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On the Low-Frequency Coupling Impedance of Transmission Line Kickers

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I. INTRODUCTION

Transmission line kickers are built as a low-pass filter with essentially lumped L & C elements, and thus are amenable to circuit analysis. In fact, the original model proposed by Nassibian and Sacherer for the analysis of the kicker coupling impedance is simply a lumped current transformer with the beam represented as the primary and the bus bar as the secondary winding.[1] Nassibian's subsequent derivation of an expression for the coupling impedance was based on considering the kicker as a uniform transmission line which is magnetically coupled to the beam but his results were limited by assuming matched terminations at both ports.In a later report, Nassibian derived expressions for the more general case of unmatched kicker terminations, but his study was limited to the real part of the coupling impedance.[2]

In contrast to the "good" agreement between experiment and theory reported by Nassibian, the results from the RHIC injection kicker impedance measurements showed significant differences with the above theories.[3] Furthermore, the impedance measurements of the kicker system[4] can only be done with one port resistor-terminated and the other changing with frequency from short to open due to the long feeding cables. The need to amend the intrinsic limitations of the Sacherer-Nassibian theories thus provided the impetus for the present study.

After summarizing the Sacherer and Nassibian theory, a generalized derivation of the longitudinal coupling impedance is presented. The kicker is still treated as a transformer-coupled uniform transmission line, but the constraints on the terminations are removed. The kicker is defined by its characteristic impedance Z_K and the propagation velocity v_K, both beam-independent parameters measurable directly on the magnet. The generalized theory of the coupling impedance has two additional free parameters: (1) the mutual inductance M and (2) the "self"-inductance of the beam L_B. The values of these parameters are obtained from "wire"-measurements, using the theoretical expressions for their interpretation. As intuitively expected, the "self"-inductance of the beam follows from the results for the kicker open at both ports.

Finally, the cell-structure of the kicker and other effects due to the non-uniformity or losses from currents in the ferrite blocks can be treated numerically in an equivalent circuit analysis using the P-Spice program by MicroSim. All circuit parameters are directly obtained from measurement and the resulting calculated coupling impedance is now in reasonable agreement with that from wire-impedance measurements.

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[1] G. Nassibian and F. Sacherer, Nucl. Instr. & Meth., vol. 159, p. 21, (1979).
[2] G. Nassibian, Report CERN/PS 84-25 (BR) (1984), and CERN/PS 85-68(BR) (1986)
[3] H. Hahn , M. Morvillo and A. Ratti, Report AD/RHIC/RD-95 (1995).
[4] H. Hahn and A. Ratti, Report AD/RHIC/RD-105 (1996).

II. SUMMARY OF NASSIBIAN AND SACHERER THEORY

Considering the kicker as a lumped current transformer, Nassibian and Sacherer gave an expression for the longitudinal coupling impedance of a C-type kicker as

$$Z = \omega^2 M^2 / \Sigma Z_K$$

with the mutual inductance between beam and kicker current, M, and the total kicker impedance contribution from the kicker inductance plus the generator impedance including cables, $\Sigma Z_K = j\omega L_K + Z_g$. The mutual inductance is quoted as

$$M = \frac{1}{2} L_K (1 + x/a)$$

with L_K the kicker inductance, a the kicker half-aperture and x the displacement from the center towards the bus bar. For a centered beam, one has $M = L_K/2$.

The effects of the finite kicker length were then taken into account by multiplying the lumped-circuit result by a transit time factor $(2/\theta) \sin \theta/2$ with θ the "electrical length" of the kicker given by

$$\theta = \omega l \sqrt{(\langle L\rangle \langle C\rangle)} = 2\pi f / f_{2\pi}$$

where $\langle L\rangle, \langle C\rangle$ the quantities per unit length. For example, the half-length RHIC injection kicker model ($l = 0.51$ m), which was extensively tested and is fully documented,[3] has an $\langle L\rangle = 1.46$ µH/m and $\langle C\rangle = 1.56$ nF/m, resulting in $f_{2\pi} = 41$ MHz.

In his subsequent report, Nassibian developed a more accurate treatment of the length effects. At frequencies well below the cell resonance,

$$\Omega_{cell} = 2/\sqrt{(L_{cell} C_{cell})}$$

with L_{cell} and C_{cell} the series inductance and shunt capacity of a cell, the kicker can be considered as a structure with uniform properties, characterized by the wave propagation velocity

$$v_K = 1/\sqrt{(\langle L\rangle \langle C\rangle)}$$

and the characteristic impedance

$$Z_K = \sqrt{(<L>/<C>)}\,.$$

For the RHIC injection kicker, one has $L_{cell} = 106$ nH and $C_{cell} = 115$ pF and thus a $f_{cell} = 91$ MHz, which sets the "low-frequency" limit. Well below this limit, the kicker has a characteristic impedance of $Z_K \sim 30\ \Omega$ and $v_K/c \sim 0.07$.

Assuming a uniform structure, and neglecting small effects due to electric coupling and the particle transit time, Nassibian's analysis of the magnetic coupling yielded separate expressions for real and imaginary part of the longitudinal coupling impedance of a "C- type" kicker with matched terminations at both ports

$$\begin{aligned} \mathrm{Re}Z &= (M/L_K)^2\, Z_K(1 - \cos k_K l) \\ \mathrm{Im}Z &= (M/L_K)^2\, Z_K(k_K l - \sin k_K l) \end{aligned}$$

where M is the mutual inductance and $k_K\, l = \theta$ the "electrical length" of the kicker as defined above. Note that the relation $Z_K\, k_K l = \omega\, L_K/c$ holds.

Later on, Nassibian derived expressions for the more general case of unmatched kicker terminations. His results for the case of a centered beam with the kicker matched at one port and the other terminated by the general impedance, $R + jX$, agree with the corresponding expressions in the Appendix of this report, but since his study was limited to the real part of the impedance, their usefulness is quite restricted.

III. GENERALIZED THEORY OF KICKER COUPLING IMPEDANCE

In this section the generalized expressions for the longitudinal coupling impedance of transmission line kickers are presented. In the low- frequency range here considered, the kicker acts as a transmission line with uniform, albeit anisotropic properties.[5] The kicker and the beam are treated as magnetically coupled transmission lines, for which the differential equations are well known. By limiting the considerations to the extreme relativistic case, where the space charge effect can be neglected, the beam can be represented by a "transmission line" in which the inductance per unit length, L_B/l, is determined by the coupling impedance of the un-terminated kicker, and the capacitance is negligible. One finds, with the harmonic time dependence $\exp(j\omega t)$ suppressed, the following set of differential equations in the position-dependent variables i_K, u_K, u_B representing the kicker current, kicker voltage, and beam voltage respectively

$$\frac{\mathrm{d}u_K}{\mathrm{d}s} = -jk_K Z_K i_K + j\frac{M}{L_K} k_K Z_K i_B$$

$$\frac{\mathrm{d}i_K}{\mathrm{d}s} = -jk_K u_K / Z_K$$

$$\frac{\mathrm{d}u_B}{\mathrm{d}s} = j\frac{M}{L_K} k_K Z_K i_K - j\frac{L_B}{L_K} k_K Z_K i_B$$

Assuming an extreme relativistic, filamentary beam current of unit strength

$$i_B = e^{-jks}$$

one obtains the coupling impedance from

$$Z = -\int_0^l \frac{\mathrm{d}u_B}{\mathrm{d}s} e^{jks} \mathrm{d}s$$

were $k = \omega/c$, $k_K = \omega\sqrt{<L><C>}$ and $Z_K = \sqrt{<L>/<C>}$.

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[5] H. Hahn, Report AD/RHIC/RD-66 (1994) and
H. Hahn and E. B. Forsyth, EPAC 94, London, vol.3, p. 2550.

The solution of the above differential equations are found without difficulty, for example by means of the MACSYMA program, together with the boundary conditions established by the kicker input and output terminations, R_i and R_o

$$u_K(s=0) = -R_i i_K(s=0)$$
$$u_K(s=l) = R_o i_K(s=l)$$

The general expression for the coupling impedance is somewhat lengthy, but reduces at low frequencies, where $k \ll k_K$, to a manageable size. Several special cases must be expected in the field or are accessible to measurement and thus are of theoretical interest. The case of one port terminated with the characteristic impedance and the other with a general impedance representing the pulser and connecting cables is recorded in the Appendix. A few simpler cases relevant to bench measurements are discussed here:

1. Case of $R_i = \infty$ and $R_o = \infty$

$$Z = jZ_K \frac{L_B}{L_K}(1-\kappa^2)\theta + j2Z_K\left(\frac{M}{L_K}\right)^2 \frac{1-\cos\theta}{\sin\theta}$$

with the magnetic coupling coefficient

$$\kappa^2 = \frac{M^2}{L_K L_B}$$

This case, i.e. with both kicker ports open, clearly shows the inadequacy of the Nassibian-Sacherer theory which predicts a singularity in the limit of $\theta \to 0$, in contrast to the present theoretical result

$$\lim_{\theta \to 0} Z = jZ_K \frac{L_B}{L_K}\theta = j\omega L_B$$

and the experimental result shown in Fig. 6 of reference 4. As theoretically required, the lambda-half (i.e. $\theta = \pi$) resonance occurs at ~20 MHz, but note that ferrite losses obfuscate the results at higher frequencies.

2. Case of $R_i = Z_K$ and $R_o = Z_K$

$$Z = Z_K \left(\frac{M}{L_K}\right)^2 (1 - \cos\theta) + jZ_K \frac{L_B}{L_K} \left\{ (1 - \kappa^2)\theta + \kappa^2 \sin\theta \right\}$$

It is to be noted, that the above expressions for the real part of the coupling impedance is identical to Nassibian's result, whereas the imaginary part differs. At very low frequencies and in the limit of θ »1 one finds

$$\lim_{\theta \to 0} Z = jZ_K \frac{L_B}{L_K} \theta = j\omega L_B$$

$$\lim_{\theta \to \infty} Z = jZ_K \frac{L_B}{L_K} (1 - \kappa^2)\theta = j\omega L_B (1 - \kappa^2)$$

Comparing these limits with wire- measurements yields L_B and κ since the parameters Z_K and L_K are known from previous kicker measurements. The values thus obtained can then be checked by comparison with the following two cases.

3. Case of $R_i = Z_K$ and $R_o = 0$

$$Z = \frac{1}{2} Z_K \left(\frac{M}{L_K}\right)^2 (1 - \cos 2\theta) + jZ_K \frac{L_B}{L_K} \left\{ (1 - \kappa^2)\theta + \frac{1}{2}\kappa^2 \sin 2\theta \right\}$$

4. Case of $R_i = Z_K$ and $R_o = \infty$

$$Z = \frac{1}{2} Z_K \left(\frac{M}{L_K}\right)^2 (3 - 4\cos\theta + \cos 2\theta) + jZ_K \frac{L_B}{L_K} \left\{ (1 - \kappa^2)\theta + 2\kappa^2 \sin\theta - \frac{1}{2}\kappa^2 \sin 2\theta \right\}$$

The low and high frequency limits in case 3 and 4 are identical to those in case 2, i.e. with fully matched terminations at both ends, a fact which can be used to check the internal consistency of the numerical results for L_B and κ obtained from the wire-measurements. In Fig. 1 and 2 the coupling impedance of the half-length kicker model, which is fully described in reference 4, corresponding to the three cases is shown. The results can be fitted with $L_B = 132$ nH and $(1-\kappa^2) = 0.325$, leading to

$$L_B/L_K = 0.18, \; M/L_K = 0.34, \text{ and } \kappa = 0.82.$$

Theory and measured results for the coupling impedance are in reasonable agreement as long as the "electrical length" of the kicker is less than one free-space wavelength. At higher frequencies, the assumption of magnetic coupling is still valid, but the kicker is no longer the ideal lossless, uniform transmission line and a more general treatment, for example based on equivalent circuits, is required.

The Nassibian expressions are discussed by Ng in the 1987 Fermilab Summer School,[6] where it is pointed out, that they do *not* satisfy the Hilbert transformation, "the reason of which is not clear at the moment." In contrast, the present results satisfy these constraints in view of their *ab ovo* derivation. It is also clear that the present treatment, but without making the approximation of $k \ll k_K$, must be applied to stripline beam position monitors in order to obtain correct results.

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[6] K-Y. Ng, AIP Conf. Proc. 184, vol 1, p. 519 (1989).

