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A Study Of Luminosity Parameters

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A STUDY OF LUMINOSITY PARAMETERS

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I Purpose Of This Study.

II Definitions Of Quantities Associated With the Luminosity.

III Various Limits Of Luminosity formulae.

IV The Overlap Integral.

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I Purpose Of This Study

Essentially the purpose of this study is twofold: a) We want to see how the various parameters that are essential to the luminosity, (i.e. that the luminosity has a strong dependence on), can be varied so as to give one fixed value of the luminosity; (b) We want to "see" how the luminosity can reasonably be expected to behave with respect to distance from the interaction region.

Intrabeam scattering is also a very important factor that must be (and is by several other people) taken into account. The papers by Piwinski[1], Bjorken and Mtingwa[2] are in depth theoretical studies of this phenomena. However in the paper of Bjorken and Mtingwa exchange terms have not been included in the Coulomb scattering amplitude. The exchange terms may well make a significant contribution for non-zero crossing angle and therefore should be investigated. In this work I don't investigate intrabeam scattering. Rather I will look at points a) and b) above.

II Definitions Of Quantities Associated With The Luminosity

Firstly the theory of the luminosity has been worked on by many people and is fairly well understood. The basic theory that can be used here can be found in the paper by Lloyd Smith[3] and in a note by A.G. Ruggiero[4].

In this study we concern ourselves with bunched beams, beginning by defining various quantities that the luminosity is dependent on.

Circumference $\equiv 2\pi R$

$f_{rev} \equiv$ frequency of revolution of beam

$\alpha \equiv$ crossing angle of beams

$\sigma_\ell \equiv$ rms Bunch length

$B \equiv$ Number of Bunches

$N \equiv$ Number of IONS / Bunch

$\epsilon^* \equiv$ Normalized Emittance $\equiv \epsilon \beta \gamma$

$\epsilon \equiv$ Emittance at 100 GeV/AMU for 95% of beam with gaussian distribution, such that

$$\sigma^2 = \frac{\epsilon \beta(s)}{6\pi} \quad \text{where } \beta_s = \beta^* + \frac{s^2}{\beta^*}$$

$L \equiv$ Luminosity

III - Various Limits Of Luminosity Formula

Reminder - We are using Gold at 100 GeV/A as a reference case.

For Short bunches + Head-ON - COLLISIONS

$$L = N^2 B f_{rev} / 4\pi \sigma_H \sigma_V \qquad \text{III - 1}$$

For LONG Bunches + COLLISIONS AT A Non-Zero Angle

$$L = N^2 B f_{rev} / 2\pi \alpha \sigma_\ell \sigma_V \qquad \text{III - 2}$$

FOR HEAD-ON COLLISIONS III-1 can be rewritten as (in terms of the normalized emittance)

$$L = \frac{3 N^2 B f_{rev}}{2 \beta^* (\beta\gamma) \epsilon^*} \sim \frac{N^2}{\epsilon^*} \qquad \text{III - 3}$$

here ϵ^* is the normalized emittance and we are assuming that beams 1 + 2 contain equal numbers of particles per bunch.

$\frac{N}{\epsilon^*}$ ≡ An invariant depending on:

a) The brilliance of the Source

b) The space charge limitation at the "bottleneck" point somewhere between the Source and the Collider.

Here the bottleneck is at the injection into the booster - between the Tandem and AGS and

$$\frac{N}{\epsilon^*} = (\beta\gamma^2)\frac{\pi B_f \Delta V}{2 r_0 F_0} \frac{A}{\varphi^2} \qquad \text{III-4}$$

So that the luminosity is given by

$$L \sim \frac{N^2}{\epsilon^*} = (\beta\gamma^2)\frac{\pi B_f \Delta V}{2 r_0 F_0}\frac{A}{\varphi^2} N \qquad \text{III-5}$$

where

$A \equiv$ Atomic Number

$\varphi \equiv$ Charge Status

$B_f \equiv$ bunching factor

$\Delta V \equiv \beta$-tune dispersion (tune shift)

$r_0 \equiv 1.535 \times 10^{-6}$ cm the classical proton radius

$F_0 \equiv 1$ for non-relativistic beams $(\beta \ll 1, \gamma \sim 1)$.

Note the dependence of the luminosity on the tune shift ΔV. Although this is quite important it will not be the focus of this note.

IV THE OVERLAP INTEGRAL

For bunched beams the luminosity is, in general, given by

$$L = N_1 N_2 f_{encounter} F_b \qquad \text{IV-1}$$

where The overlap Integral

$$F_b \equiv \frac{2}{(2\pi)^{3/2}\left(\sigma_{\ell_1}^2+\sigma_{\ell_2}^2\right)^{1/2}} \int_{-\infty}^{+\infty} \frac{dS \exp\left\{-2S^2\left(\frac{1}{\sigma_{\ell_1}^2+\sigma_{\ell_2}^2}+\frac{\alpha^2/4}{\sigma_{x_1}^2+\sigma_{x_2}^2}\right)\right\}}{\left(\sigma_{x_1}^2+\sigma_{x_2}^2\right)^{1/2}\left(\sigma_{z_1}^2+\sigma_{z_2}^2\right)^{1/2}}$$

IV-2

and $f_{encounter} = B f_{rev}$.

