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# Beam-Beam Collisions and Crossing Angles in RHIC

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# Beam-beam collisions and crossing angles in RHIC

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*Abstract*

This paper evaluates the strength of head on and parasitic beam-beam collisions in RHIC when the crossing angle is zero. A non-zero crossing angle is not required in normal operation with 120 bunches, thanks to the early separation of the two beams. The RHIC lattice is shown to easily accommodate even conservatively large crossing angles, for example in beam dynamics studies, or in future operational upgrades to as many as 360 bunches per ring. A modest loss in luminosity is incurred when gold ions collide at an angle after 10 hours of storage.

## 1 BEAM-BEAM TUNE SHIFT PARAMETERS

When two identical Gaussian beams collide, the horizontal and vertical beam-beam tune shift parameters are given by

$$\xi_{H,V} = \frac{r}{2\pi\gamma} \frac{N\,\beta_{H,V}}{\sigma_{H,V}(\sigma_H + \sigma_V)} \qquad (1)$$

where $N$ is the single bunch population, the classical radius $r$ is $r_p = 1.5347 \times 10^{-18}$ meters for protons and $r_{Au} = 48.992 \times 10^{-18}$ meters for gold, $\beta_{H,V}$ is the beta function in the appropriate plane, and $\gamma$ is the Lorentz factor. Assuming from here on that the beam is round ($\beta_H = \beta_V, \sigma_H = \sigma_V$), the transverse beam size is given (in the relativistic limit) by

$$\sigma = \sqrt{\frac{\epsilon_N \beta}{6\pi\gamma}} \qquad (2)$$

where $\epsilon_N$ is the "6$\pi$" normalized emittance used at RHIC. Equation 1 is succinctly rewritten as

$$\xi = \frac{3Nr}{2\,\epsilon_N} \qquad (3)$$

Note that the tune shift parameter is independent of energy ($\gamma$), and independent of $\beta$. The tune shift of small amplitude particles is equal to the parameter, $\xi$, no matter what the azimuthal location of the collision, if the beams are round and if they collide head-on.

Nominally there are $N = 10^{11}$ protons per bunch, with a 95% normalized transverse emittance of $\epsilon_N = 20\pi$ microns. When gold ions are stored, there are $N = 10^9$ ions per bunch, with an emittance that rises from $\epsilon_N = 10\pi$ microns at injection to $\epsilon_N = 40\pi$ microns at the end of a 10 hour store. Centered on these nominal parameters, it is convenient to numerically parameterize the proton and gold tune shift parameters as

$$\xi_p = 0.00366 \frac{N}{10^{11}} \frac{20\pi\mu\mathrm{m}}{\epsilon_N} \qquad (4)$$

and

$$\xi_{Au} = 0.00117 \frac{N}{10^9} \frac{20\pi\mu\mathrm{m}}{\epsilon_N} \qquad (5)$$

For comparison purposes, strong beam-beam effects are noticed in proton colliders when $\xi = 0.004$, with 6 head-on collisions per turn [1].

## 2 BEAM SEPARATION GEOMETRY

There are two beam separation dipole magnets, DX and D0, between each interaction point (IP) and the first quadrupole of the interaction region triplet. The large bore magnet DX, nearest to the IP, is common to both beams. A drift follows, allowing the two trajectories to diverge far enough to enter one or the other of side-by-side D0 dipoles. The D0 magnets remove most – but not all – of the angular divergence applied by DX. Each D0 is immediately followed by three triplet quadrupoles, for a total of eight magnets in a single cryostat. Table 1 lists the nominal geometrical parameters of this region.

| Quantity | Units | Value |
|---|---|---|
| DX magnetic length | [m] | 3.70 |
| DX bending radius | [m] | 196.17 |
| DX bend angle | [mrad] | 18.86 |
| DX bend center (from IP) | [m] | 11.65 |
| D0 magnetic length | [m] | 3.60 |
| D0 bending radius | [m] | 237.06 |
| D0 bend angle | [mrad] | −15.19 |
| D0 bend center (from IP) | [m] | 22.30 |

Table 1: DX and DO dipole parameters, when the crossing angle is zero. Bend center locations are measured in meters from the IP.

If a TOTAL central collision crossing angle of $\alpha$ is required, then the DX and D0 bend angles, $\theta_X(\alpha)$ and $\theta_0(\alpha)$, need to be adjusted so the beam has the correct angle and displacement when entering the first quadrupole. These constraints are met, to a very good approximation, if

$$\begin{aligned}\theta_X(\alpha) &= \theta_X(0) - \frac{\alpha}{2}\frac{s_0}{s_0 - s_X} & (6)\\ &= 18.86 - 1.047\,\alpha \quad [\mathrm{mrad}] \\ \theta_0(\alpha) &= \theta_0(0) + \frac{\alpha}{2}\frac{s_X}{s_0 - s_X} & (7)\\ &= -15.19 + 0.547\,\alpha \quad [\mathrm{mrad}]\end{aligned}$$

(8)

where $s_X$ and $s_0$ are the bending center locations listed in Table 1. Note that the absolute value of the bend angles, and the fields of DX and D0, both decrease with increasing crossing angle. A typical value of $\alpha$ in the various scenarios discussed below is 1 milliradian.

