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Parameters Of The RF System For The "Weak-Focusing" Lattices

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PARAMETERS OF THE RF SYSTEM FOR THE "WEAK-FOCUSING" LATTICES

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rf REQUIREMENTS
(Parabolic Distribution)

Bunch half length $= \sqrt{5}\sigma_L$

Bunch phase half width $\phi = \sqrt{5}\ \sigma_L\ h/R$

Bunch half height $\Delta_E = \sqrt{5}\ \delta_E$

Bunch area/amu $S = 5\pi\ \sigma_L\ \delta_E\ \gamma\ E_o/c \quad = \dfrac{\gamma\ E_o}{2\ h\ f_o}\ \Delta_E\ \phi$

In the small-amplitude approximation and stationary

$$\phi = \left(\frac{8\pi\ |\eta|\ h^3\ f_o^{\ 2}}{\gamma\ E_o eV}\ \frac{A}{Q}\right)^{1/4} \sqrt{S}$$

$$\Delta_E = \frac{2h\ f_o}{\gamma\ E_o}\ \frac{S}{\phi}$$

Bucket half height required, stationary

$$\Delta_B = \frac{\Delta_E}{\sin \phi/2}$$

Bucket Area/amu required

$$A_B = \frac{4}{\pi}\ \frac{\gamma\ E_o}{h\ f_o}\ \Delta_B$$

Voltage required

$$V = \frac{\pi}{2}\ \frac{h\ |\eta|\ \gamma\ E_o}{e}\ \frac{A}{Q}\ \Delta_B^{\ 2}$$

$$V \approx 8\pi\ \frac{|\eta|\ h^3\ f_o^{\ 2}}{e\ \gamma\ E_o}\ \frac{A}{Q}\ \frac{S^2}{\phi^4} \quad (\phi \ll \pi)$$

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Parameter Variations with h_{rf}
(Equipartition)
$\gamma = 100$, Au

Lattice	N_B	h_{rf}	δ_E	S eV•sec	L $cm^{-2}sec^{-1}$	Diamond cm rms	V MV
	$\times 10^9$	$\times 57$	$\times 10^{-4}$		$\times 10^{26}$		
			α = 0 mrad				
15/120°	1	12	11.3	3.8	10.5*	29	2.2
	1	6	9.8	6.6	14.*	57	0.85
12/90°	1	12	6.9	2.3	5.6	29	2.1
	1	6	6.0	4.0	7.4	57	0.81
	1	1	4.2	16.9	15.1*	343	0.066
9/120°	1	6	5.9	4.0	8.4	57	0.95
			α = 2 mrad				
15/120°	2	12	12.9	4.4	11.	12	3.0
	2	6	11.3	7.6	6.7	14	1.1
	1	6	9.8	6.6	1.9	13	0.85
12/90°	2	12	7.9	2.7	7.6	14	2.8
	2	6	6.9	4.6	4.8	18	1.1
	2	1	4.8	19.4	1.2	17	0.087
	1	6	6.0	4.0	1.4	16	0.81
9/120°	2	6	6.7	4.6	5.1	17	1.2

* $\Delta\nu_{BB} > 0.003$

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MOMENTUM SPREAD AT TRANSITION

The momentum spread at transition scales like

$$\delta_E \propto \left(\frac{h^2}{\gamma_{tr}^2} \frac{V^2}{\dot{B}} \cos^2 \Phi_s\right)^{1/6}$$

with $V \sin \phi_s = 2\pi \; R \; \rho \; \dot{B}$

Assuming the same rf system, the lattices with $\gamma_{tr} \approx 25$ require at transition about 15% more momentum aperture than one with $\gamma_{tr} \approx 38$.

An acceptable rf system for the $\gamma_{tr} \approx 25$ lattices is obtained by using (primed quantities):

$$h' = \frac{1}{2} h; \quad V' = \frac{1}{5} V; \quad \dot{B}' = \frac{1}{4} \dot{B}; \quad \phi_s' \approx \frac{5}{4} \phi_s$$

leading to $\delta_E' = 0.7 \; \delta_E$

The resulting physical aperture requirement due to momentum spread is

$(X'_{p\,max} = 1.57 \text{ m}, \; X_{p\,max} = 0.7 \text{ m})$

$$\sigma_H' = 1.57 \; \sigma_H$$

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SUGGESTED rf PARAMETERS

$$f_{rf} = 6 \times 57 \times f_o = 26.7 \text{ MHz}$$

$$V_{max} = 1 \text{ MV}$$

$$V_{acceleration} = 200 \text{ kV}$$

$$\text{Acceleration time} = 2 \text{ min.}$$

Questions:

- What is dynamics of intrabeam scattering at operating point.

- Parzen will calculate $L = L(t)$
  $\sigma = \sigma_L(t)$

- Slowest beam growth is expected, if full voltage is reached at the end of the acceleration cycle, since

$$\tau_E^{-1} \propto \frac{N_B}{\varepsilon \; S \; \delta_E^2}; \qquad \tau_H^{-1} \propto \frac{N_B}{\varepsilon^2 \; S}$$

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CHOICE OF TRANSITION ENERGY

Due to intrabeam scattering the momentum spread of the bunch increases until $\Delta_E = \Delta_B$. If this limit is exceeded, the particles are lost.

At constant voltage

$$\Delta_B^2 \propto \frac{1}{h \gamma |\eta|} = \frac{\gamma_{tr}}{h |\gamma/\gamma_{tr} - \gamma_{tr}/\gamma|}$$

The bucket height requirements vary with energy according to

$$\Delta_E \propto \frac{1}{\sqrt{\gamma}} \qquad \text{(equipartition)}$$

$$\Delta_E(\gamma=12) > 1.4 \Delta_E(\gamma=100) \qquad \text{(Parzen)}$$

Equivalent performance over energy range, (i.e. $L \propto \gamma$) requires

$$\gamma_{tr}^2 = \gamma_1 \gamma_2 \frac{(\Delta_1/\Delta_2)^2 \gamma_1 + \gamma_2}{\gamma_1 + (\Delta_1/\Delta_2)^2 \gamma_2}$$

For $\gamma_1=12$ and $\gamma_2=108$ follows the optimized transition energy $(\Delta_1/\Delta_2 \approx 1.4)$:

$$\gamma_{tr} = 36 \times 0.6 \approx 22$$