We want to evaluate this integral in general over the interaction region

$$-10\text{ m} \le S \le 10\text{ m}$$

Upon Consideration IV-2 becomes

$$\frac{F}{b} = \frac{4K_a}{(2\pi)^{3/2}} \int_0^{\infty} \frac{dS}{\left[\frac{\epsilon_{x_1}}{6\pi}\left(\beta^*_{x_1} + \frac{S^2}{\beta^*_{x_1}}\right) + \frac{\epsilon_{x_2}}{6\pi}\left(\beta^*_{x_2} + \frac{S^2}{\beta^*_{x_2}}\right)\right]^{1/2} \left[\frac{\epsilon_{z_1}}{6\pi}\left(\beta^*_{z_1} + \frac{S^2}{\beta^*_{z_1}}\right) + \frac{\epsilon_{z_2}}{6\pi}\left(\beta^*_{z_2} + \frac{S^2}{\beta^*_{z_2}}\right)\right]^{1/2}}$$

$$\times \exp\left\{-2S^2\left(K_a^2 + \frac{\alpha^2/4}{\frac{\epsilon_{x_1}}{6\pi}\left(\beta^*_{x_1} + \frac{S^2}{\beta^*_{x_1}}\right) + \frac{\epsilon_{x_2}}{6\pi}\left(\beta^*_{x_2} + \frac{S^2}{\beta^*_{x_2}}\right)}\right)\right\}$$

IV-3

where

$$K_a = \frac{1}{(\sigma_{\ell_1}^2 + \sigma_{\ell_2}^2)^{1/2}} .$$

IV-3'

IV-3 Can readily be written in the following form :

$$\frac{F}{b} = \sqrt{\frac{C_2 K_a^2}{\pi^3 B_1 B_2}} \int_0^{\pi/2} \frac{\sec^2\theta \, d\theta \, \exp\left\{-C_2 \tan^2\theta \left(K_a^2 + \frac{\alpha^2/4}{A_1 + \frac{B_1 C_2}{2}\tan^2\theta}\right)\right\}}{\left[\left(\frac{C_2}{2}\right)^2 \sec^4\theta + P\right]^{1/2}}$$

IV-4

where

$$A_1 = \frac{\epsilon_{x_1} \beta_{x_1}^* + \epsilon_{x_2} \beta_{x_2}^*}{6\pi}, \quad A_2 = \frac{\epsilon_{z_1} \beta_{z_1}^* + \epsilon_{z_2} \beta_{z_2}^*}{6\pi}$$

$$B_1 = \frac{1}{6\pi}\left(\frac{\epsilon_{x_1}}{\beta_{x_1}^*} + \frac{\epsilon_{x_2}}{\beta_{x_2}^*} \right), \quad B_2 = \frac{1}{6\pi}\left(\frac{\epsilon_{z_1}}{\beta_{z_1}^*} + \frac{\epsilon_{z_2}}{\beta_{z_2}^*} \right)$$

$$C_1 = \frac{A_1 A_2}{B_1 B_2}, \quad C_2 = \frac{A_1 B_2 + B_1 A_2}{B_1 B_2}$$

$$P = C_1 - \left(\frac{C_2}{2}\right)^2$$

For arbitrary S then upper limit is $\theta = \tan^{-1}\left(\sqrt{\frac{2S^2}{C_2}}\right)$.

However for this study we will use formula IV-2, plotting the integrand vs distance from the center of the interaction region.

V Results

Numerous results were obtained for studies a) and b). These results may be obtained upon request from myself.

In the following pages I give the results for one simple case for a)* and several cases for b).

References

1) A. Piwinski, Proc. 9th Int. Conf. On High Energy Accelerators.

2) J. D. Bjorken & S. K. Mtingwa, Fermilab-Pub-82/47-Thy, July, 1982.

3) L. Smith, LBL-PEP NOTE-20, April, 1972

4) A.G. Ruggiero, RHIC-PG-4, BNL, November, 1983.