## 3 PARASITIC COLLISIONS

The ratio of harmonic numbers in RHIC is 7:1, consistent with the acceleration and storage radio frequencies of 28.15 MHz ($h_{acc} = 360, \lambda_{acc} = 10.650$ m) and 197.02 MHz ($h_{store} = 2520, \lambda_{store} = 1.521$ m). Since the harmonic number at injection is 360, in principle it is possible to arrange for identical collision patterns at all 6 IPs with 3,6,12,15,18,24,30,36,60,72,90,120,180, or 360 evenly spaced bunches. After a minor upgrade the nominal number of bunches in each of the 2 RHIC rings will become 120 (minus 5 or 6 bunches for the abort gap).

If 120/180/360 bunches are stored, the azimuthal distance between beam-beam crossings is 3/2/1 times $\lambda_{coll} = 5.325$ meters – half the acceleration wavelength. While in principle it is possible to collide 180 or 360 bunches – for example, for the purposes of beam dynamics experiments – this will require the rise time of the injection kickers to be reduced below the nominal specification of 100 nanoseconds. Such a scenario is operationally impractical even with faster injection kickers, since the detectors would also need a major electronics upgrade to take data at this bunch spacing.

The two beams enter separate beam pipes at a crotch 15.70 meters from the IP. Thus, there will be 0/1/2 parasitic beam-beam collisions on each side of the IP when 120/180/360 bunches are present. It is natural to label the locations of these parasitic collisions "1" and "2", according to the distance from the IP measured in units of $\lambda_{coll}$, with "0" being the head-on collision at the IP.

The total beam separation $d$ at each of the collision points is recorded in Table 2, when the crossing angle is zero. Using equation 2, the beam size $\sigma_{20}$ for a standard normalized emittance of $20\pi$ microns is also given for protons and gold, at injection and storage. It is the relative size of the total beam separation, measured in units of the beam size $\sigma$, that is the relevant parameter when evaluating the potency of parasitic beam-beam collisions. Hence, Table 2 also records $d/\sigma_{20}$ in all cases.

| Quantity | Units | "0" | "1" | "2" |
|---|---|---|---|---|
| Location $s$ | [m] | 0.00 | 5.33 | 10.65 |
| Total separation $d$ | [m] | .0000 | .0000 | .0037 |
| Injection $\beta$ | [m] | 10.0 | 12.84 | 21.34 |
| Storage $\beta$ | [m] | 1.00 | 29.41 | 114.4 |
| Proton inject $\sigma_{20}$ | [mm] | 1.03 | 1.17 | 1.51 |
| Proton store $\sigma_{20}$ | [mm] | 0.11 | 0.60 | 1.19 |
| Gold inject $\sigma_{20}$ | [mm] | 1.63 | 1.85 | 2.38 |
| Gold store $\sigma_{20}$ | [mm] | 0.18 | 0.96 | 1.88 |
| Proton inject $d/\sigma_{20}$ | | 0.00 | 0.00 | 2.45 |
| Proton store $d/\sigma_{20}$ | | 0.00 | 0.00 | 3.11 |
| Gold inject $d/\sigma_{20}$ | | 0.00 | 0.00 | 1.55 |
| Gold store $d/\sigma_{20}$ | | 0.00 | 0.00 | 1.97 |

Table 2: Parasitic collision parameters, with no crossing angle. The central and parasitic collisions are labeled "0" through "2", according to their distance from the IP. A standard emittance of $20\pi$ microns is assumed throughout. The Lorentz factor at injection(storage) for protons is $\gamma = 31.17(268.2)$, while for gold ions at injection(storage) it is $\gamma = 12.6(108.4)$.

The horizontal and vertical beta functions in the region of interest – before the triplet – behave as if they are in a pure drift, except for negligible edge focusing at the ends of the DX magnet. They are therefore equal, and given by

$$\beta(s) \;=\; \beta^* + \frac{s^2}{\beta^*} \tag{9}$$

where $s$ is the azimuthal distance from the IP, and $\beta^*$ is the beta function at the IP. The relative luminosity of a head-on parasitic collision is just

$$\frac{L}{L^*} \;=\; \frac{\beta^*}{\beta} \tag{10}$$

since the luminosity scales as the inverse square of the beam size. Assuming that $\beta^* = 1.0$ meter at storage, collision "1" only generates about 3.4% as many background events as the good events from the central collision at the IP, even with no crossing angle. However, the background contamination increases rapidly – initially quadratically – as $\beta^*$ increases to 10.0 meters, the value assumed for injection.