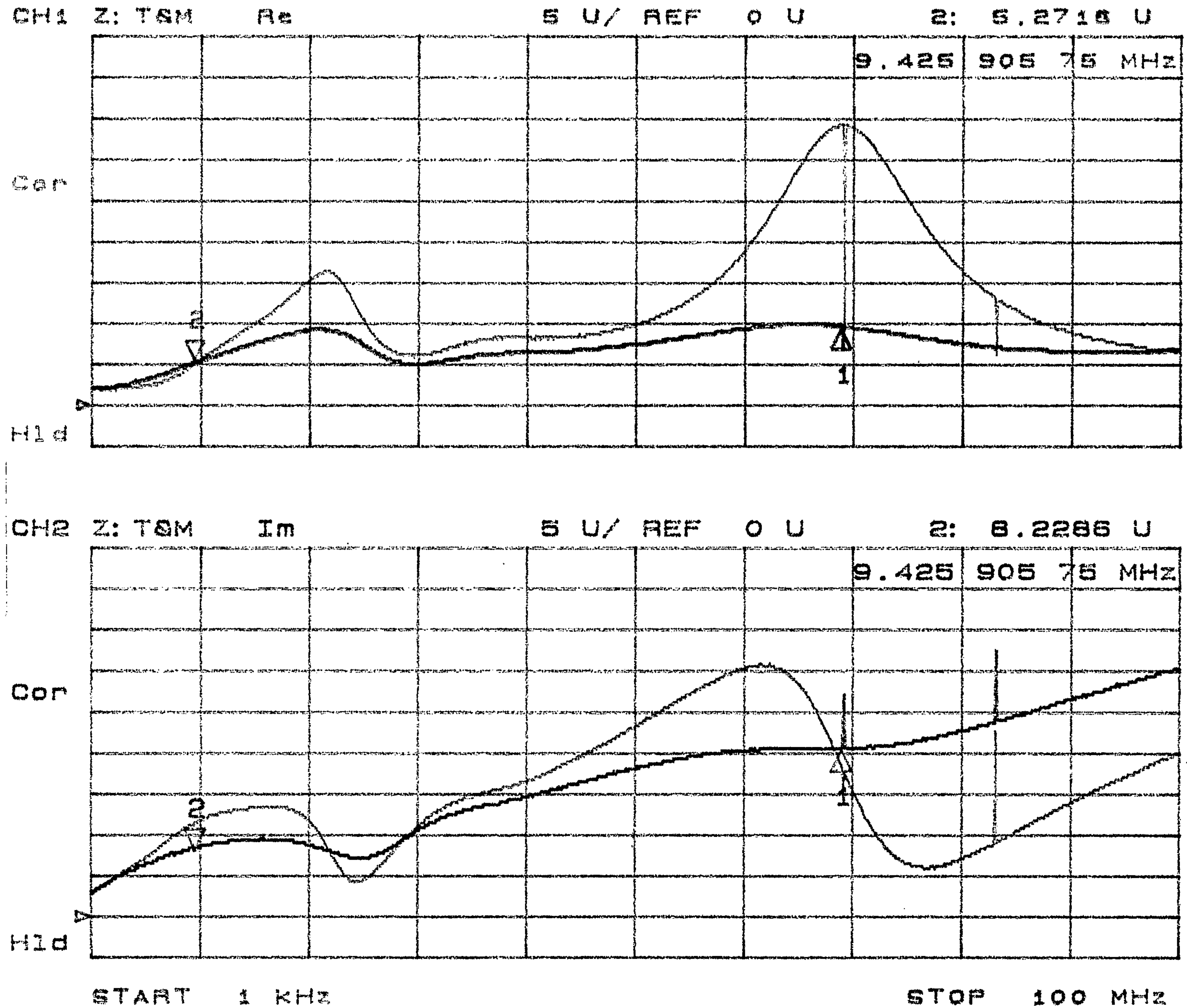

Fig. 1. Kicker coupling impedance, with both ports terminated (heavy line) and with ports open and terminated (light line).

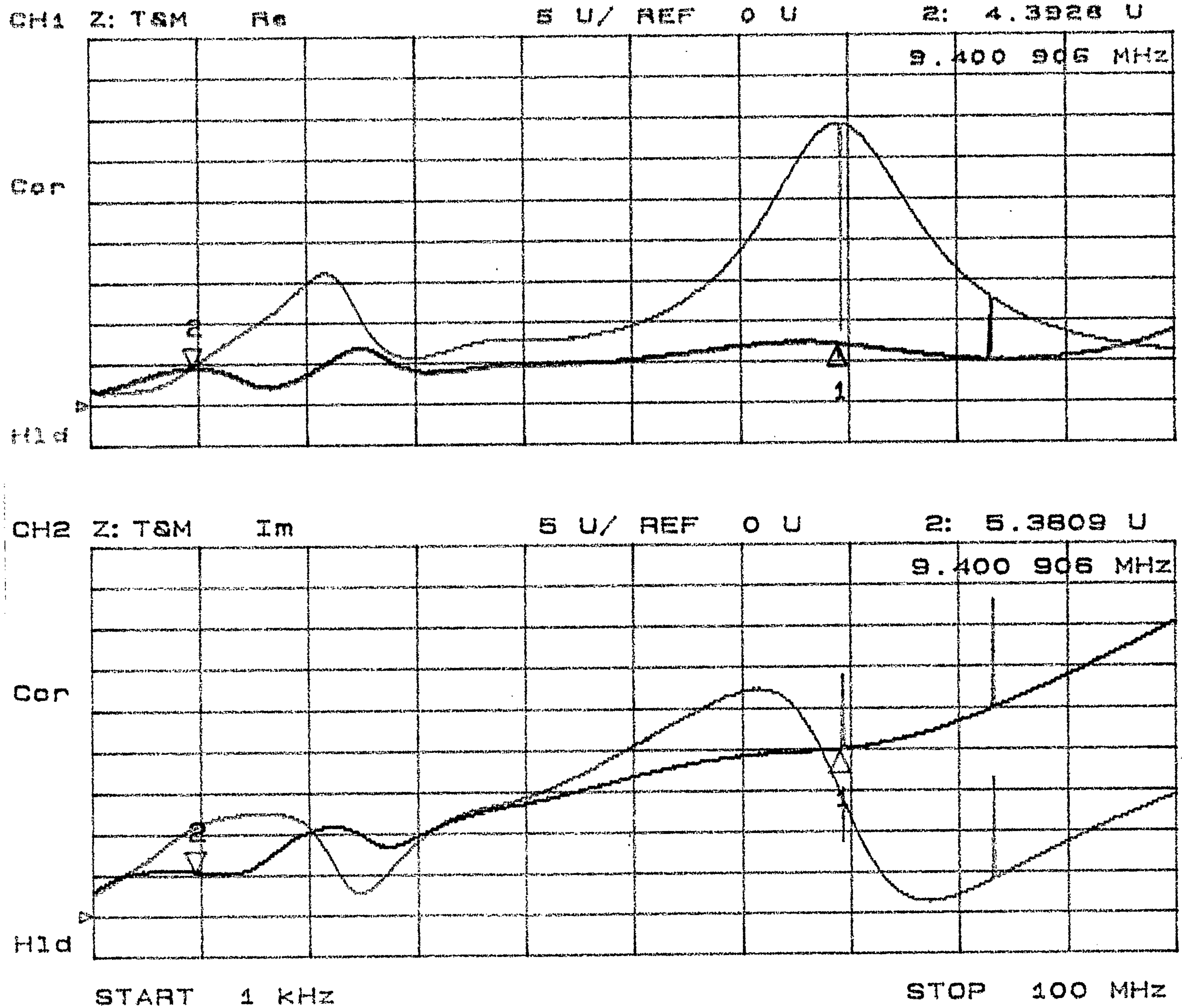

Fig. 2. Kicker coupling impedance with one output port terminated and the other shorted (dark line) or open (light line).

IV. EQUIVALENT CIRCUIT ANALYSIS

The analytical treatment of the kicker is instructive but limited and can be complemented with the use of equivalent circuits, which opens the problem to powerful computer programs such as P-Spice. Obviously, the accuracy of the results depends on the quality of the assumptions and a judicious choice of the circuit is essential. A simple example of a kicker equivalent circuit was developed by Voelker and Lambertson,[7] but a more general circuit is necessary to correctly represent a transmission line kicker.

In a first attempt at developing an equivalent circuit for the RHIC injection kicker, its ferrite/dielectric cell structure was represented by a cascade of uniform transmission lines with different characteristic impedance and propagation velocity.[5] The configuration of the final kicker design was changed and now has localized dielectric blocks only between busbar and frame whereas the sides are formed with continuous ferrite blocks.[8] The cascaded transmission line approach is thus no longer appropriate and a better model seems to be a 6-section low pass filter with lumped elements, shown in Fig. 3. In this figure the coupling to the beam is also indicated, so that the coupling impedance can be computed by the P-Spice program.

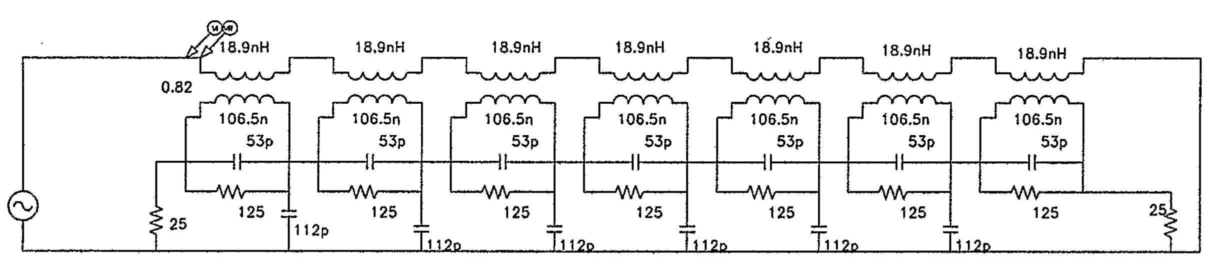

Fig. 3. Equivalent circuit representation of the kicker coupling impedance.

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[7]F. Voelker and G. Lambertson, Proc. 1989 IEEE Conf. Particle Accelerators, Chicago, vol.2, p.851 (1989).
[8]H. Hahn, N. Tsoupas, and J. E. Tuozzolo, Report AD/RHIC/RD-105 (1997).

The circuit parameters are determined by direct measurements on the kicker. The series inductors are obtained by shorting the output port and a measurement of the input impedance, shown in Fig. 4. The total inductance at 1 MHz is 745.6 nH, leading to the value of 106.5 nH for each of the 7 inductors. The shunt capacity was measured with the output port open to be ~790 pF, or 132 pF for each of the 6 capacitors. This value contains a large contribution from the ends and it was adjusted to 112 pF to render the quarter-wavelength resonance at 9.7 MHz. At frequencies up to this resonance, the uniform transmission line assumption is valid, and thus $Z_{in} = Z_K \tan(f/f_{\pi/2})$ yields a kicker impedance of 28.6 Ω and the propagation velocity of 0.06 c. At frequencies above this resonance, the ferrite losses become significant and strongly damp the residual structure resonances due to the mismatched 25 Ω termination, seen in Fig. 5. The relatively pronounced resonance at ~65 MHz is independent of the kicker length and is only correlated to the length of the individual ferrite blocks in the sides. This "ferrite" resonance is adequately described by the 125 Ω series resistor and the 53 pF capacitor in parallel with the series inductor.

Having established the equivalent circuit of the kicker alone, one can add the beam as a series of magnetically coupled inductors, the values of which are determined from the wire measurements in Fig. 1 and 2 as $L_B/7$ = 18.9 nH and κ = 0.82 as discussed above. The coupling impedance computed by the P-Spice program at low frequencies, i.e. below ~100 MHz, is shown in Figs. 6 through 9 where the real and imaginary part of the impedance is directly compared with the results from the "wire" measurements, taken from Fig. 1 and 2. The agreement is reasonable considering the possible errors in measurement and the limitations of the model. It would seem obvious that the equivalent circuit analysis is superior to any analytical treatment. Once established, it provides the tool to investigate the effects caused by changes or errors, such as mismatched terminations due to the added half-length end ferrites or strongly frequency-dependent terminations due to the pulser with its ~75 m long connecting cables.

