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A Study Of Luminosity Parameters

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A STUDY OF LUMINOSITY PARAMETERS

L. E. Roberts

(BNL, February 2, 1984)

- I Purpose of This Study.
- II Definitions Of Quantities Associated With the Luminosity.
- III Varions Limits Of Luminosity formulae.
- IV The Overlop Integral.
- V Some Results Are Listed.

I Purpose Of This Study

Essentially the purpose of this study is twofold: a) We want to see how the various farameters that are essential to the luminosity, (h.e. that the luminosity has a strong dependence on), can be varied so as to give one fixed value of the luminosity; (b) We want to "see" how the luminosity

Ton reasonably he expected to behave with respect to distance from the interaction region.

Introbeam scattering is also a very important footor that must be (and is by seneral other people) token into account. The papers by Pivinshe, Broken into Mingwa are in depth theretical studies of her phenomena. However in the paper of Broken and Itingwa exchange terms have not been included in the Coulomb scattering amplitude. The exchange terms noy well make a significant Contribution for

non-zero Grossing angle and therefore should be investigated. In this work I don't investigate introbeam exattering. Rother I will look at points a) and b) above.

II Definitions. Of Quantities Associated with The

Firstly the theory of the luminosity has been worked on by many people and is fairly well understood. The basic theory that can be used here can be found in the paper by Lloyd Smith and in a note by A.G. Ruggiero 4).

In this study we Concern ourselves with functed hearns, beginning by defining various quantities that the luminosity is dependent on.

Circomference = ZTR

frev = frequency of revolution of beam & = (rossing angle of beams

Te = rms Bunch length

B = Number of Bunches

N = Number of Ions/Bunch

E = Normalized Emittance = EBJ

E = Emittance at Loo Gev/AMU

for 95% of beam withingaussian distribution, Such that

 $\sigma^{2} = \frac{\epsilon \beta(s)}{6\pi} \quad \text{where } \beta_{s} = \beta^{*} + \frac{s^{2}}{\beta^{*}}$

L= Luminosity

III - Varions Limits Of Luminosity Formula

Reminder-We are using Gold at 100 Gev/A as a reference case.

bunches

For Short + Head-ON-Goltssons

$$L = N^2 B f_{rev} / 4\pi \sigma_H \sigma_V$$

$$III - 1$$

For Long Bunches + Collisions AT A Non-Zero Angle

亚-2

FOR HEAD-ON COLLISIONS III-1 Con be rewritten as

(in terms of the normalized emittance)

四-3

here E* is the normalized emittance and we are assuming that fearns 1+2 Contain equal numbers of Particles

NEX = An invariant defending on:

a) The brilliance of the Source

6) The Space charge limitation

at the "bottleneck" point Somewhere between the Source and the Collider.

Here the bottleneck is at the injection into the booster-between the Tandem and

$$\frac{N}{\epsilon^*} = (\beta \lambda_5) \frac{11 \beta^{2} \Delta \lambda}{5 \ell^{2}} \frac{\Delta}{\delta_5}$$

so that the luminosity is given by

$$L \sim \frac{N^2}{\epsilon^*} = (\beta \gamma^2) \frac{\pi B_f \Delta V}{2 \Gamma_0 F_0} \frac{A}{\varphi^2} N \qquad III-5$$

where

A = Atomic Number

P = Charge Status

Bf = bonching factor

△V= B-tone dispersion

(tune Shift)

lo = 1.535 × 10-6 cm the classical Proton radius

F = 1 for non-relativistic beams (BCC1, Y-1)

Note the dependence of the lominosity on the

Lone Shift DV. Although this is Quite important it

will not be the focus of this mate.

