gamma_{tr}$$



$$\delta \gamma_{tr} \equiv z \dot{\gamma} \tau_{th}$$

If the growth time at  $t = \pm \frac{1}{2} \tau_{th}$  is much shorter than  $\tau_{th}$ , then  $\gamma_{tr}$  has to jump.

$$Z_n/n = 5 + i Z_{sc}/n$$

Scheme	A	B	C
$\delta\gamma_{tr}$	24	4.3	4.3
$T_{th}$	4 (sec)	0.7	7.2
$G(\frac{1}{2}T_{th})$	812 (sec <sup>-1</sup> )	904	929
$Z_{sc}/n$ at $t = T_{th}$	0.8 (ohm)	1.7	1.9

$$Z_n/n = 2 + i Z_{sc}/n$$

Scheme	A	B	C
$\delta\gamma_{tr}$	7.6	1.7	1.8
$T_{th}$	1.3 (sec)	0.3	3
$G(\frac{1}{2}T_{th})$	377 (sec <sup>-1</sup> )	496	532
$Z_{sc}/n$ at $t = T_{th}$	0.8	1.7	1.9

$$Z_n/n = 0 + i Z_{sc}/n$$

Scheme	A	B	C
$\delta\gamma_{tr}$	2/2 *	0.9/2 *	1/2 *
$T_{th}$	0.35 (sec)	0.15	1.8
$G(\frac{1}{2}T_{th})$	171 (sec <sup>-1</sup> )	345	393
$Z_{sc}/n$ at $t = T_{th}$	0.8	1.7	1.9

\* Divided by 2 since instability start at  $t=0$  instead of  $t = -T_{th}$

# Crossing the Transition Energy

(16)

	D		E	
$S = 0.2 \text{ eV/A-sec}$	15/120°	12/90°	9/120°	
$\gamma_T$	39.0	25.1	<del>26.9</del> 23.6	
$f_{RF}$	53.487 MHz	26.743	26.743	
$V$	1.0 MV	0.2	0.2	
$\phi_s$	5.45°	6.82	6.82	
charact. time	31.8 msec	60.4	66.3	55.7
bunch length	$\pm 0.40 \text{ nsec}$	0.92	0.88	0.96
bunch spread	$\pm 0.51 \%$	0.34	0.33	0.35
Betatron Emittance				
@ $\delta = \gamma_T$	0.39	0.97	0.85	0.97
95% of beam	$\pi \text{ mm-mrad}$			
	$Z_n/n = 2 +$	$Z_{sc}/n$		
$8 \gamma_{tr}$		4	3	
$\tau_{th}$		2.8 (sec)	2	
$G(\frac{1}{2} \tau_{th})$		483 (sec <sup>-1</sup> )	544	
$Z_{sc}/n$ at $t = \frac{1}{2} \tau_{th}$		1.6 (ohm)	2	

# Recommendations

(17)

1. Lower  $\gamma_{tr}$
2. Study how to jump  $\gamma_{tr}$
3. More detailed analysis.