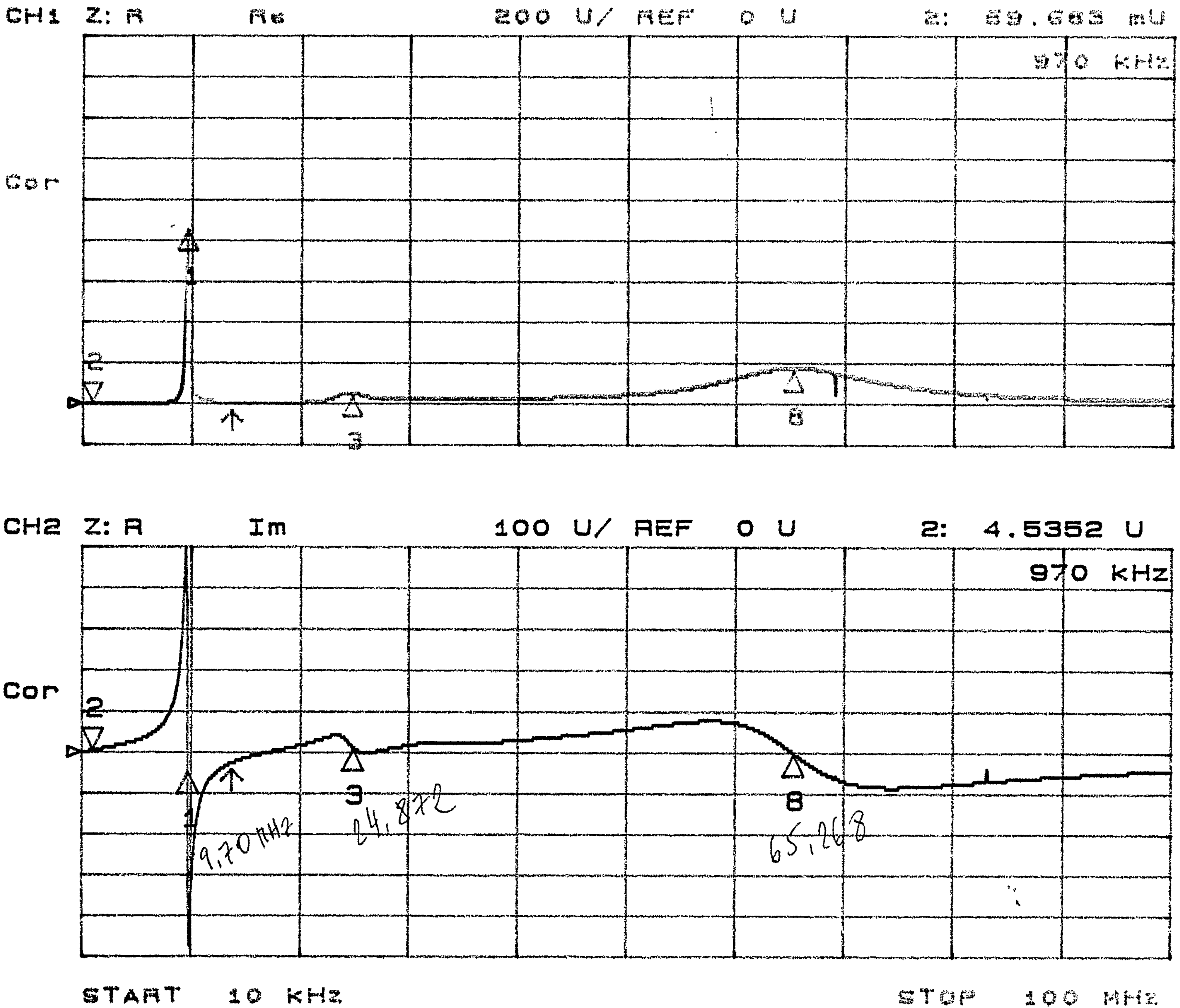


Fig. 4. Input impedance of shorted kicker.

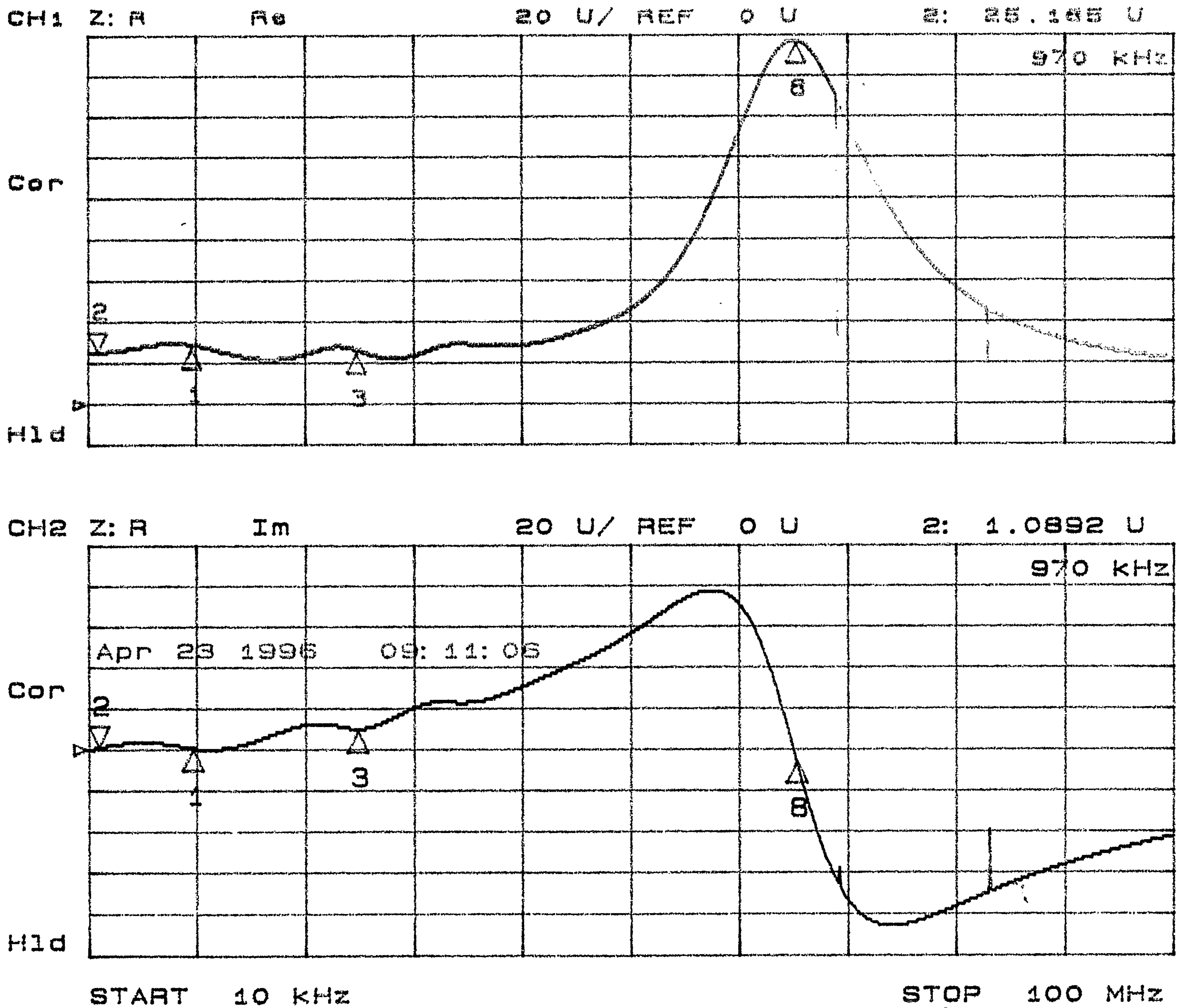


Fig. 5. Input impedance of kicker terminated with 25 Ω.

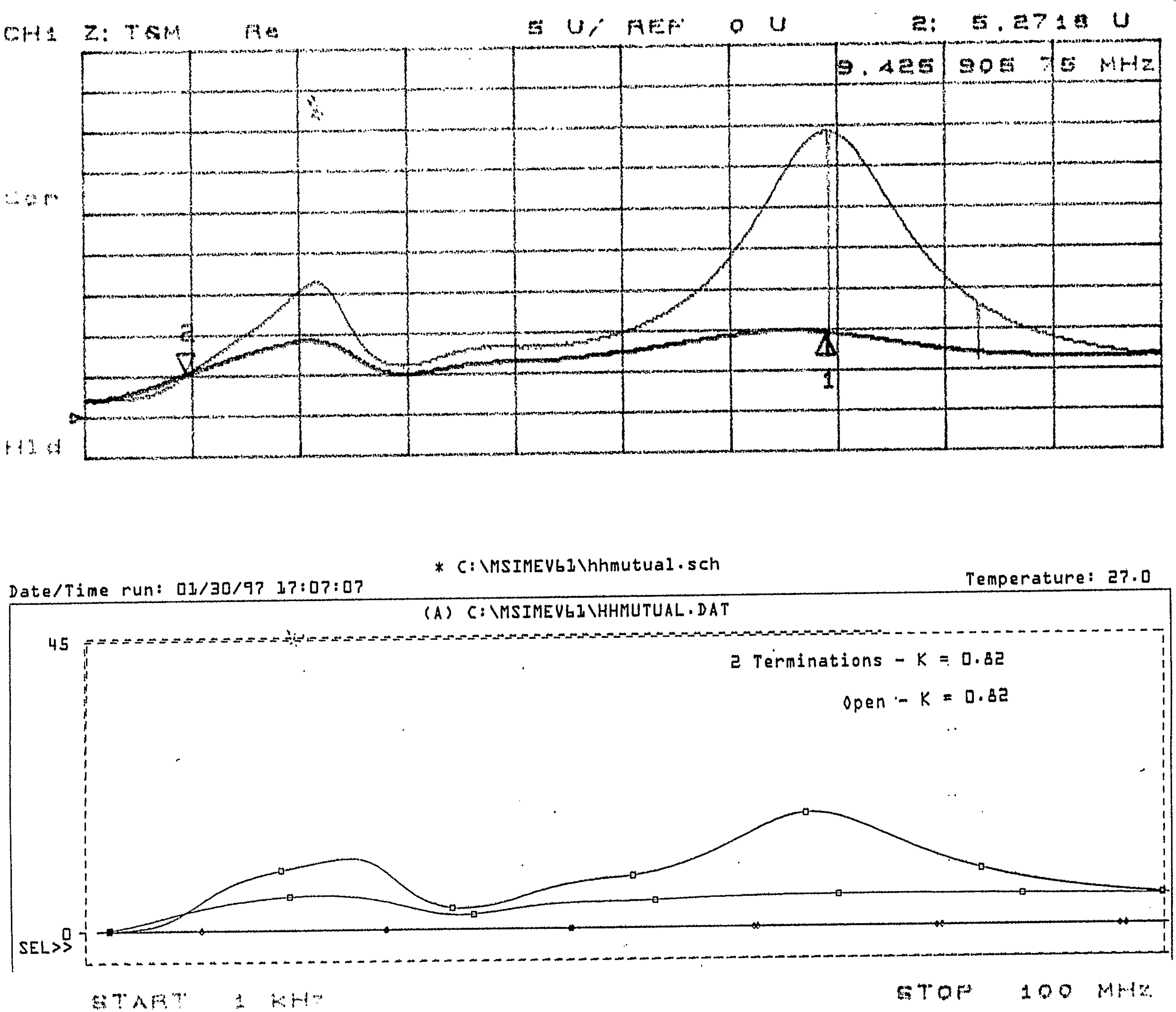


Fig. 6. Comparison of measured and Spice-calculated real part of kicker coupling impedance for one port terminated and the other port open or terminated.

