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Converging Towards a Solution on ? vs 1/?

W. MacKay

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Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

DISCLAIMER

Converging towards a Solution on γ vs $1/\gamma$

Waldo MacKay, 9 June, 1998

Dear Mario,

OK. We appear to be converging towards agreement. I'm sorry I have been occupied with other jobs and have not had time to ponder the SG force until recently. While going over your last message, I see that you are now willing to accept the famous (you would say infamous)

$$f_z = \gamma\mu^* \frac{\partial B_z}{\partial z} + \gamma\mu^* \frac{\beta}{c}\frac{\partial B_z}{\partial t}.$$

I now must admit that perhaps there are ways to play with the cavity to get the desired result: $\Delta U \propto \gamma$. Going back to my long paper [internal note: RHIC/AP/153] where I evaluated the energy gain through the cylindrical cavity, I see that my result for the energy gain was not exactly proportional to $1/\gamma$, but was

$$\Delta U = (-1)^{m/2} \frac{\mu^\diamond B_0}{\gamma} \frac{R}{1-R^2} \left[\cos\phi_0 - (-1)^n \cos\left(\frac{n\pi}{R} + \phi_0\right)\right],$$

with

$$R = \frac{n\pi\beta c}{\omega l} = \frac{\beta}{\sqrt{1 + \left(\frac{X'_{0m} l}{n\pi a}\right)^2}}.$$

So the energy gain was actually proportional to

$$\frac{1}{\gamma}\frac{R}{1-R^2} = \frac{\beta\sqrt{1+\left(\frac{X'_{0m} l}{nb}\right)^2}}{\gamma\left(\frac{X'_{0m} l}{n\pi a}\right)^2 + \gamma^{-1}}$$

clearly X'_{0m} is nonzero; however, if we build the cavity for energies below some selected maximum value of $\gamma_{\max} mc^2$, we can get a contribution which is roughly proportional to γ. (Duh!) For example, if we take the TE_{012} mode which gives a similar B_z profile along the axis, and we look at $\gamma_{\max} = 100$, then we would want to have

$$\left(\frac{3.832 l}{2\pi a}\right) \ll \frac{1}{\gamma_{\max}} = 0.01,$$

or

$$\frac{l}{a} \ll 0.0164.$$

While this requires a rather funny pancake-shaped cavity, it does give an analytic existence proof of a cavity which would work at least on paper for a rather large value of gamma. Perhaps one might consider higher order modes with $n > 2$ to decrease the required aspect ratio of the cavity. Or better yet, design a reentrant cavity, such as you folks have already done. In fact such a cavity might indeed not be limited to $\gamma < \gamma_{\max}$.