IN THE OVERLAP INTEGRAL

For bunched beams the luminosity is, in general, given by

W-1

where The overlap Integral

$$F_{b} = \frac{2}{(2\pi)^{3/2}(\sigma_{\ell_{1}}^{2} + \sigma_{\ell_{2}}^{2})} \int_{-\infty}^{+\infty} \frac{dS \exp \left\{-2S^{2}\left(\frac{1}{\sigma_{\ell_{1}}^{2} + \sigma_{\ell_{2}}^{2}} + \frac{\alpha^{2}/4}{\sigma_{\chi_{1}}^{2} + \sigma_{\chi_{2}}^{2}}\right)\right\}}{(\sigma_{\chi_{1}}^{2} + \sigma_{\chi_{2}}^{2})^{1/2}(\sigma_{\chi_{1}}^{2} + \sigma_{\chi_{2}}^{2})^{1/2}}$$

IV - 2

and fencounter = Bfrev.

we want to evaluate this integral in general over the interaction region -10 m = S = 10 m

upon Consideration IV-2 becomes

$$F_{b} = \frac{4 \kappa_{a}}{(2\pi)^{3/2}} \int_{0}^{\infty} \frac{dS}{\left[\frac{\epsilon_{x_{1}}}{\epsilon_{\pi}} \left(\beta_{x_{1}}^{*} + \frac{S^{2}}{\beta_{x_{1}}^{*}}\right) + \frac{\epsilon_{x_{2}}}{\epsilon_{\pi}} \left(\beta_{x_{2}}^{*} + \frac{S^{2}}{\beta_{x_{2}}^{*}}\right)\right] \int_{0}^{\infty} \frac{\epsilon_{z_{1}}}{\epsilon_{\pi}} \left(\beta_{z_{1}}^{*} + \frac{S^{2}}{\beta_{z_{2}}^{*}}\right) + \frac{\epsilon_{z_{2}}}{\epsilon_{\pi}} \left(\beta_{z_{2}}^{*} + \frac{S^{2}}{\beta_{z_{2}}^{*}}\right)\right]^{1/2}$$

$$\times \exp \left\{ -2 s^{2} \left(\kappa_{\alpha}^{2} + \frac{\alpha^{2}/4}{\frac{\epsilon_{x_{1}}}{6\pi} (\beta_{x_{1}}^{*} + \frac{S^{2}}{\beta_{x_{1}}^{*}}) + \frac{\epsilon_{x_{2}}}{6\pi} (\beta_{x_{2}}^{*} + \frac{S^{2}}{\beta_{x_{2}}^{*}})} \right\}$$

 \overline{W} – 3

where

$$K_{\alpha} = \frac{1}{\left(\sigma_{\ell_1}^2 + \sigma_{\ell_2}^2\right)^{1/2}}.$$

W-3'

IV-3 can readily be written in the following

$$F_{b} = \sqrt{\frac{C_{z} k_{a}^{2}}{\pi^{3} B_{z} B_{z}}} \left\{ \frac{\sec^{2} \Theta d\Theta \exp \left\{-C_{z} ton \Theta \left(k_{a}^{2} + \frac{\alpha^{2}/4}{A_{z} + B_{z} C_{z} ton^{2} \Theta}\right)\right\}}{\left[\left(\frac{C_{z}}{2}\right)^{2} Sec^{4} \Theta + P\right]^{1/2}} \right\}$$

W-4

. Where
$$A_{1} = \frac{\epsilon_{x_{1}} \beta_{x_{1}}^{*} + \epsilon_{x_{2}} \beta_{x_{2}}^{*}}{6\pi}, \quad A_{2} = \frac{\epsilon_{z_{1}} \beta_{z_{1}}^{*} + \epsilon_{z_{2}} \beta_{z_{2}}^{*}}{6\pi}$$

$$B_{1} = \frac{1}{2} \left(\epsilon_{x_{1}} + \epsilon_{x_{2}} \beta_{x_{2}}^{*} + \epsilon_{x_{2}} \beta_{x_{2}}^{*}$$

$$B_{1} = \frac{1}{6\pi} \left(\frac{\epsilon_{x_{1}}}{\beta_{x_{1}}^{*}} + \frac{\epsilon_{x_{2}}}{\beta_{x_{z}}^{*}} \right) \quad , \quad B_{z} = \frac{1}{6\pi} \left(\frac{\epsilon_{z_{1}}}{\beta_{z_{1}}^{*}} + \frac{\epsilon_{z_{2}}}{\beta_{z_{2}}^{*}} \right)$$

$$C_1 = \frac{A_1 A_2}{B_1 B_2} \qquad , \quad C_2 = \frac{A_1 B_2 + B_1 A_2}{B_1 B_2}$$

$$P = C_1 - \left(\frac{C_2}{2}\right)^2$$

For arbitrary s then

upper limit 15
$$\theta = \tan^{-1}\left(\sqrt{\frac{2s^2}{c_2}}\right)$$

However for this study we will use formula IV-2, plotting the integrand VS distance from the Center of the interaction region.