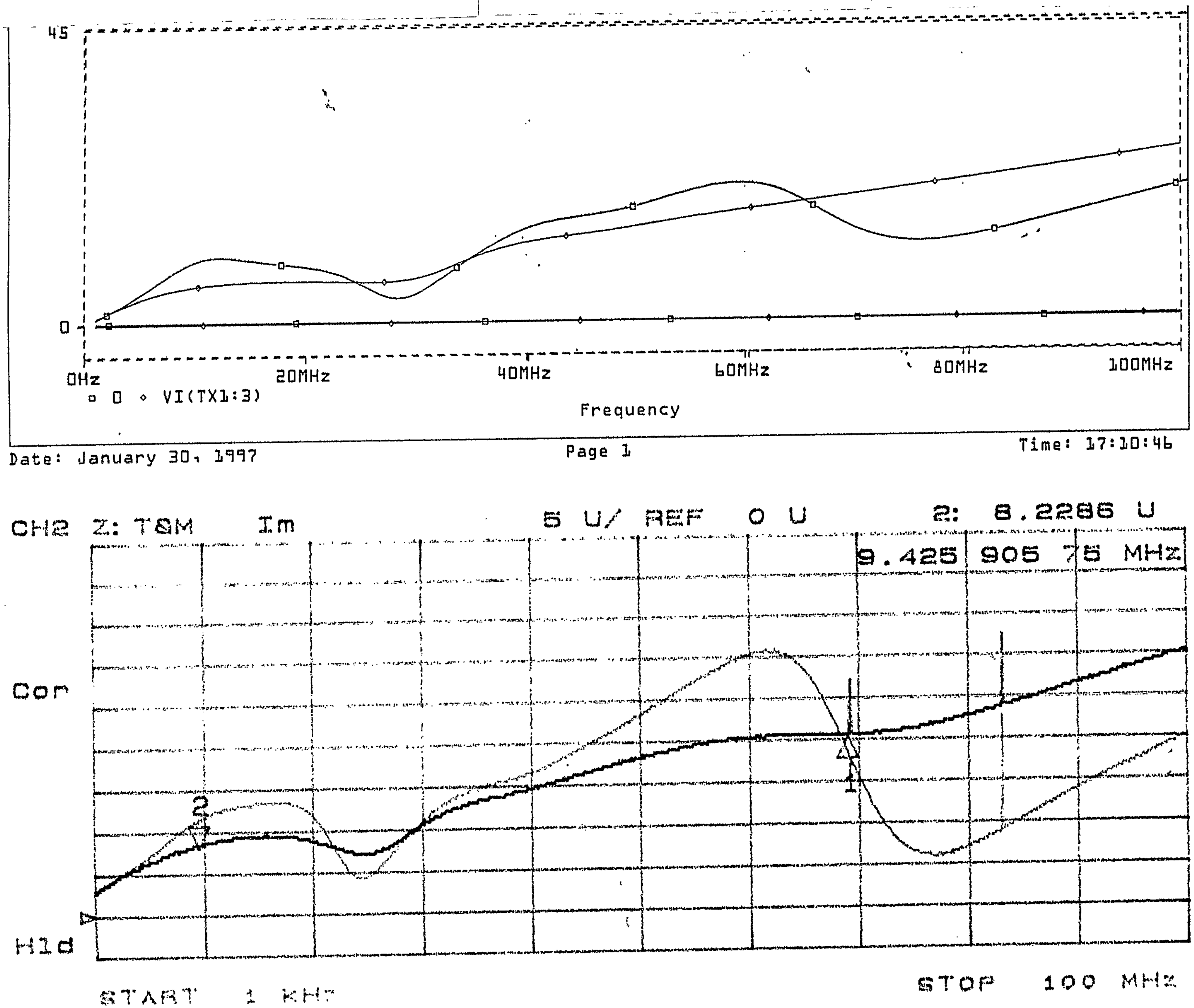
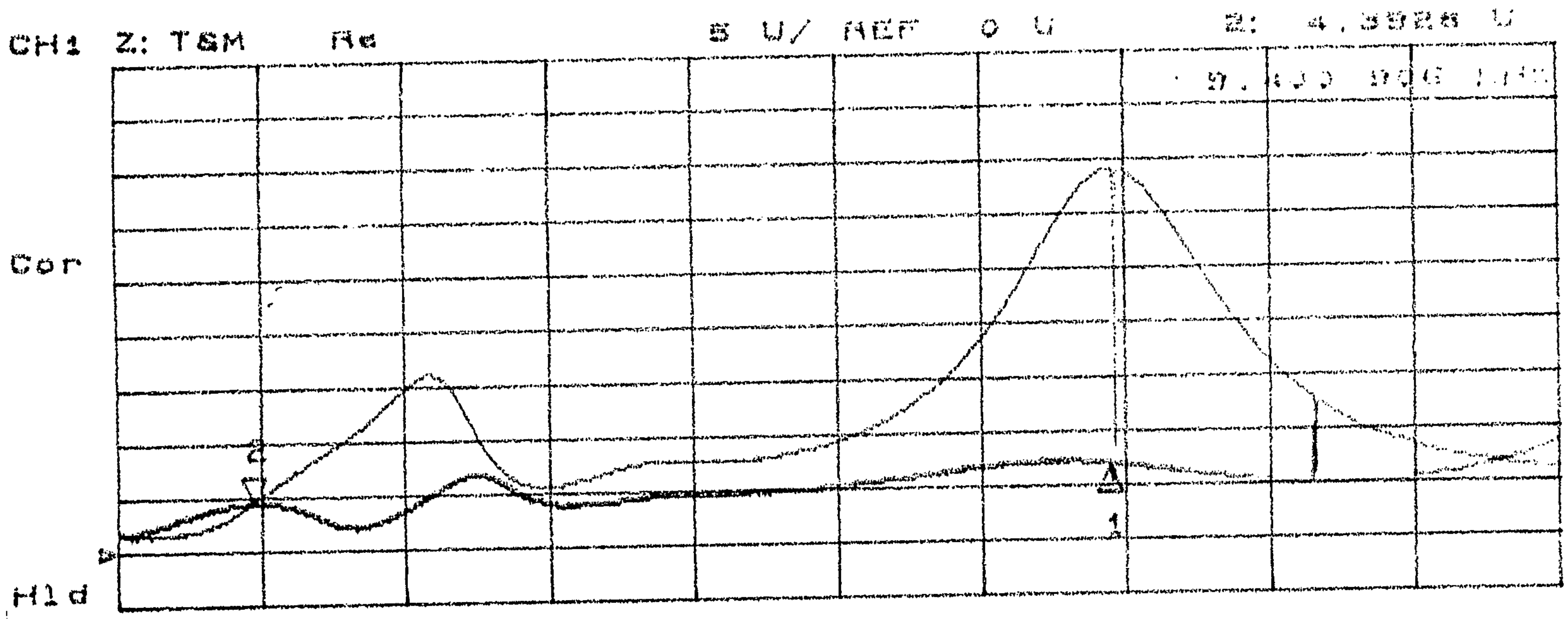


Fig. 7. Comparison of measured and Spice-calculated imaginary part of kicker coupling impedance for one port terminated and the other port open or terminated.



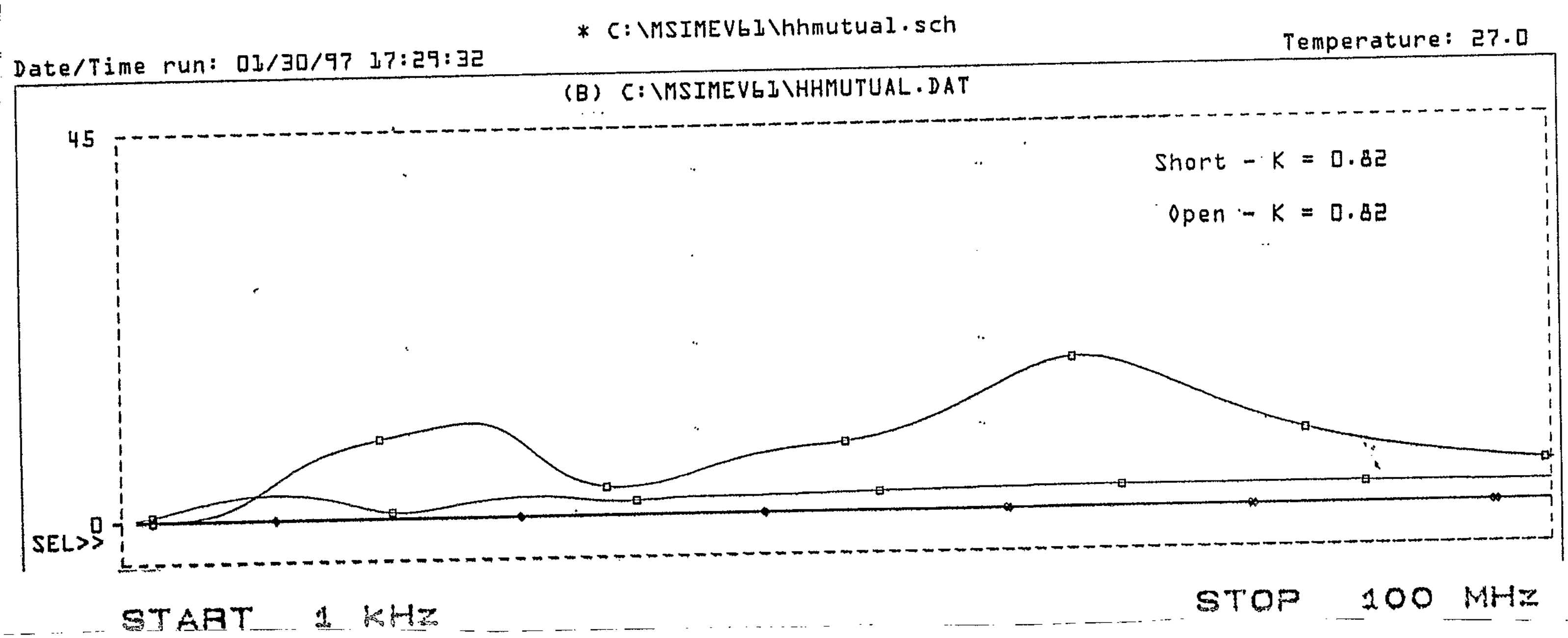


Fig. 8. Comparison of measured and Spice-calculated real part of kicker coupling impedance for one port terminated and the other open or shorted.

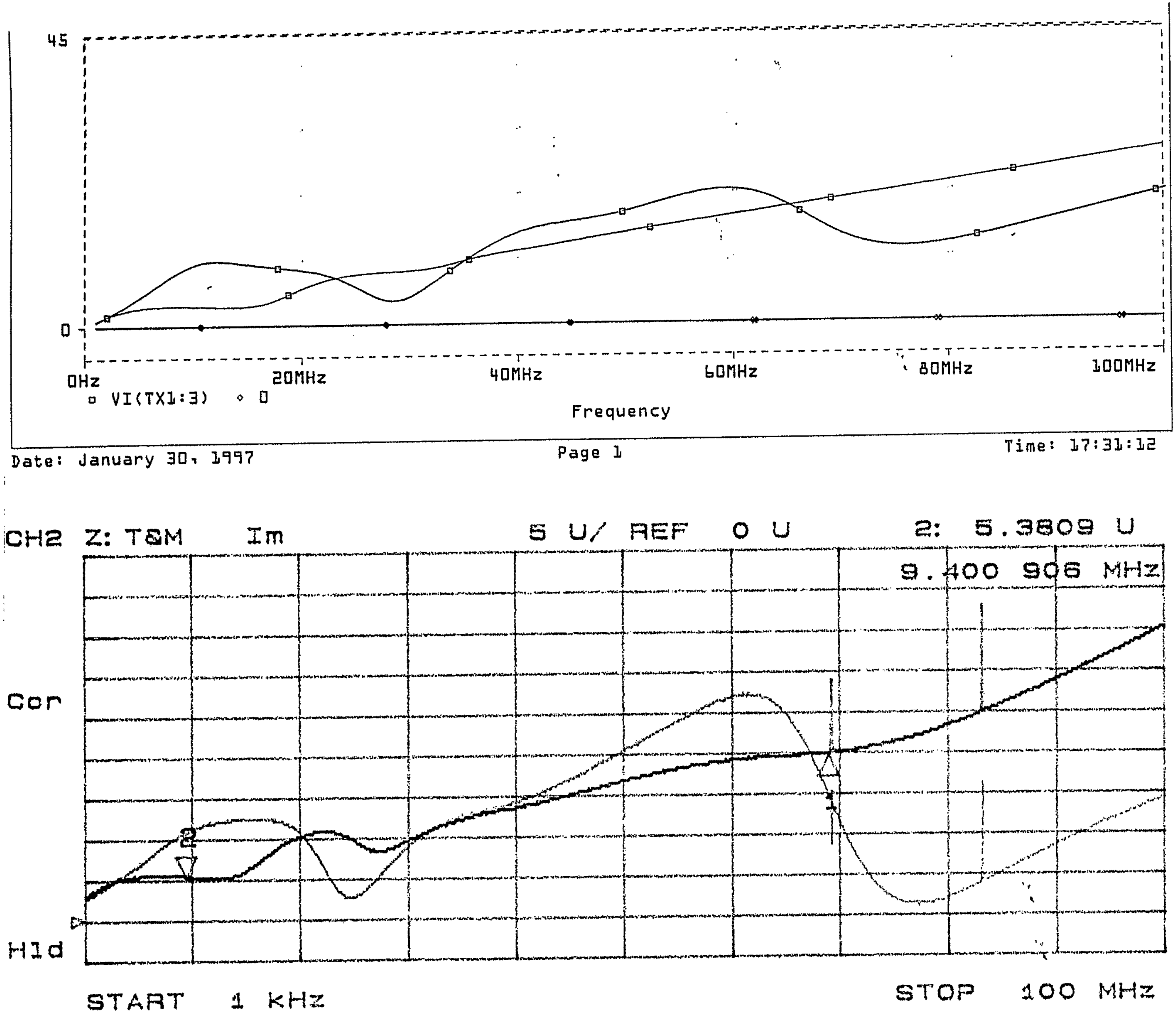


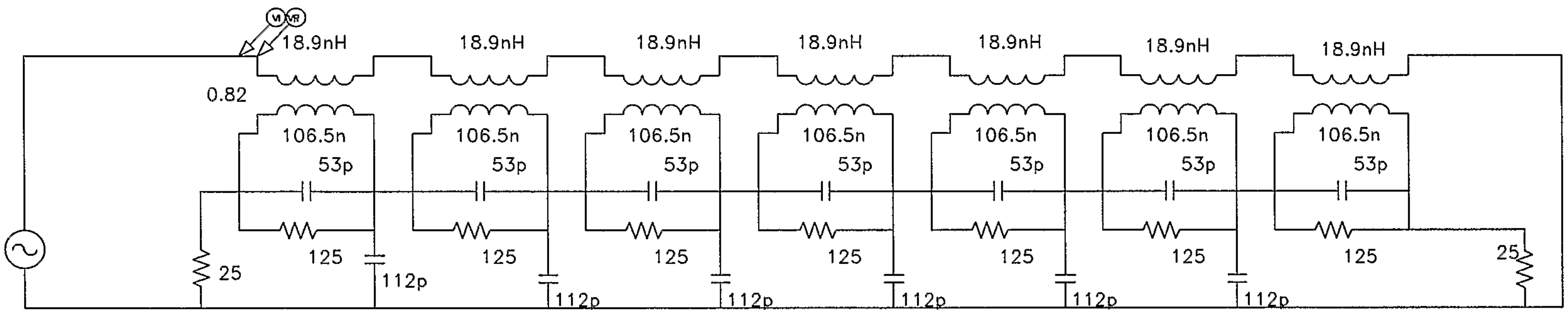
Fig. 9. Comparison of measured and Spice calculated imaginary part of kicker coupling impedance for one port terminated and the other open or shorted.