## 4 CROSSING ANGLES

When 180 bunches are stored, only parasitic collision "2" is active. The total separation – between about 1.5 and 3.1 $\sigma_{20}$, according to Table 2 – is in the maximally bad intermediate regime. Not only do such separations make a significant contribution to the total tune shift, but they also drive both odd and even order resonances. The nominal proton tune shift parameter $\xi_p \approx 0.0037$ is large enough to disallow this – or any head-on – parasitic collision. Protons *must* have a crossing angle when 180 bunches collide. The nominal gold tune shift parameter $\xi_{Au} \approx 0.0011$ is relatively small. Gold ions *possibly* do not need a crossing angle when 180 bunches collide.

When 360 bunches are stored, parasitic collisions occur at "1" and "2". Protons *must* have a crossing angle when 360 bunches collide. If gold ions collide without a crossing angle, they see a total of three head-on collisions of equal strength per IP, plus two ugly "2" collisions. Gold ions *probably* need a crossing angle when 360 bunches collide.

The small amplitude tune shift due to a single long range collision scales like

$$|\Delta Q_{LR}| \simeq 2\xi \left(\frac{\sigma'^*}{\alpha}\right)^2 \tag{11}$$

where

$$\sigma'^* = \frac{\sigma^*}{\beta^*} \tag{12}$$

is the root mean square angular size at the central collision point [2]. For this reason it is conservative to make the total crossing angle

$$\alpha = 7\,\sigma'^*_{20} \tag{13}$$

where, as before, a standard normalized emittance of 20 $\pi$ microns is assumed. Table 3 lists the crossing angle that this leads to, for protons and gold at injection and at collision. It also lists the modified bend angles of the DX and D0 dipoles, according to equations 6 and 7.

| Quantity | Proton inject | Proton store | Gold inject | Gold store |
|---|---|---|---|---|
| ANGLES | | | | |
| Beam $\sigma'^*_{20}$ [mrad] | 0.10 | 0.11 | 0.16 | 0.18 |
| Crossing $\alpha$ [mrad] | 0.70 | 0.77 | 1.14 | 1.26 |
| DX $\theta_X$ [mrad] | 18.13 | 17.81 | 17.67 | 17.54 |
| D0 $-\theta_0$ [mrad] | 14.45 | 14.38 | 13.99 | 13.87 |
| Length $\sigma_z$ [m] | 0.353 | 0.072 | 0.467 | 0.206 |
| $L(\alpha)/L(0)$ | 0.993 | 0.970 | 0.987 | 0.811 |

Table 3: Various angles, and luminosity performance, when RHIC beams collide at an angle. The worst case bunch length has been used for gold ions in storage, after 10 hours of intra beam scattering.

A potentially serious side effect is the loss of luminosity that a crossing angle incurs. This is given by

$$\frac{L(\alpha)}{L(0)} = \frac{1}{\sqrt{1 + (\alpha\sigma_z/2\sigma'^*_{20})^2}} \tag{14}$$

where $\sigma_z$ is the root mean square length of a bunch [3]. Table 3 indicates that negligible luminosity is lost when protons cross at an angle, but that as much as 19% of the nominal luminosity is lost at the end of a 10 hour gold ion store.

## 5 CONCLUSIONS

A crossing angle is not required when 120 or fewer bunches are stored. Although a crossing angle is required when 180 nominal proton bunches collide, it might be possible for 180 gold bunches to collide head-on. A crossing angle is required when 360 bunches of protons, or gold ions, collide.

The largest crossing angle is required for gold ions. A conservative estimate is $\alpha \simeq 1.26$ milliradians. This is easily achieved by reducing the magnetic field of DX and D0 dipoles by 7.0% and 8.7%, respectively.

Gold ions might see a luminosity decrease of about 19% at the end of a 10 hour store. This loss can be minimized by using a less conservative crossing angle – or by decreasing the storage time.

All of these statements assume a fixed standard normalized emittance of $20\pi$ microns.

## 6 REFERENCES

[1] "Beam-Beam Interaction Effects in the Fermilab Collider", Donna Siergiej, PhD Thesis, University of New Mexico, March 1995.

[2] "Conceptual Design" of the Superconducting Super Collider, p. 198 et seq., SSC-SR-202, March 1986.

[3] "Conceptual Design" of the Superconducting Super Collider, p. 104, SSC-SR-202, March 1986.