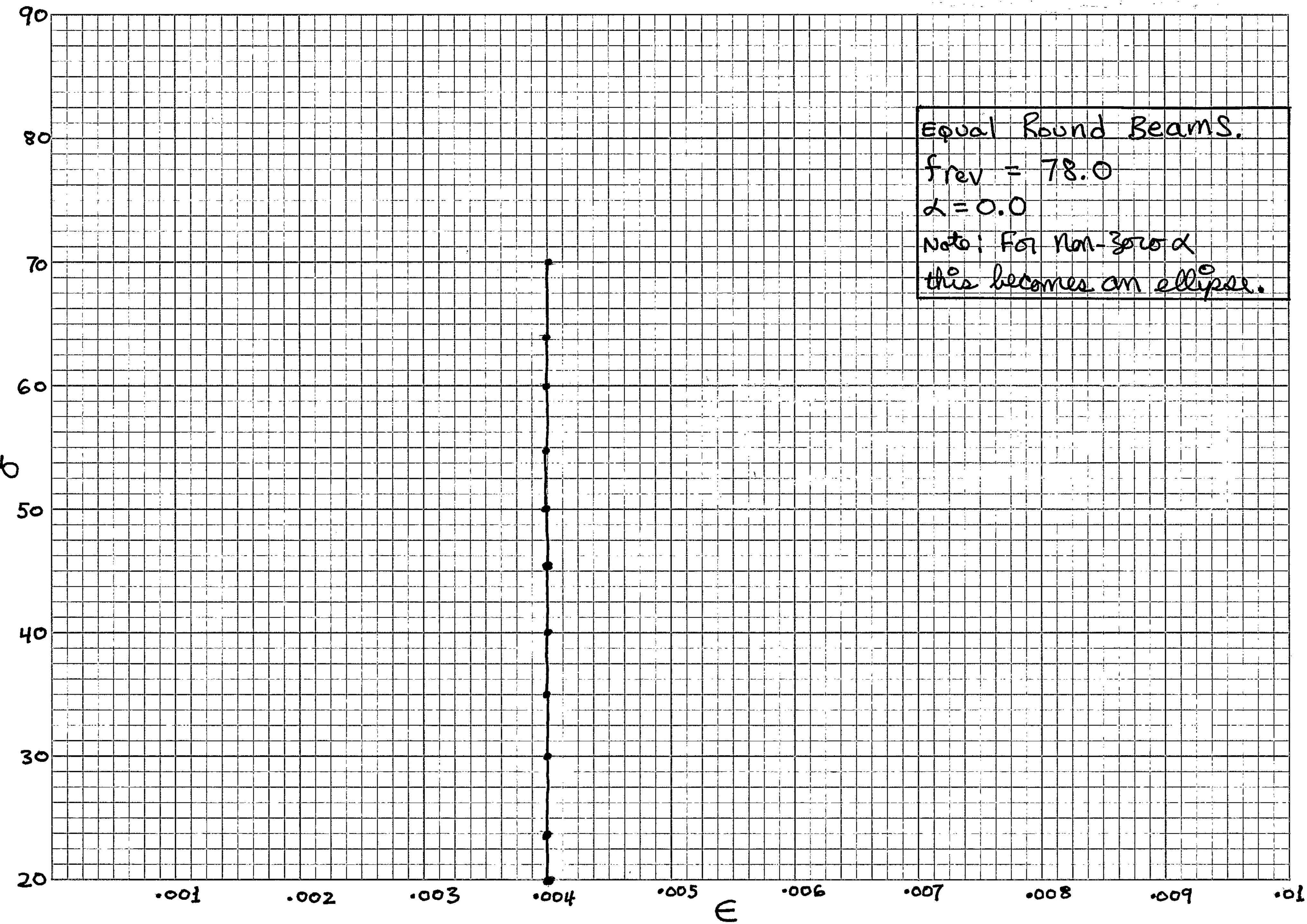
* I give a plot of σ vs ϵ for $\alpha = 0$.

Results

of

Study

a)