APPENDIX

The general expression for the coupling impedance is best handled via the computer program MACSYMA. The case of one port terminated, $R_o = Z_K$, and the input port represented by the general impedance $R_i = R + jX$ is of practical importance, as it reflects the typical situation of the kicker system. Here one finds

$$
\begin{aligned}
Z = {} & \frac{1}{2} Z_K \left(\frac{M}{L_K} \right)^2 \Big\{ Z_K^2 (1 - \cos 2\theta) + 4 Z_K R (1 - \cos\theta) \\
& + (R^2 + X^2)(3 - 4\cos\theta + \cos 2\theta) - 2 Z_K X (2\sin\theta - \sin 2\theta) \Big\} / D \\
& + j Z_K \frac{L_B}{L_K} \theta \\
& - j \frac{1}{2} Z_K \left(\frac{M}{L_K} \right)^2 \Big\{ Z_K^2 (2\theta - \sin 2\theta) + 4 Z_K R (\theta - \sin\theta) \\
& + (R^2 + X^2)(2\theta - 4\sin\theta + \sin 2\theta) - 2 Z_K X (1 - 2\cos\theta + \cos 2\theta) \Big\} / D
\end{aligned}
$$

with the denominator $D = (Z_K + R)^2 + X^2$


18.9nH 18.9nH 18.9nH 18.9nH 18.9nH 18.9nH 18.9nH
0.82
106.5n 106.5n 106.5n 106.5n 106.5n 106.5n 106.5n
53p 53p 53p 53p 53p 53p 53p
125 125 125 125 125 125 125
25 25
112p 112p 112p 112p 112p 112p 112p

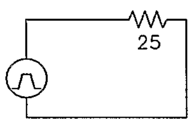

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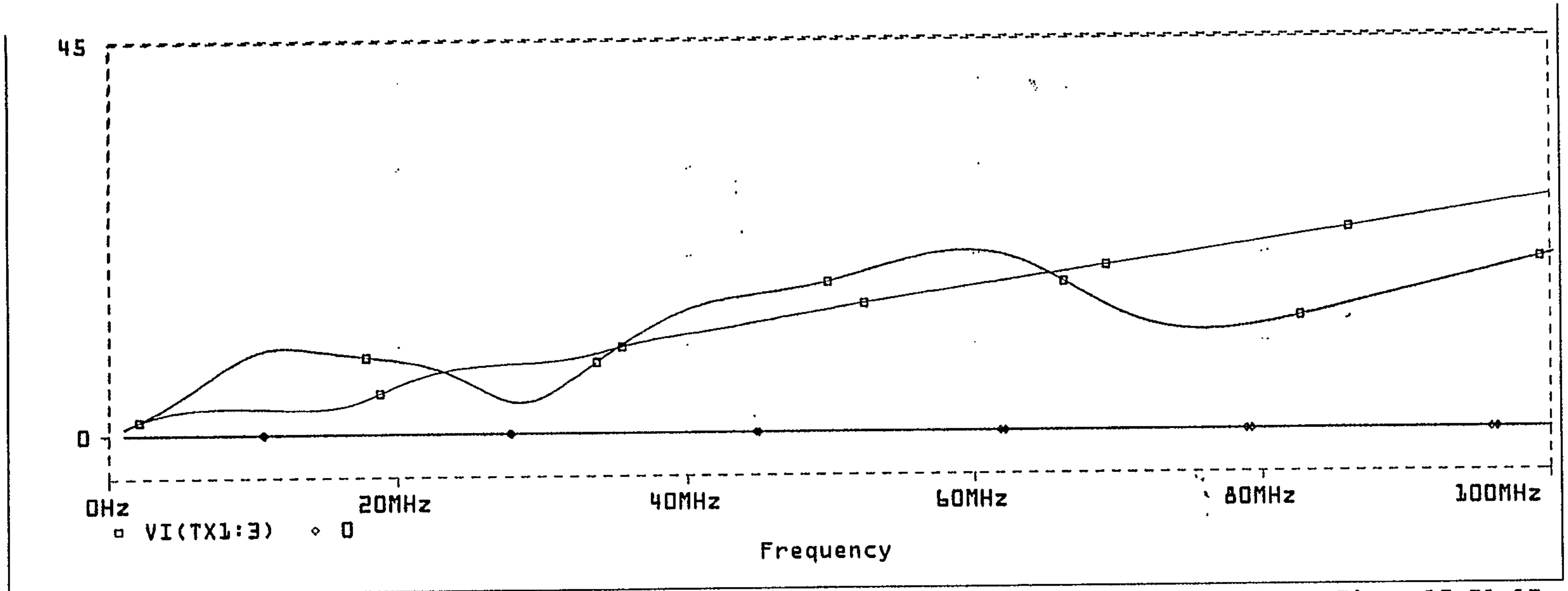
Fig 3



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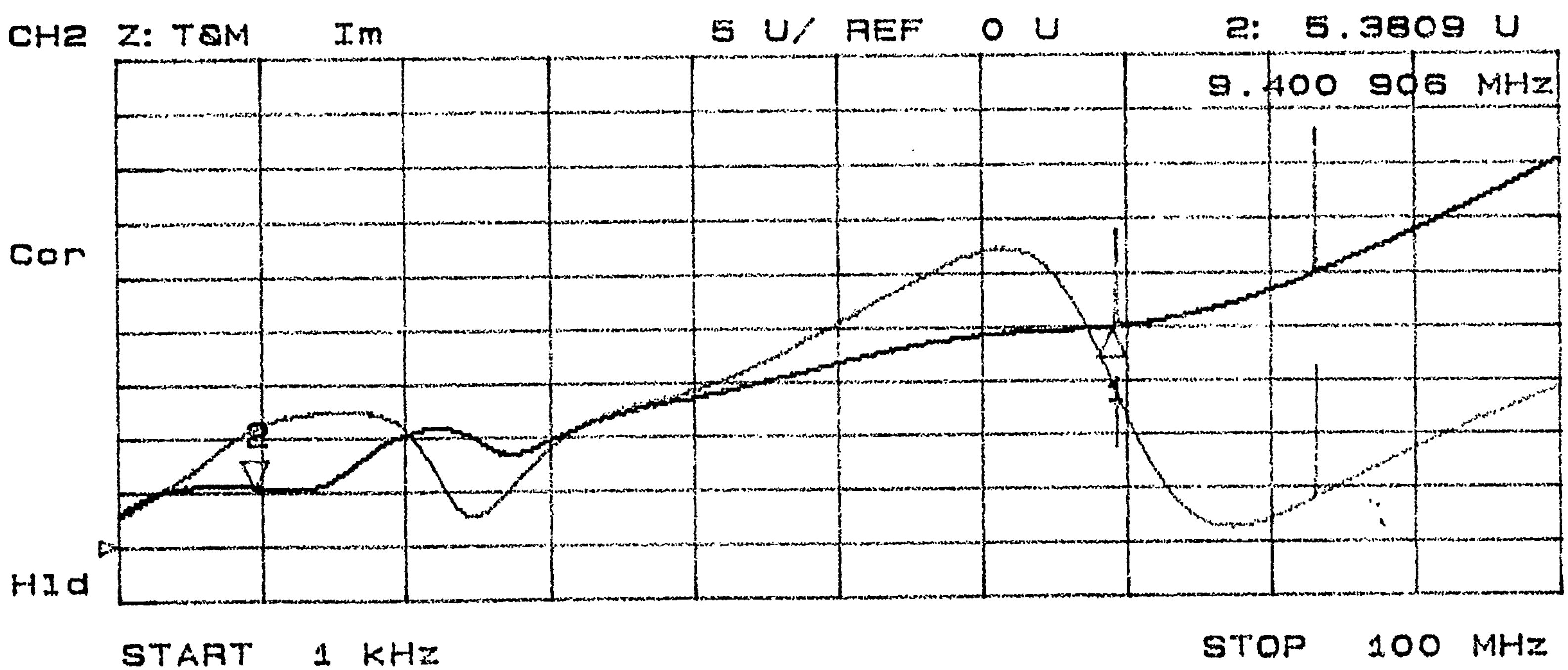
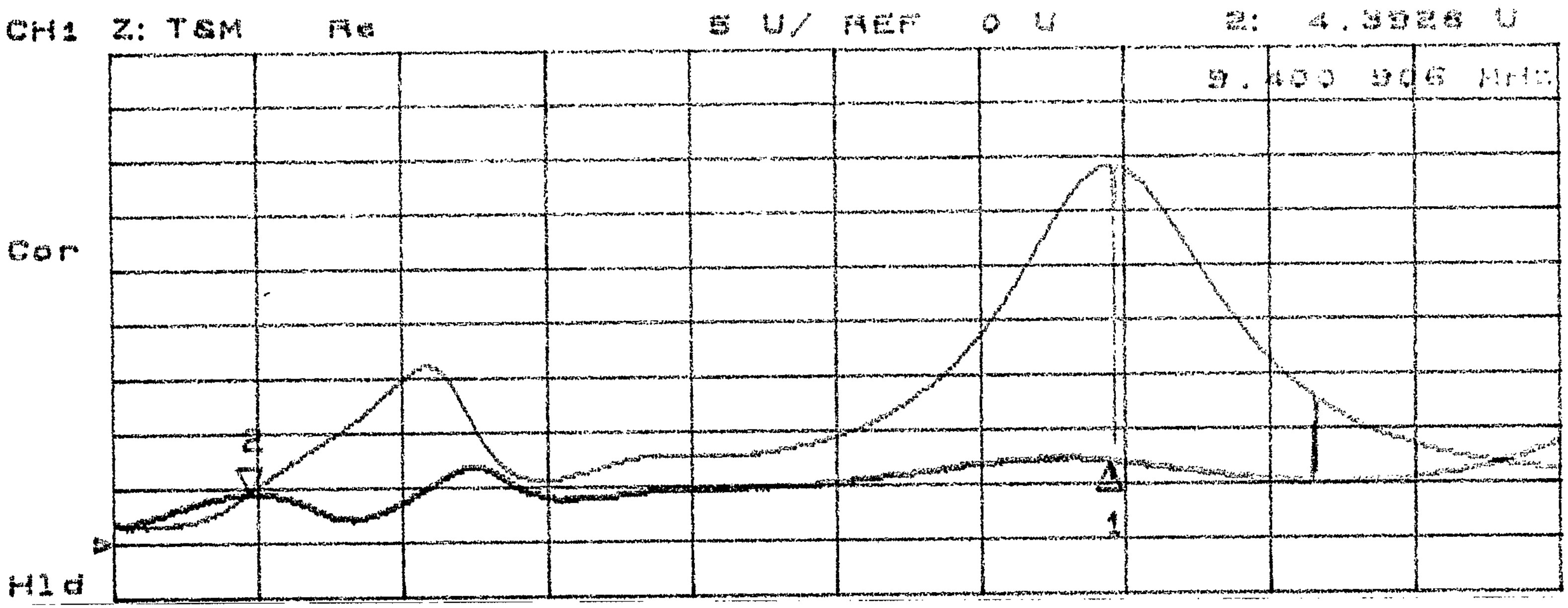
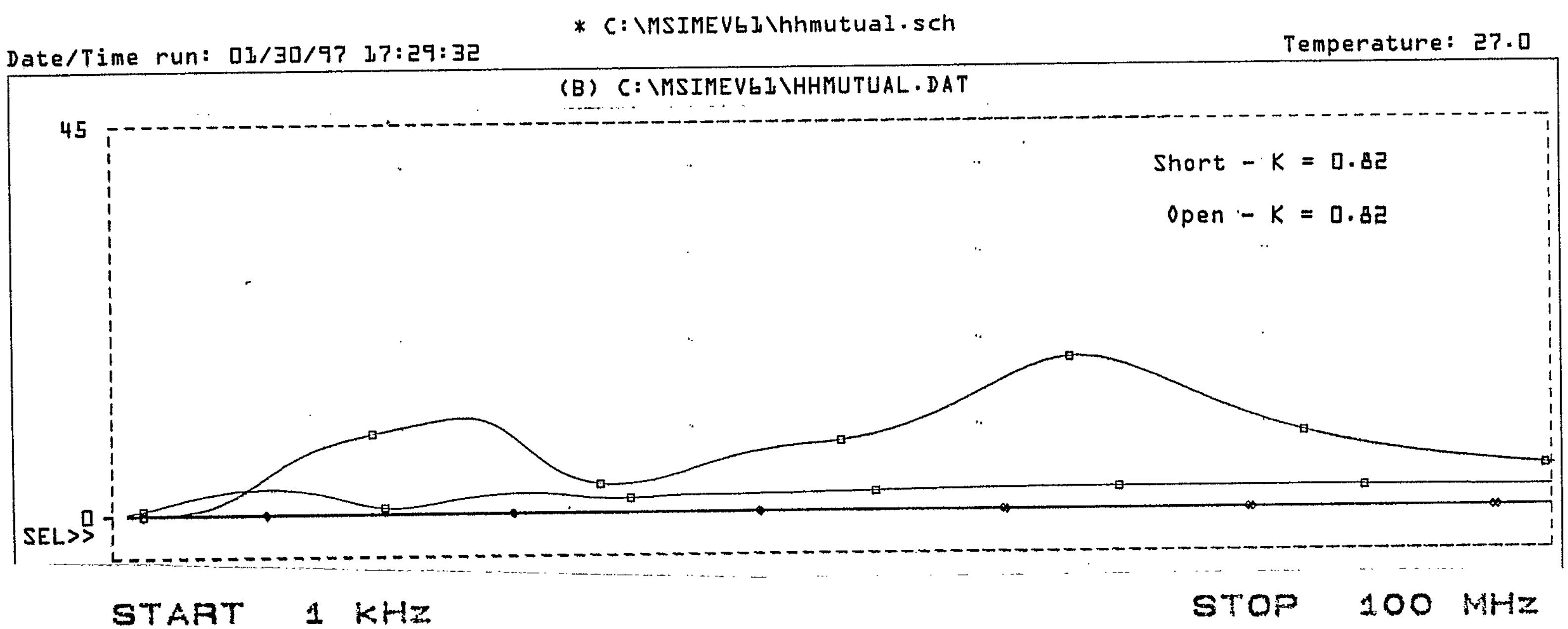