V Results

Numerons results were obtained for studies a) and b). These results may be obtained upon request from myself. In the following pages of give the results for one simple Case for a)* and several Cases for b).

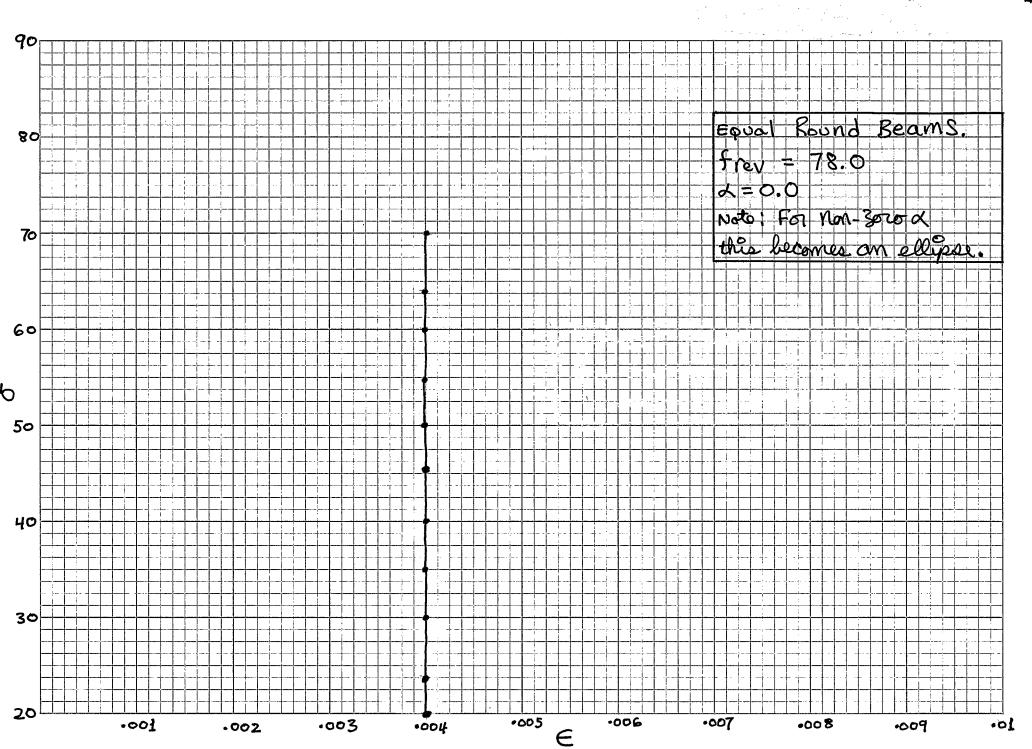
References

- 1) A. Piwinski, Proc. 9th Int. Conf. On High Energy. Accelerators.
- 2) J. D. BJorken & S. K. Mtingwa, Fermulab-Put--82/47-Thy, July, 1982.
- 3) L. Smith, LBL-PEP NOTE-20, April, 1972
- 4) A.G. Ruggero, RHIC-PG-4, BNL, November, 1983.
- * d give a plot of o vs E for a=0.