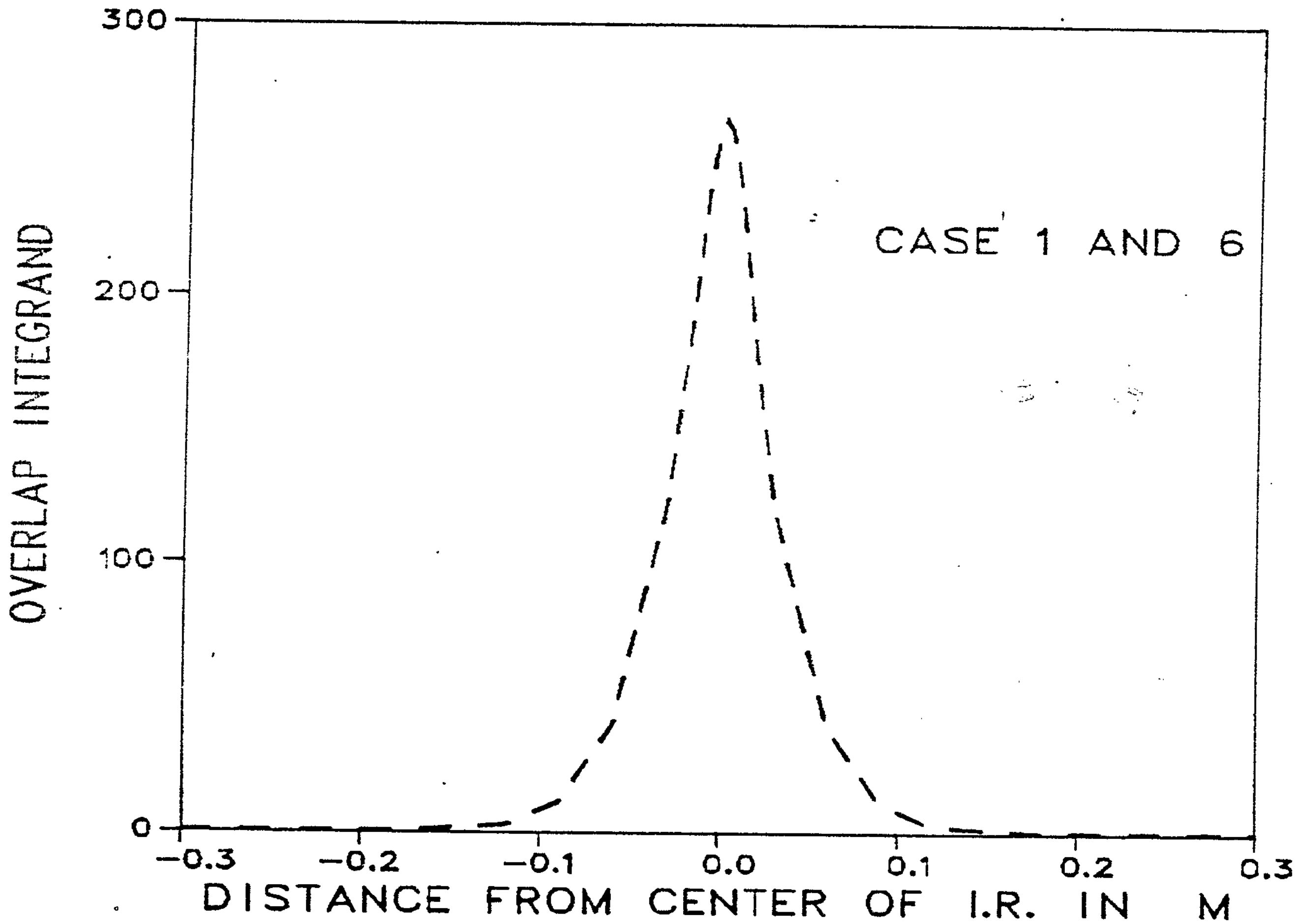

Equal Round Beams.
frev = 78.0
α = 0.0
Note: For non-zero α
this becomes an ellipse.
90
80
70
60
50
40
30
20
σ
.001
.002
.003
.004
.005
.006
.007
.008
.009
.01
ε

Results

Of

Study

b)


300
200
100
0
OVERLAP INTEGRAND
CASE 1 AND 6
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
DISTANCE FROM CENTER OF I.R. IN M

CASE # 1

α	σ_{ℓ_1}	σ_{ℓ_2}	ϵ_{x_1}	ϵ_{x_2}	$\beta^*_{x_1}$	$\beta^*_{x_2}$	ϵ_{z_1}	ϵ_{z_2}	$\beta^*_{z_1}$	$\beta^*_{z_2}$
.002	$\frac{10}{\sqrt{2}}$	$\frac{10}{\sqrt{2}}$	.1	.1	2	2	.1	.1	40	40

DATA FOR GRAPH

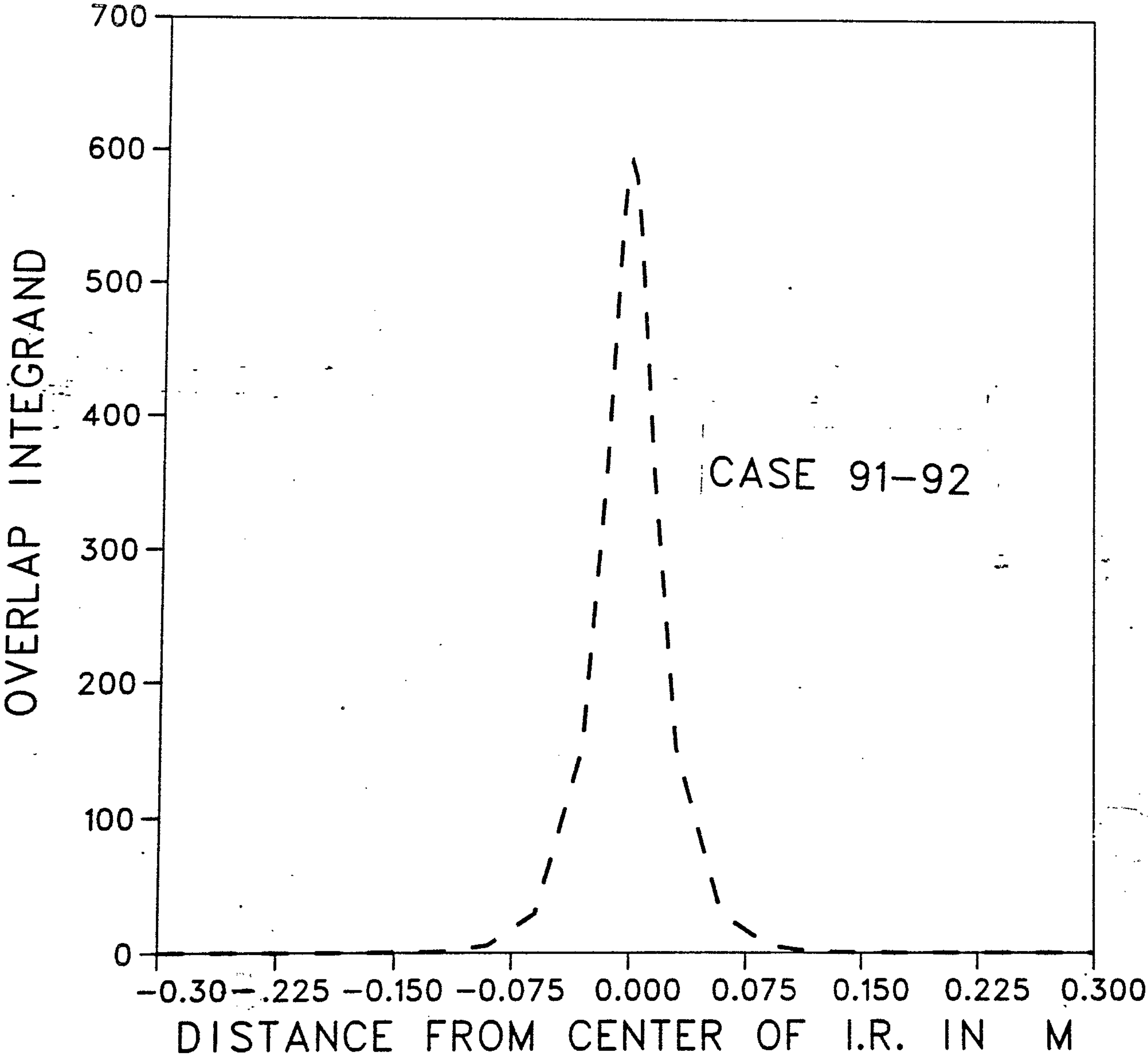
S	F_b
0.0000	.2650E+03
10.0000	.6820E+01
20.0000	.7908E-02
30.0000	.2147E-06
40.0000	.1184E-12
50.0000	.1276E-20
60.0000	.2633E-30
70.0000	.1032E-41
80.0000	.7615E-55
90.0000	.1054E-69
100.0000	.2722E-86
110.0000	.1309-104