3 I. 96
OPEN + SHORT
H - 1.85

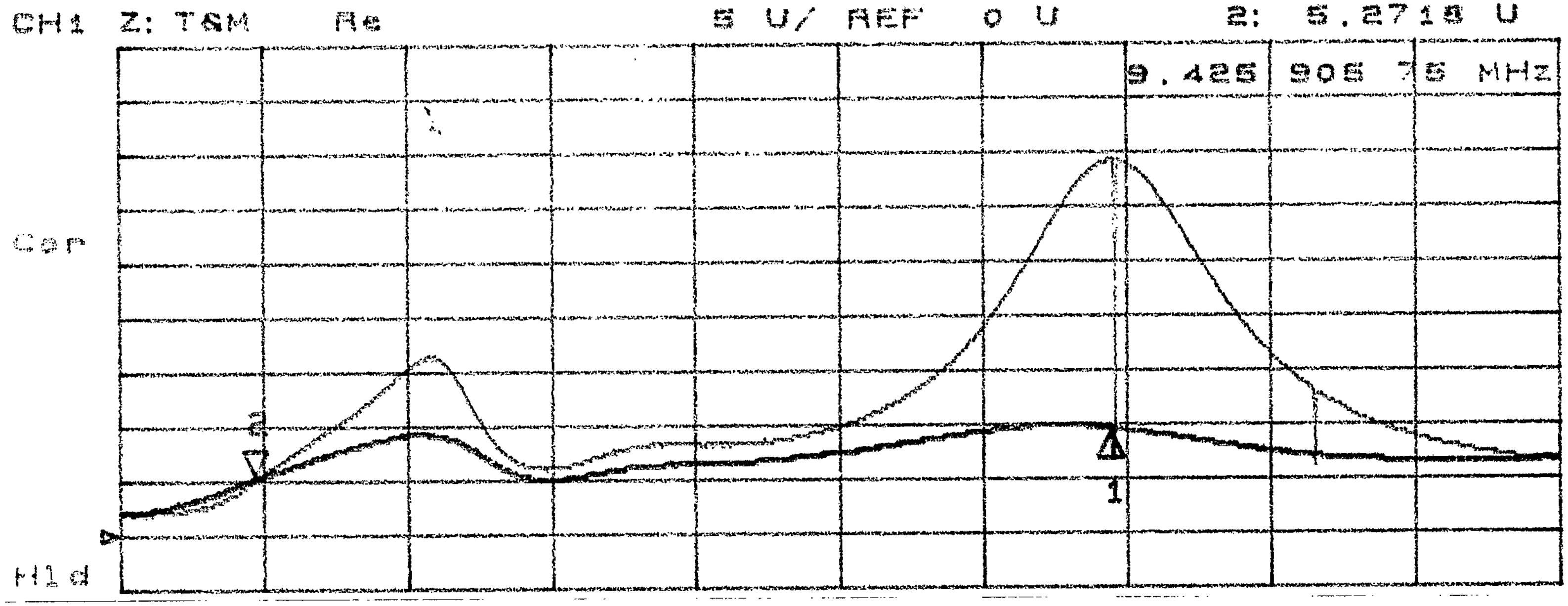


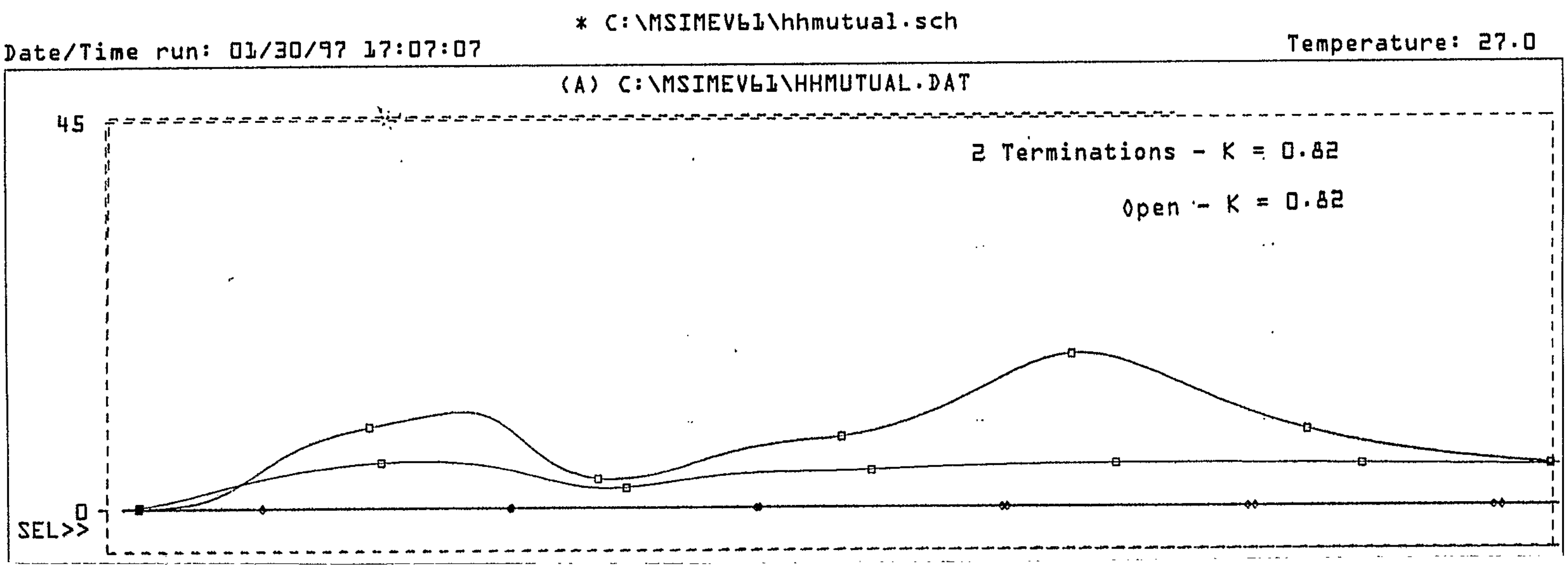


both terminals open + ~~short~~ ~~cold~~

H-1.85

31.V.96





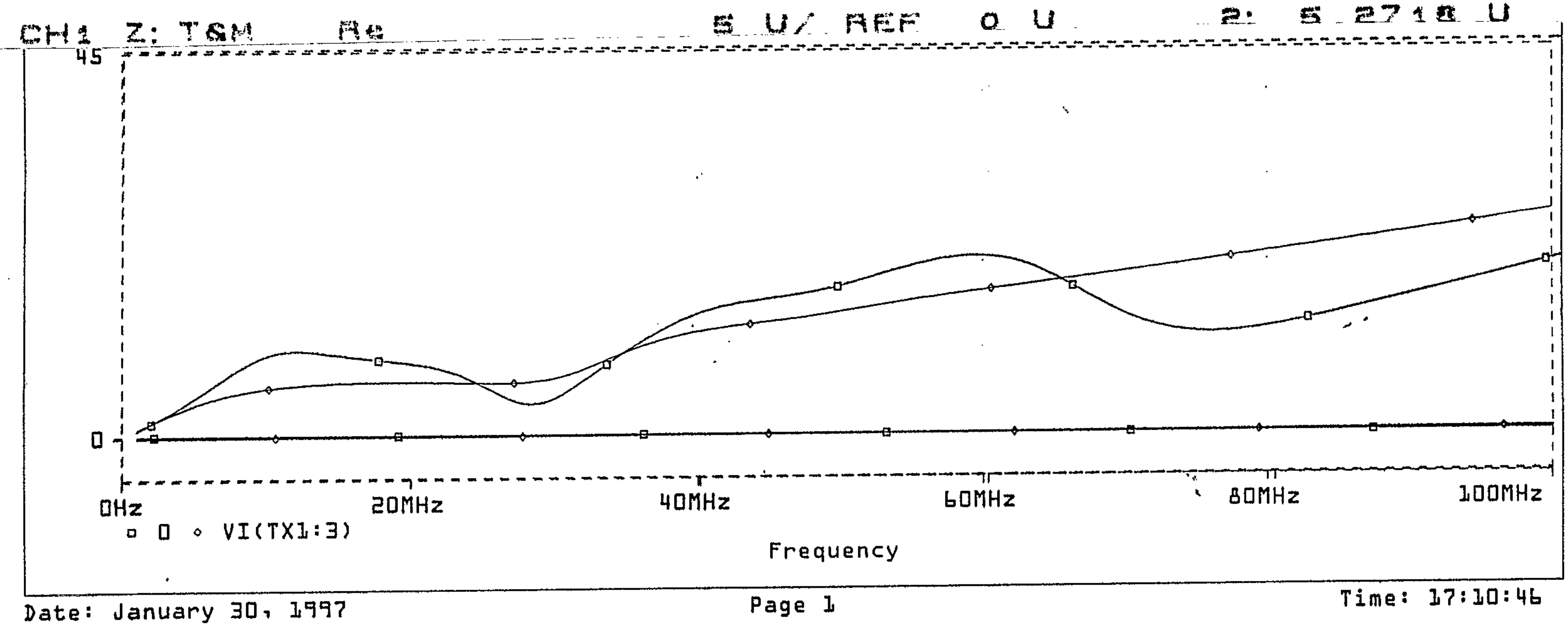
START 1 KHz STOP 100 MHz

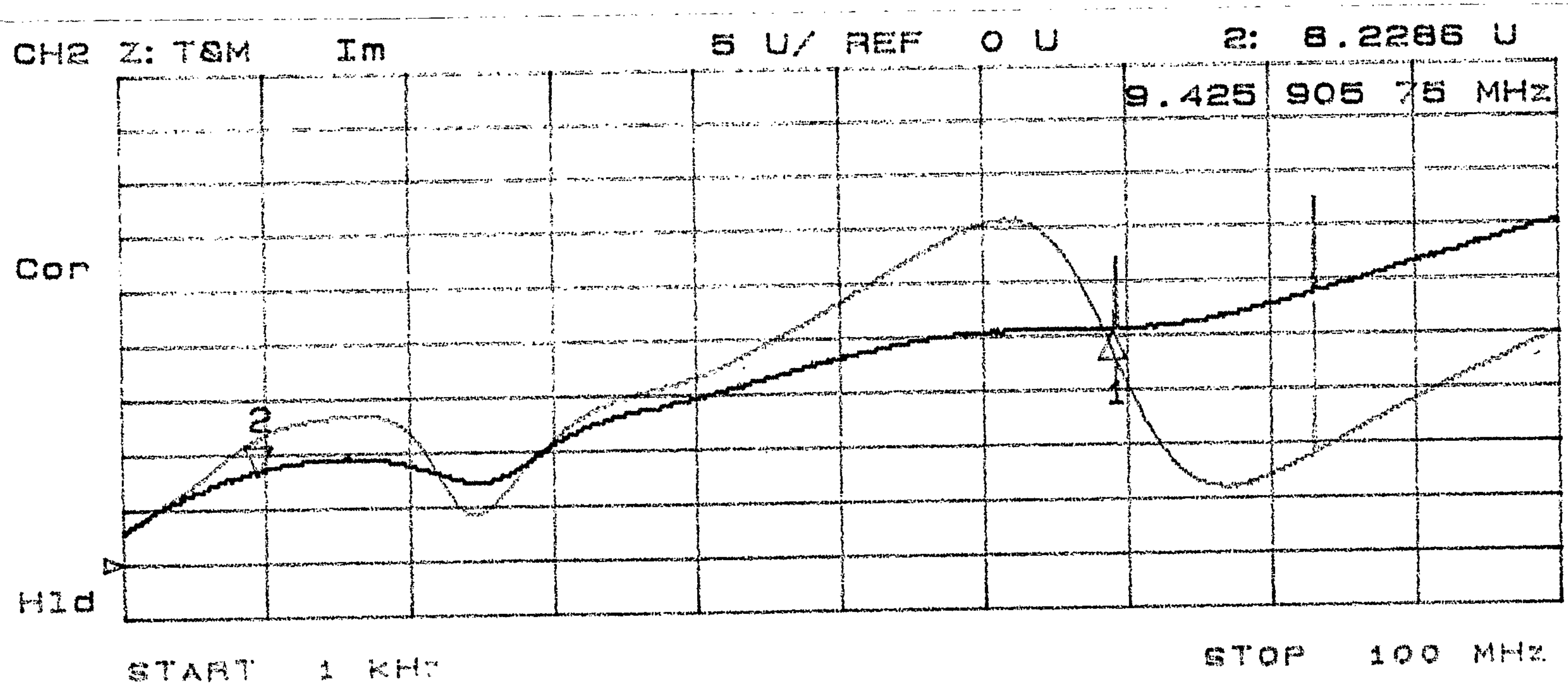
31 . 96