Results

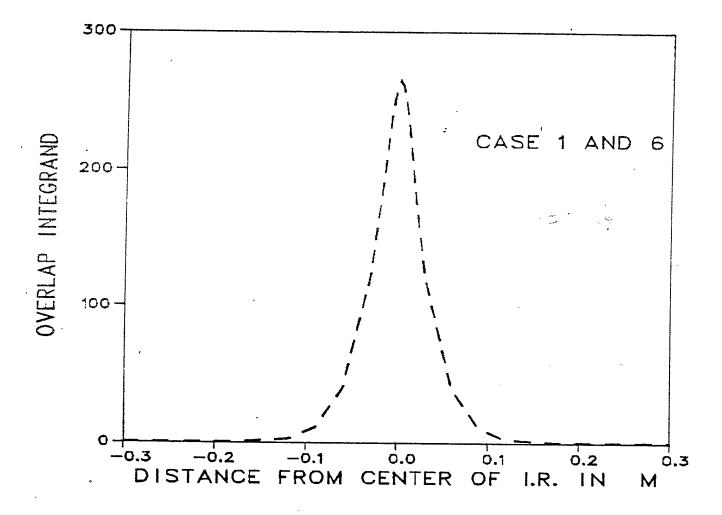
Study

a)



Results 0f

Study



×	σ_{ℓ_1}	Olz	$\epsilon_{x_{\frac{1}{2}}}$	e _{xz}	β* _X	β**	ϵ_{z_I}	ϵ_{z_z}	$\beta_{Z_{\underline{J}}}^{\underline{*}}$	$\beta_{z_2}^*$
0002		i .	l .	1	2		<u>.</u> 1			

DATA FOR GRAPH

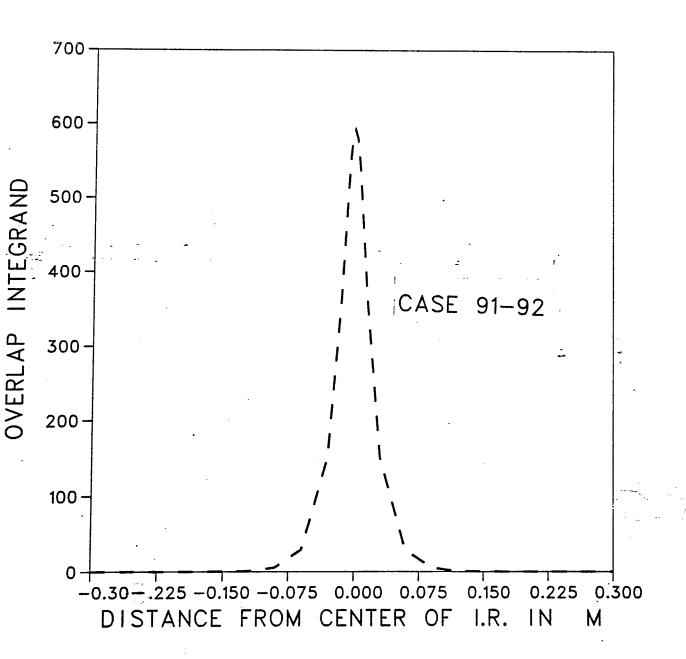
\$	F.
0.0000	.2650E+03
10.0000	-6820E+01
20.0000	.7908E-02
30.0000	.2147E-06
40.0000	.1184F-12
50.0000	.1276E-20
60.0000	-2633E-30
70.0000	.1032E-41
80.0000	.7615E=55
90.0000	.1054E-69
100.0000	.2722E-86
110.0000	.1309-104