CASE # 6

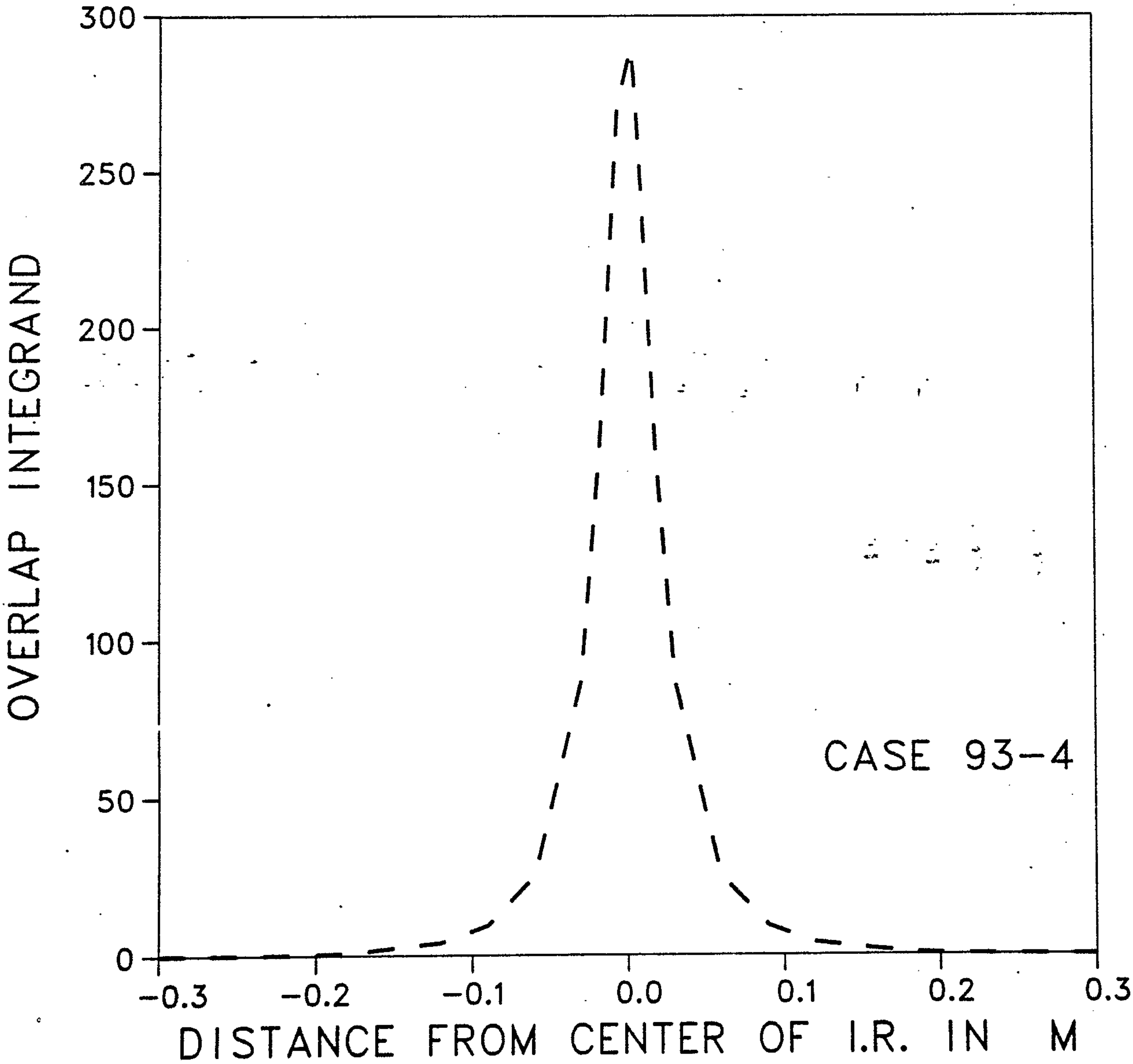
α	σ_{ℓ_1}	σ_{ℓ_2}	ϵ_{x_1}	ϵ_{x_2}	$\beta^*_{x_1}$	$\beta^*_{x_2}$	ϵ_{z_1}	ϵ_{z_2}	$\beta^*_{z_1}$	$\beta^*_{z_2}$
.004	$\frac{10}{\sqrt{2}}$	$\frac{10}{\sqrt{2}}$	.1	.1	2	2	.1	.1	40	40