both terminals to open + ~~short~~

H-185

31. V. 96





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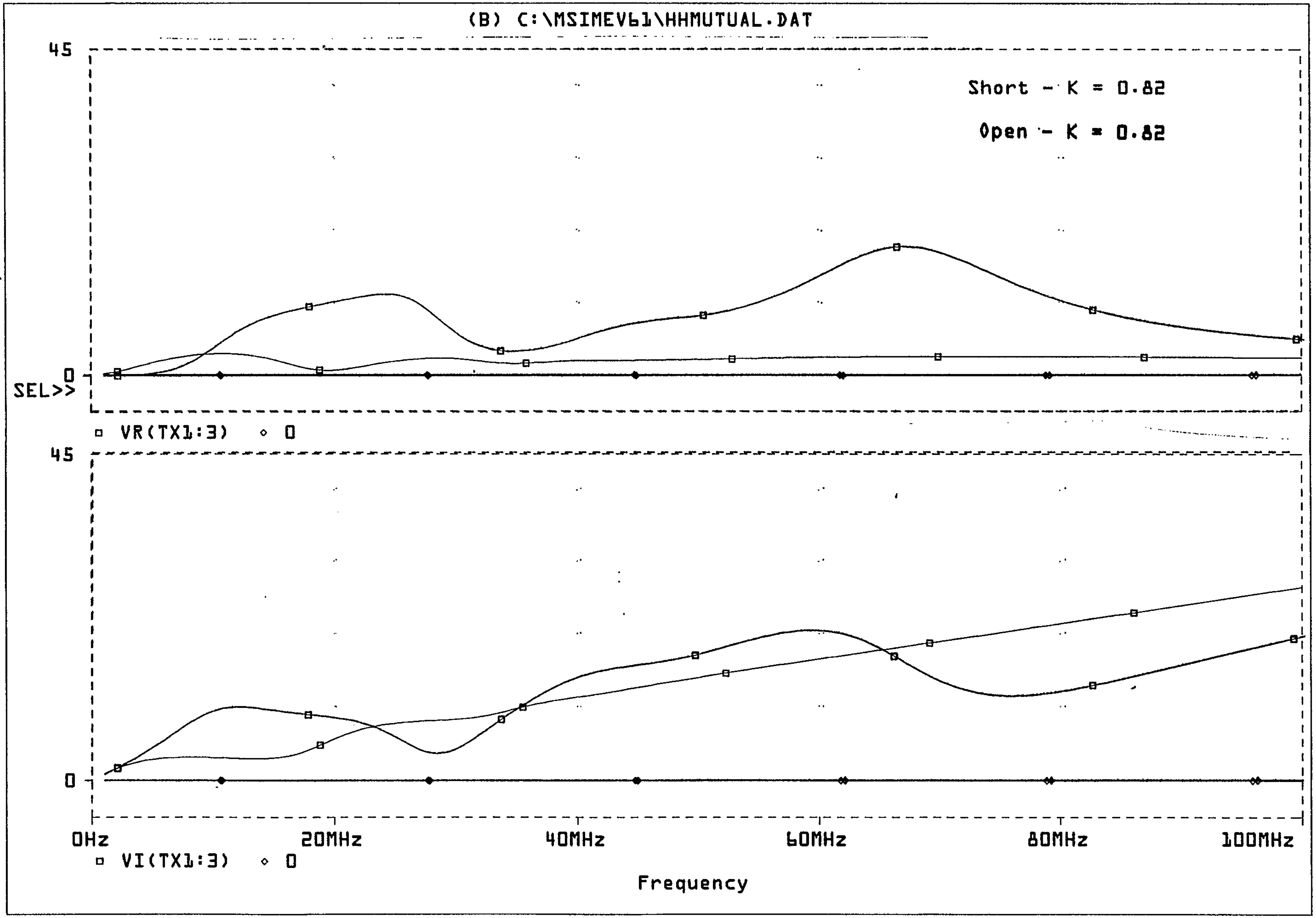

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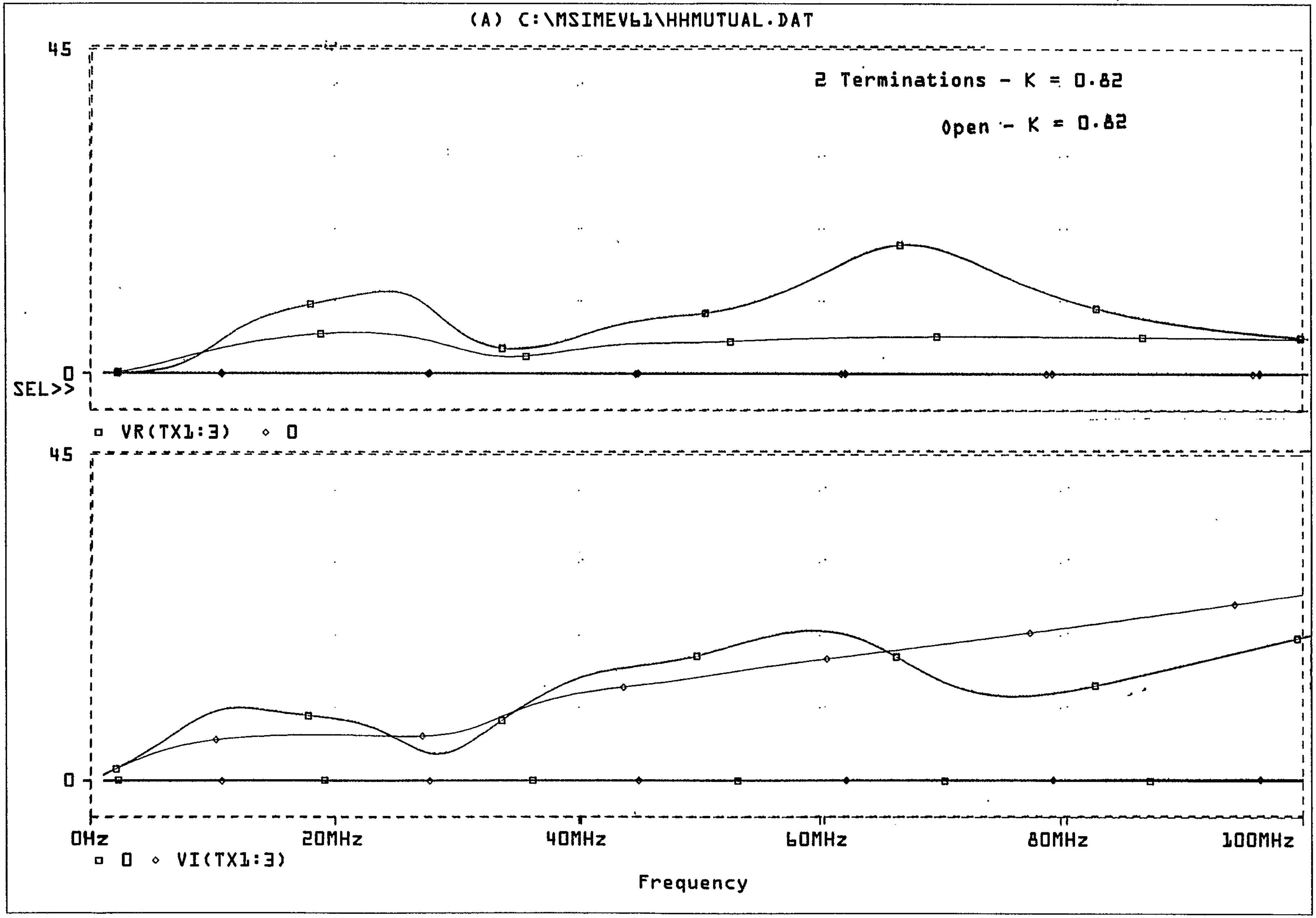
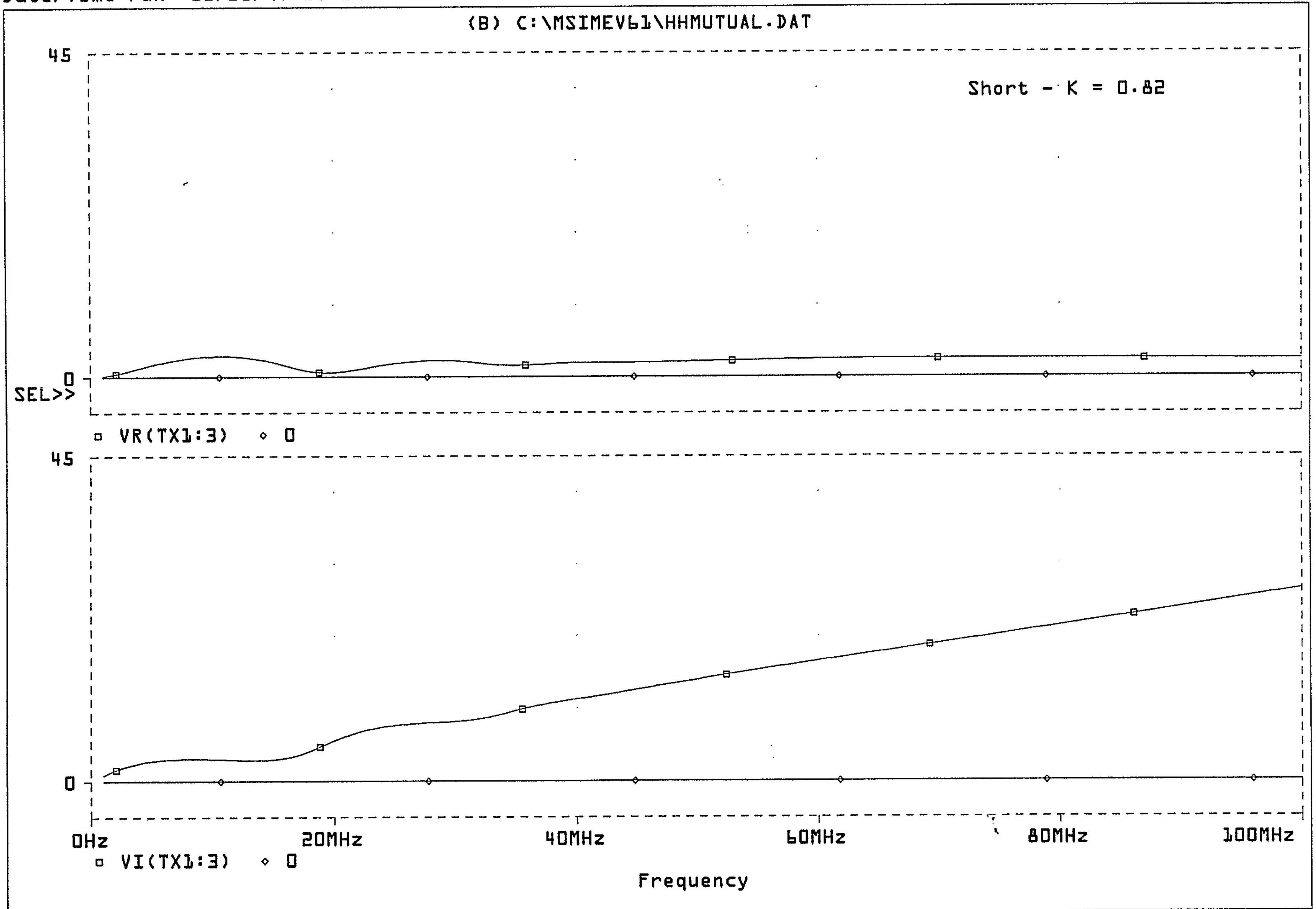
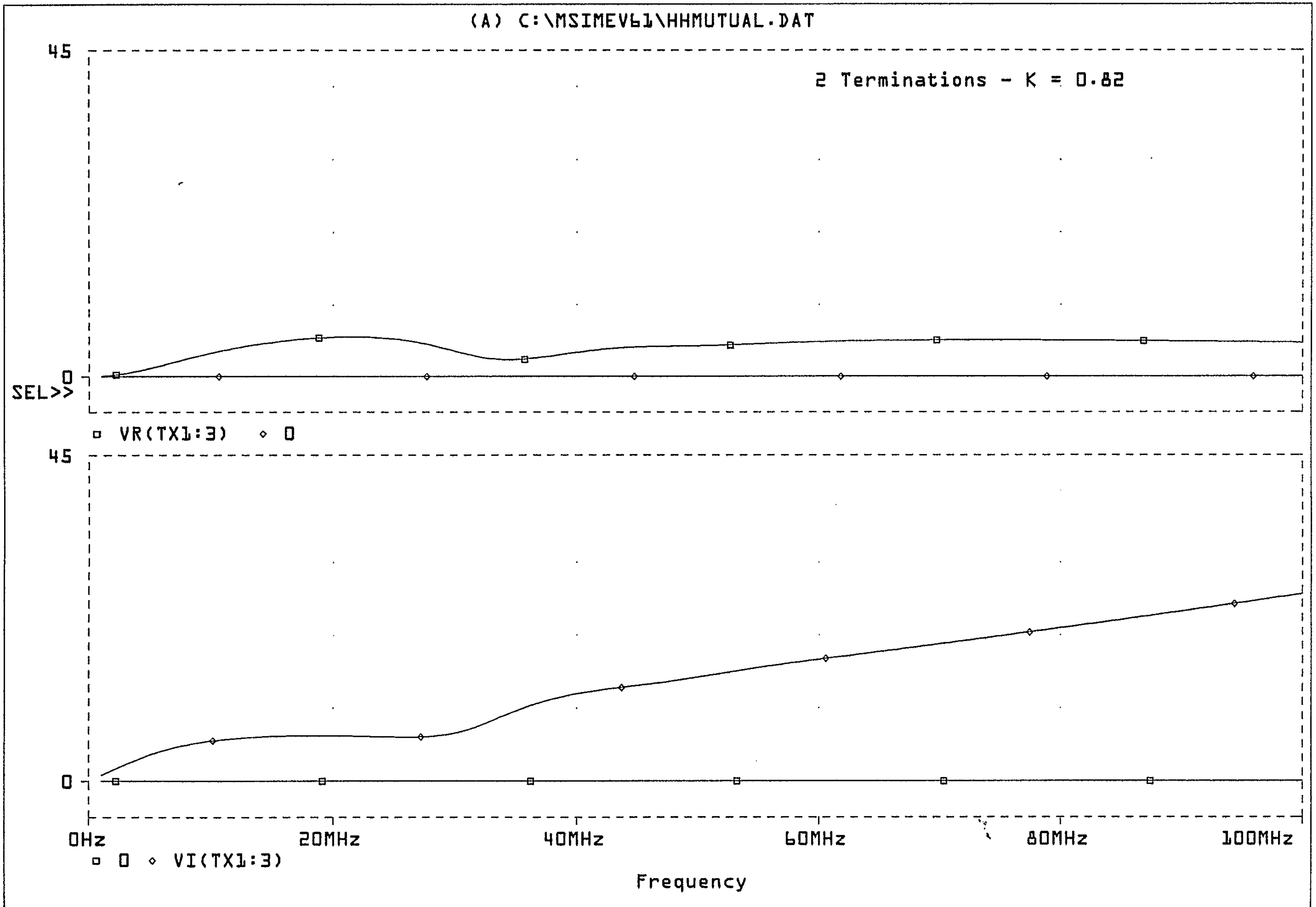



