



Brookhaven
National Laboratory

BNL-102279-2014-TECH

RHIC/AP/175;BNL-102279-2013-IR

Converging Towards a Solution on ? vs 1/?

W. MacKay

June 1998

Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Converging towards a Solution on γ vs $1/\gamma$
 Waldo MacKay, 9 June, 1998

Dear Mario,

OK. We appear to be converging towards agreement. I'm sorry I have been occupied with other jobs and have not had time to ponder the SG force until recently. While going over your last message, I see that you are now willing to accept the famous (you would say infamous)

$$f_z = \gamma\mu^* \frac{\partial B_z}{\partial z} + \gamma\mu^* \frac{\beta}{c} \frac{\partial B_z}{\partial t}.$$

I now must admit that perhaps there are ways to play with the cavity to get the desired result: $\Delta U \propto \gamma$. Going back to my long paper [internal note: RHIC/AP/153] where I evaluated the energy gain through the cylindrical cavity, I see that my result for the energy gain was not exactly proportional to $1/\gamma$, but was

$$\Delta U = (-1)^{m/2} \frac{\mu^\diamond B_0}{\gamma} \frac{R}{1-R^2} \left[\cos \phi_0 - (-1)^n \cos \left(\frac{n\pi}{R} + \phi_0 \right) \right],$$

with

$$R = \frac{n\pi\beta c}{\omega l} = \frac{\beta}{\sqrt{1 + \left(\frac{X'_{0m} l}{n\pi a} \right)^2}}.$$

So the energy gain was actually proportional to

$$\frac{1}{\gamma} \frac{R}{1-R^2} = \frac{\beta \sqrt{1 + \left(\frac{X'_{0m} l}{nb} \right)^2}}{\gamma \left(\frac{X'_{0m} l}{n\pi a} \right)^2 + \gamma^{-1}}$$

clearly X'_{0m} is nonzero; however, if we build the cavity for energies below some selected maximum value of $\gamma_{\max} mc^2$, we can get a contribution which is roughly proportional to γ . (Duh!) For example, if we take the TE₀₁₂ mode which gives a similar B_z profile along the axis, and we look at $\gamma_{\max} = 100$, then we would want to have

$$\left(\frac{3.832l}{2\pi a} \right) \ll \frac{1}{\gamma_{\max}} = 0.01,$$

or

$$\frac{l}{a} \ll 0.0164.$$

While this requires a rather funny pancake-shaped cavity, it does give an analytic existence proof of a cavity which would work at least on paper for a rather large value of gamma. Perhaps one might consider higher order modes with $n > 2$ to decrease the required aspect ratio of the cavity. Or better yet, design a reentrant cavity, such as you folks have already done. In fact such a cavity might indeed not be limited to $\gamma < \gamma_{\max}$.