DATA FOR GRAPH

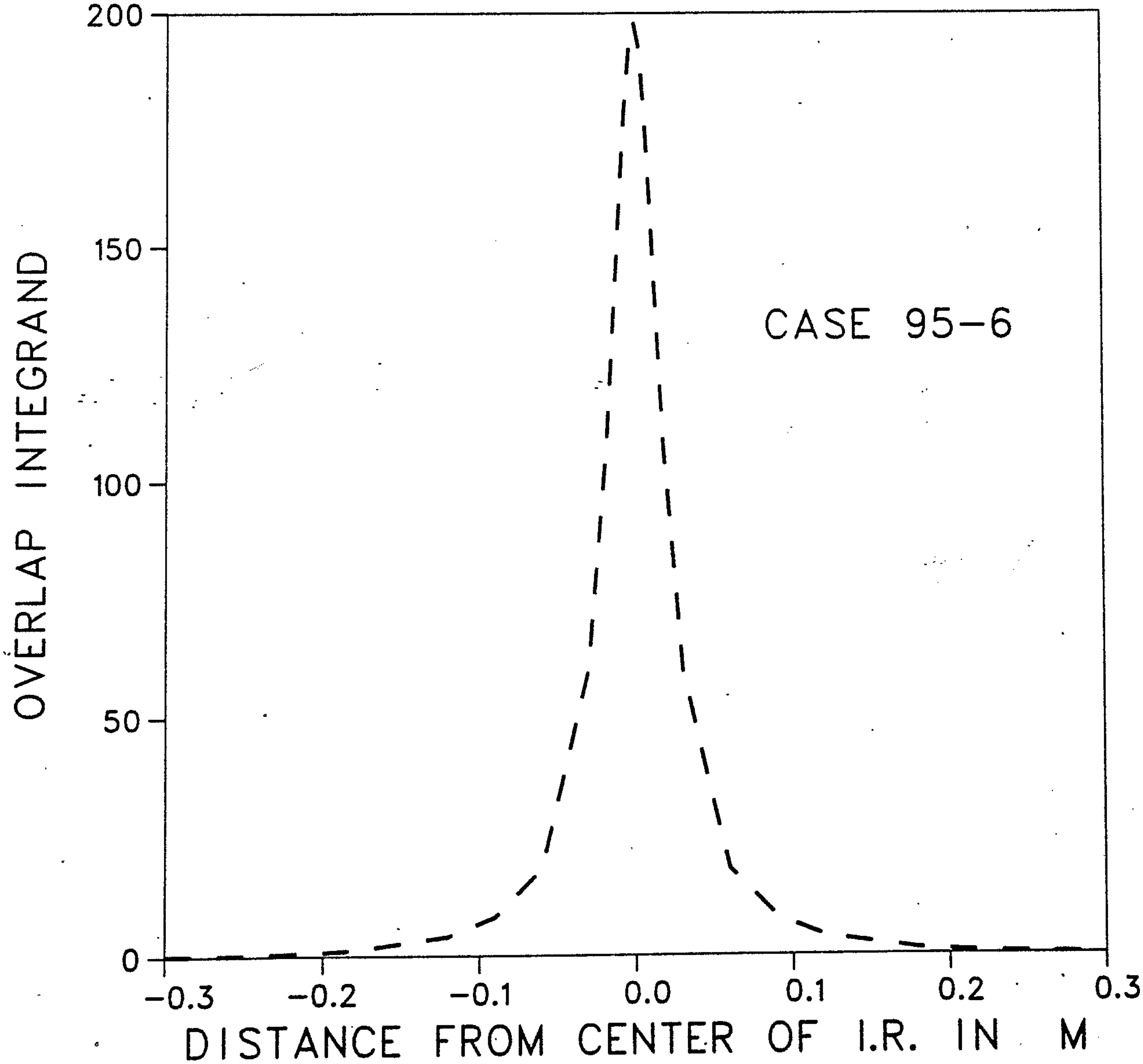
S	F_b
0.0000	.2650E+03
10.0000	.6813E+01
20.0000	.7899E-02
30.0000	.2144E-06
40.0000	.1183E-12
50.0000	.1274E-20
60.0000	.2630E-30
70.0000	.1030E-41
80.0000	.7606E-55
90.0000	.1052E-69
100.0000	.2719E-86
110.0000	.1308-104