(B) C:\MSIMEV61\HHMUTUAL.DAT
Short - K = 0.82
45
0
SEL>>
□ VR(TX1:3) ◇ 0
45
0
0Hz
20MHz
40MHz
60MHz
80MHz
100MHz
□ VI(TX1:3) ◇ 0
Frequency

* C:\MSIMEV61\hhmutual.sch

Date/Time run: 01/30/97 17:07:07 Temperature: 27.0



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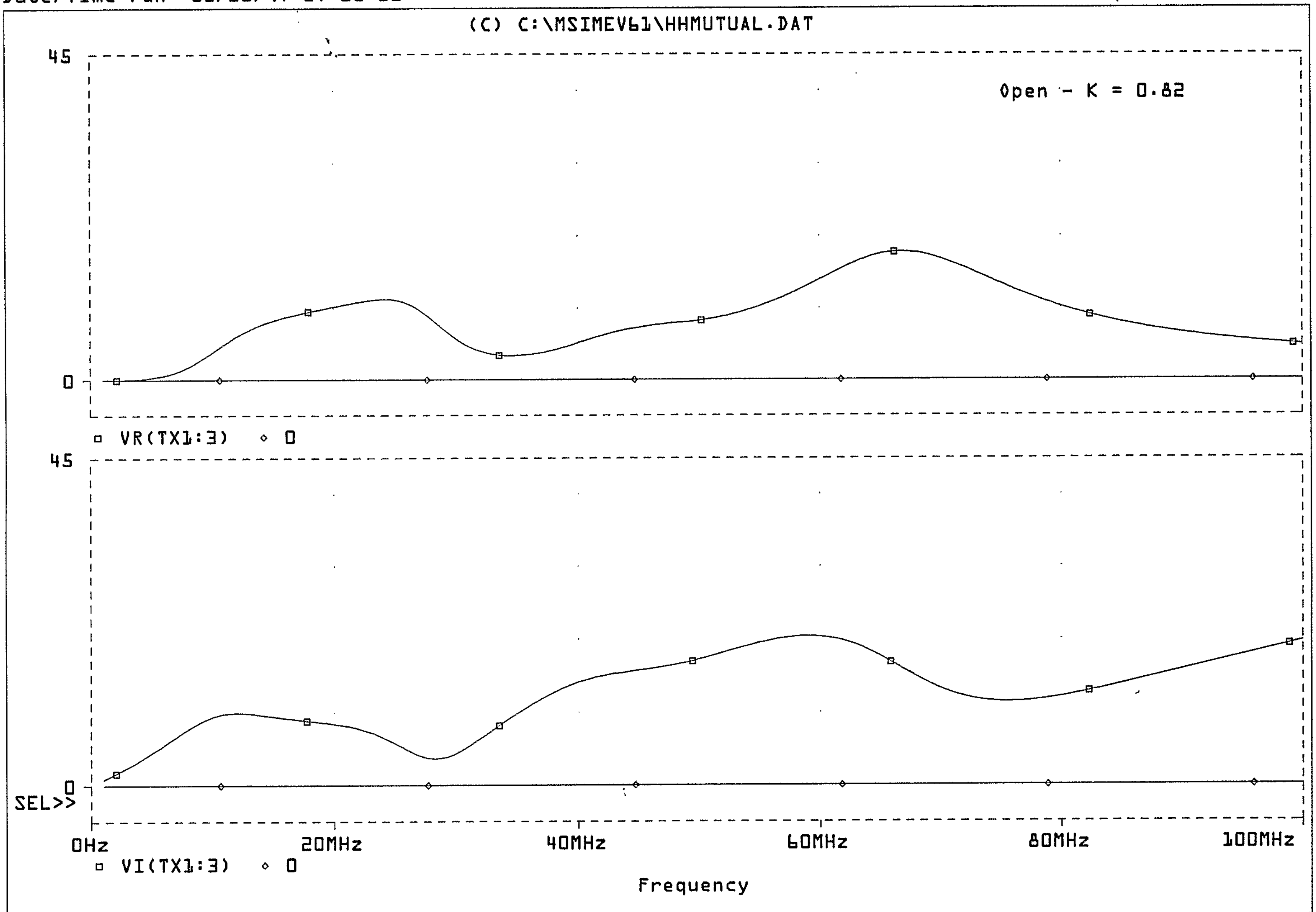
* C:\MSIMEV61\hhmutual.sch

Date/Time run: 01/30/97 17:31:36 Temperature: 27.0

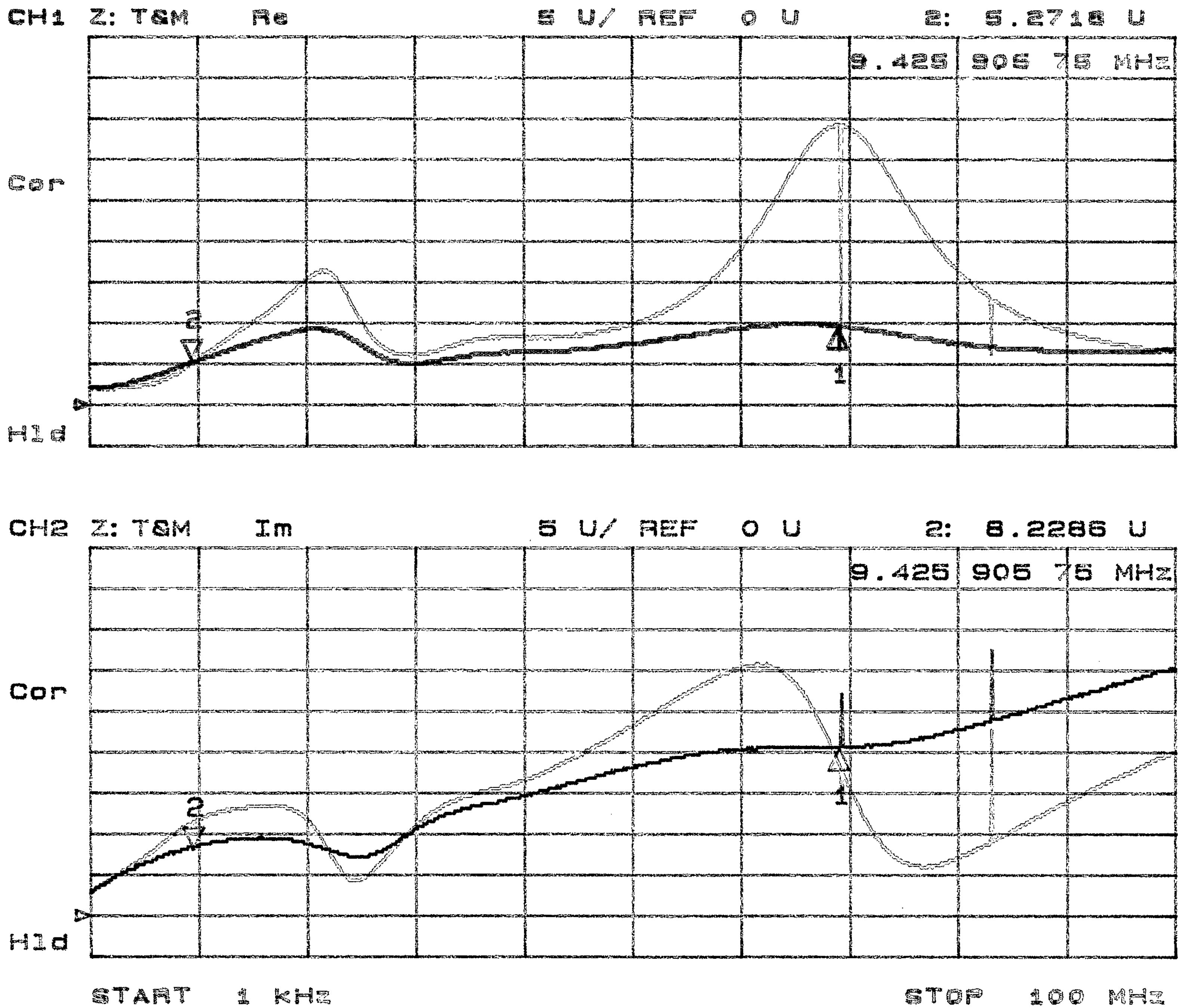


Date: January 30, 1997 Page 1 Time: 17:32:42

31 ●. 96
both terminals to open + ~~short~~
" ~~cold~~ "

H-185

31. V. 96



3[illegible]I. 96
OPEN + SHORT
H-1.85