CASE #___6

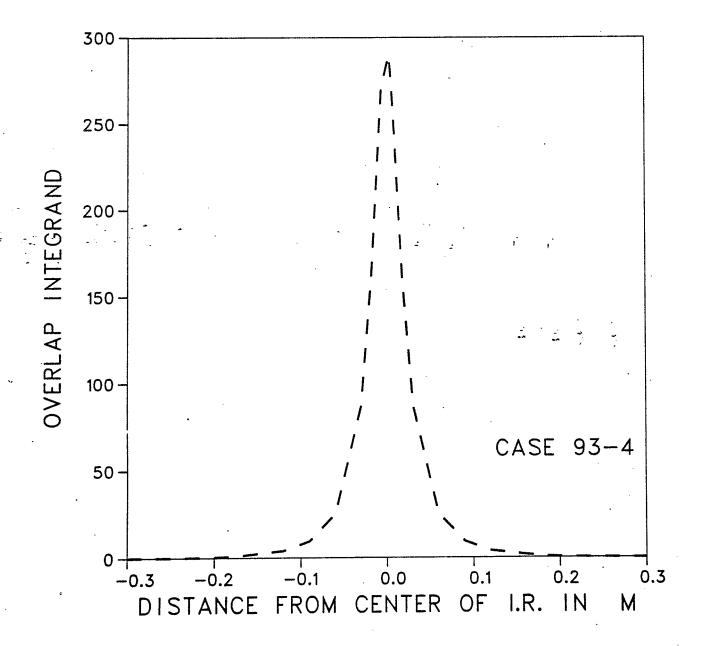
X	σ_{ℓ_1}	OZZ	$\epsilon_{x_{\frac{1}{2}}}$	E _{x2}	β _X	β* _{X2}	ϵ_{z_I}	ϵ_{z_z}	$\beta_{Z_{\underline{1}}}^{*}$	$\beta_{z_2}^*$
•004	io Vz	2/6	9 <u>1</u>	. <u>1</u>	2	2	• 1	01	40	40

DATA FOR GRAPH

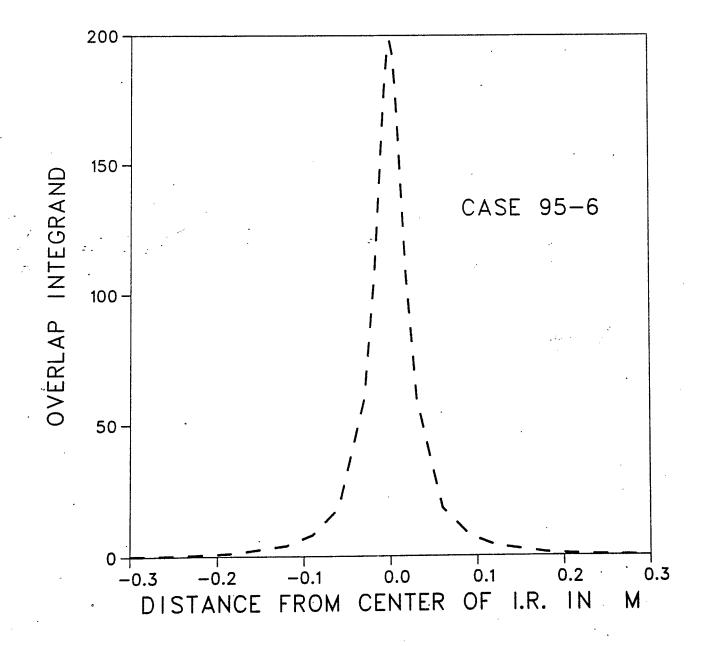
S	F
0.0000	•2650E+03
10.0000	.6813E+01
20.0000	.7899E-02
30.0000	.2144E-06
40.0000	.1183E-12
50.0000	.1274E-20
60.0000	•2630E-30
70.0000	.1030E-41
80.0000	•7606E-55
90.0000	.1052E-69
10.0.0000	.2719E-86
110.0000	.1308-104



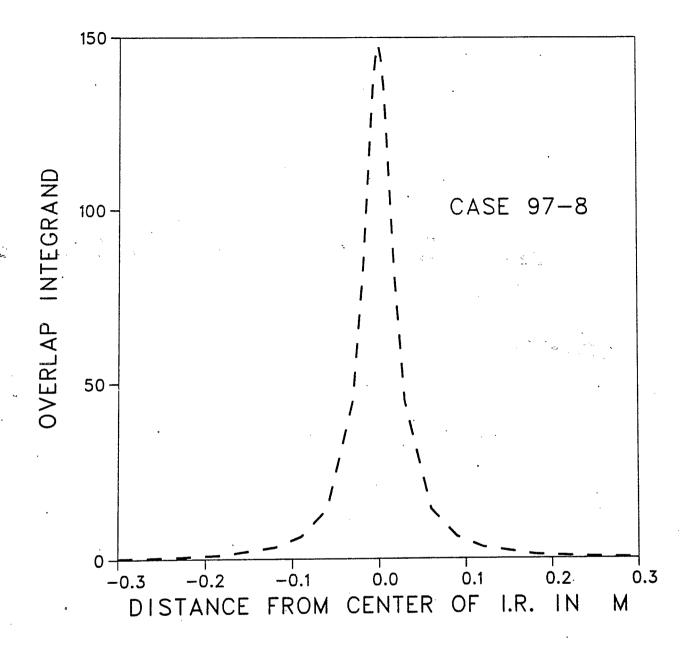
×	$O_{\ell_{\underline{1}}}$	Olz	ϵ_{x_L}	$\epsilon_{X_{Z}}$	B*±	B _{×₂}	ϵ_{z_1}	ϵ_{z_2}	β _{2,1}	β ₂₂	# Co-2e
0	2)52	의년	۰۶	٥2	2	. N	•2	2	Ņ	2	9.1
								e <u>1</u>			