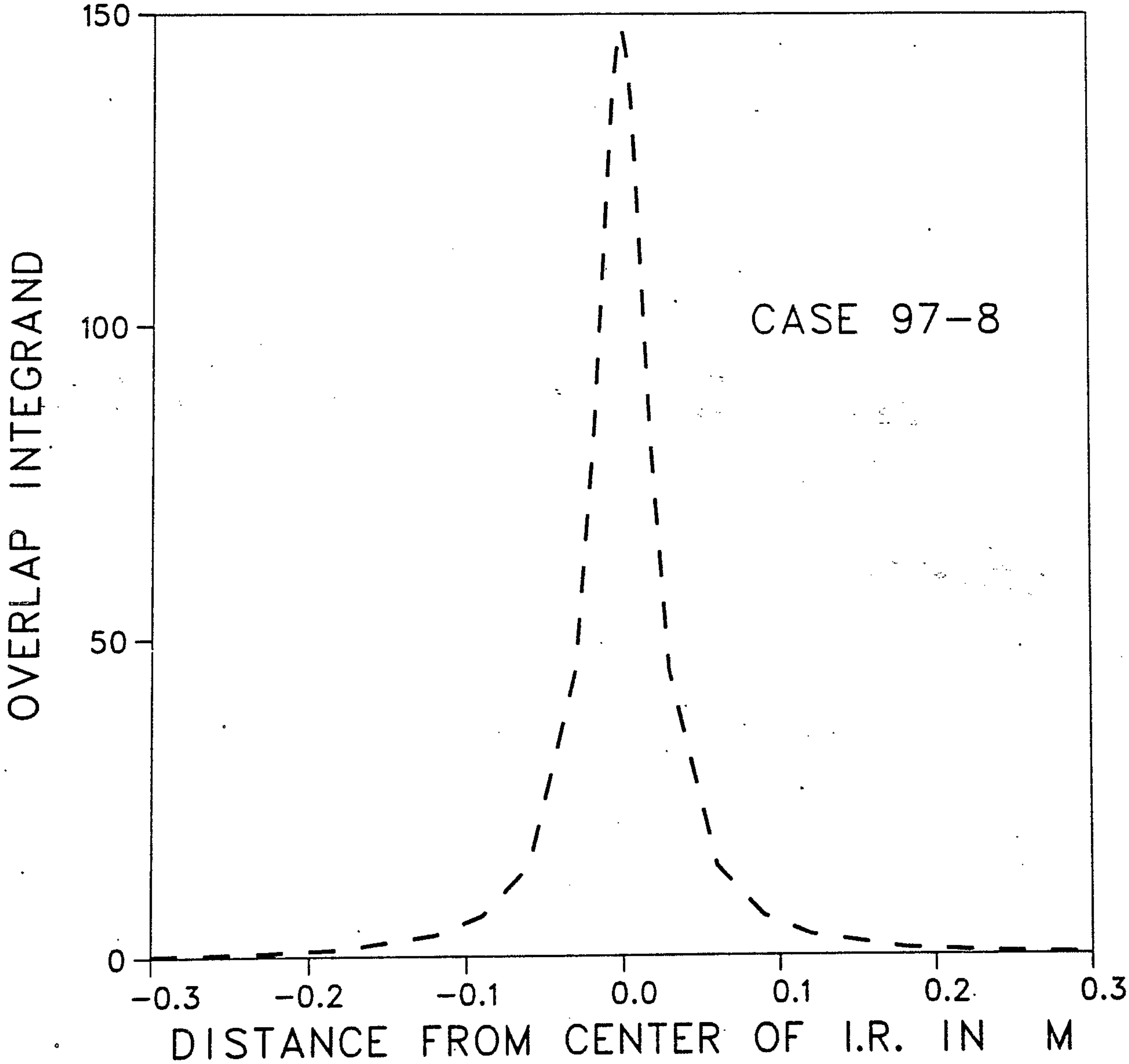
α	σ_{ℓ_1}	σ_{ℓ_2}	ϵ_{x_1}	ϵ_{x_2}	$\beta^*_{x_1}$	$\beta^*_{x_2}$	ϵ_{z_1}	ϵ_{z_2}	$\beta^*_{z_1}$	$\beta^*_{z_2}$	Case #
0	$\frac{10}{\sqrt{2}}$	$\frac{10}{\sqrt{2}}$	.2	.2	2	2	.2	.2	2	2	91
"	"	"	.1	.1	4	4	.1	.1	4	4	92