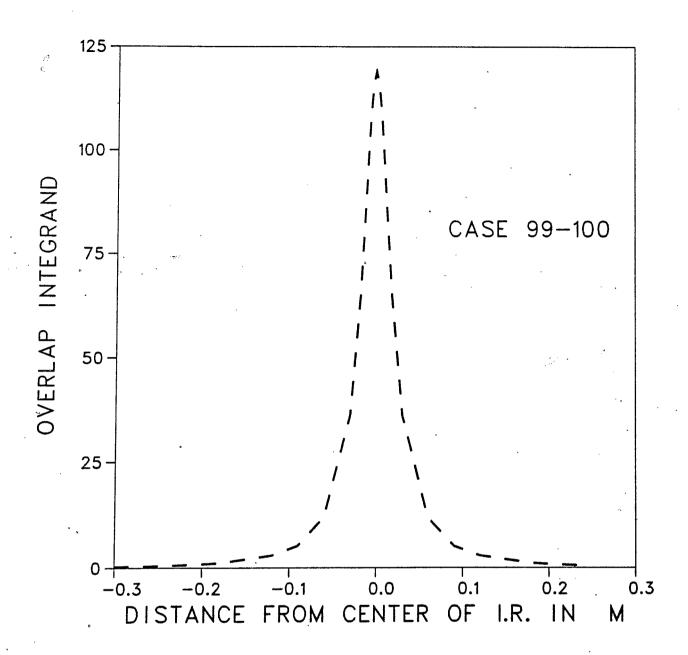
×	Oli	Olz	ϵ^{x^f}	€*2	ß _{x,i}	8*x2	€zį	€ _{Z₂}	β* _{Z,1}	β × β _{Z2}	Case #
0	20	20	وح	۰2	S	2	د ک	<i>o</i> "Z	Ŝ	2.	93
	VZ	J2	• 1	0 L	4	4	٠L	o <u>1</u>	4	4	94



×	Oli	Olz	ϵ_{x_i}	€,2	$\beta_{x_i}^*$	8*x2	€zi	€ _{Z2}	β×zi	β <mark>*</mark>	Case #
0	30	30	٠2	ه ک	2	- 2	٥٧	٥٧	2	2	95
	V2	VZ	<u>-</u>	•1	4	4	°L	.1	4	4	96



×	Oli	Olz	$\epsilon_{x_{i}}$	ϵ_{x_2}	$\beta_{x_i}^*$	8*x2	€zi	EZ2	β×zi	β <mark>*</mark> β _{Z2}	Case #
0	40	40	٥٤	د کے	Ŋ	N	٥2	۰۷	2	2	97
	VZ	VZ	<i>.</i> 1	·1	4	4	<i>o</i> .1.	-1	4	Ч	98



×	Oli	Olz	ϵ_{x_i}	€*3	$\beta_{x_i}^*$	8*x2	€zi	€ _{Z₂}	β×zį	β × β _{Z2}	Case #
0	<u>50</u>	50	٥2	02	2	Ž	σŽ	٥2	2	2	99
	VZ	J2.	01	.1	4	4	<i>.</i> 1	<u>.</u> ‡	4	4	loo