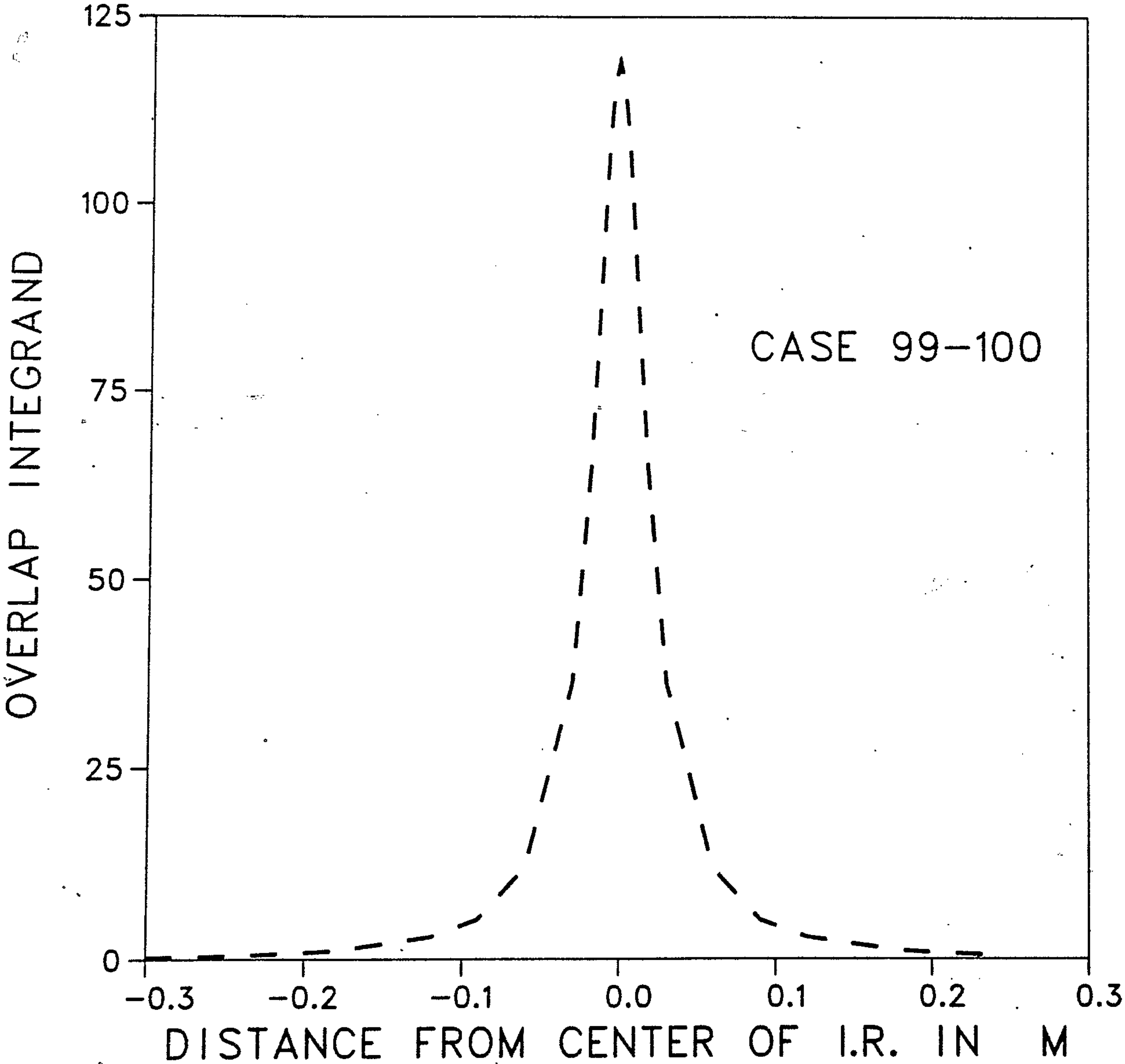
α	σ_{ℓ_1}	σ_{ℓ_2}	ϵ_{x_1}	ϵ_{x_2}	$\beta^*_{x_1}$	$\beta^*_{x_2}$	ϵ_{z_1}	ϵ_{z_2}	$\beta^*_{z_1}$	$\beta^*_{z_2}$	Case #
0	$\frac{20}{\sqrt{2}}$	$\frac{20}{\sqrt{2}}$	.2	.2	2	2	.2	.2	2	2	93
			.1	.1	4	4	.1	.1	4	4	94



α	σ_{ℓ_1}	σ_{ℓ_2}	ϵ_{x_1}	ϵ_{x_2}	$\beta^*_{x_1}$	$\beta^*_{x_2}$	ϵ_{z_1}	ϵ_{z_2}	$\beta^*_{z_1}$	$\beta^*_{z_2}$	Case #
0	$30/\sqrt{2}$	$30/\sqrt{2}$	.2	.2	2	2	.2	.2	2	2	95
			.1	.1	4	4	.1	.1	4	4	96



α	σ_{ℓ_1}	σ_{ℓ_2}	ϵ_{x_1}	ϵ_{x_2}	$\beta^*_{x_1}$	$\beta^*_{x_2}$	ϵ_{z_1}	ϵ_{z_2}	$\beta^*_{z_1}$	$\beta^*_{z_2}$	Case #
0	$\frac{40}{\sqrt{2}}$	$\frac{40}{\sqrt{2}}$	.2	.2	2	2	.2	.2	2	2	97
			.1	.1	4	4	.1	.1	4	4	98



α	σ_{ℓ_1}	σ_{ℓ_2}	ϵ_{x_1}	ϵ_{x_2}	$\beta^*_{x_1}$	$\beta^*_{x_2}$	ϵ_{z_1}	ϵ_{z_2}	$\beta^*_{z_1}$	$\beta^*_{z_2}$	Case #
0	$\frac{50}{\sqrt{2}}$	$\frac{50}{\sqrt{2}}$	.2	.2	2	2	.2	.2	2	2	99
			.1	.1	4	4	.1	.1	4	4	100