

# Parameters Of The RF System For The "Weak-Focusing" Lattices

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PARAMETERS OF THE RF SYSTEM  
FOR THE "WEAK-FOCUSING" LATTICES

H. HAHN

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rf REQUIREMENTS  
(Parabolic Distribution)

Bunch half length  $= \sqrt{5} \sigma_L$

Bunch phase half width  $\phi = \sqrt{5} \sigma_L h/R$

Bunch half height  $\Delta_E = \sqrt{5} \delta_E$

Bunch area/amu  $S = 5\pi \sigma_L \delta_E \gamma E_o / c = \frac{\gamma E_o}{2 h f_o} \Delta_E \phi$

In the small-amplitude approximation and stationary

$$\phi = \left( \frac{8\pi |\eta| h^3 f_o^2 A}{\gamma E_o eV Q} \right)^{1/4} \sqrt{S}$$

$$\Delta_E = \frac{2h f_o}{\gamma E_o} \frac{S}{\phi}$$

Bucket half height required, stationary

$$\Delta_B = \frac{\Delta_E}{\sin \phi/2}$$

Bucket Area/amu required

$$A_B = \frac{4}{\pi} \frac{\gamma E_o}{h f_o} \Delta_B$$

Voltage required

$$V = \frac{\pi}{2} \frac{h |\eta| \gamma E_o}{e} \frac{A}{Q} \Delta_B^2$$

$$V \approx 8\pi \frac{|\eta| h^3 f_o^2 A}{e \gamma E_o Q \phi^4} S^2 \quad (\phi \ll \pi)$$

Parameter Variations with  $h_{rf}$   
(Equipartition)  
 $\gamma = 100$ , Au

Lattice	$N_B$	$h_{rf}$	$\delta_E$	S eV·sec	L $\text{cm}^{-2}\text{sec}^{-1}$	Diamond cm rms	V MV
	$\times 10^9$	$\times 57$	$\times 10^{-4}$		$\times 10^{26}$		
$\alpha = 0 \text{ mrad}$							
15/120°	1	12	11.3	3.8	10.5*	29	2.2
	1	6	9.8	6.6	14.*	57	0.85
12/90°	1	12	6.9	2.3	5.6	29	2.1
	1	6	6.0	4.0	7.4	57	0.81
	1	1	4.2	16.9	15.1*	343	0.066
9/120°	1	6	5.9	4.0	8.4	57	0.95
$\alpha = 2 \text{ mrad}$							
15/120°	2	12	12.9	4.4	11.	12	3.0
	2	6	11.3	7.6	6.7	14	1.1
	1	6	9.8	6.6	1.9	13	0.85
12/90°	2	12	7.9	2.7	7.6	14	2.8
	2	6	6.9	4.6	4.8	18	1.1
	2	1	4.8	19.4	1.2	17	0.087
	1	6	6.0	4.0	1.4	16	0.81
9/120°	2	6	6.7	4.6	5.1	17	1.2

\*  $\Delta v > 0.003$   
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## MOMENTUM SPREAD AT TRANSITION

The momentum spread at transition scales like

$$\delta_E \propto \left( \frac{h^2}{\gamma_{tr}^2} \frac{V^2}{\dot{B}} \cos^2 \phi_s \right)^{1/6}$$

with  $V \sin \phi_s = 2\pi R \rho \dot{B}$

Assuming the same rf system, the lattices with  $\gamma_{tr} \approx 25$  require at transition about 15% more momentum aperture than one with  $\gamma_{tr} \approx 38$ .

An acceptable rf system for the  $\gamma_{tr} \approx 25$  lattices is obtained by using (primed quantities):

$$h' = \frac{1}{2} h; \quad V' = \frac{1}{5} V; \quad \dot{B}' = \frac{1}{4} \dot{B}; \quad \phi'_s \approx \frac{5}{4} \phi_s$$

leading to

$$\delta'_E = 0.7 \delta_E$$

The resulting physical aperture requirement due to momentum spread is

$$(X'_{p \max} = 1.57 \text{ m}, X_{p \max} = 0.7 \text{ m})$$

$$\sigma'_H = 1.57 \sigma_H$$

SUGGESTED rf PARAMETERS

$$f_{\text{rf}} = 6 \times 57 \times f_0 = 26.7 \text{ MHz}$$

$$V_{\text{max}} = 1 \text{ MV}$$

$$V_{\text{acceleration}} = 200 \text{ kV}$$

$$\text{Acceleration time} = 2 \text{ min.}$$

Questions:

- What is dynamics of intrabeam scattering at operating point.
- Parzen will calculate  $L = L(t)$   
 $\sigma = \sigma_L(t)$
- Slowest beam growth is expected, if full voltage is reached at the end of the acceleration cycle, since

$$\tau_E^{-1} \propto \frac{N_B}{\epsilon S \delta_E^2}; \quad \tau_H^{-1} \propto \frac{N_B}{\epsilon^2 S}$$

# CHOICE OF TRANSITION ENERGY

Due to intrabeam scattering the momentum spread of the bunch increases until  $\Delta_E = \Delta_B$ . If this limit is exceeded, the particles are lost.

At constant voltage

$$\Delta_B^2 \propto \frac{1}{h \gamma |\eta|} = \frac{\gamma_{tr}}{h |\gamma/\gamma_{tr} - \gamma_{tr}/\gamma|}$$

The bucket height requirements vary with energy according to

$$\Delta_E \propto \frac{1}{\sqrt{\gamma}} \quad (\text{equipartition})$$

$$\Delta_E(\gamma=12) > 1.4 \Delta_E(\gamma=100) \quad (\text{Parzen})$$

Equivalent performance over energy range, (i.e.  $L \propto \gamma$ ) requires

$$\gamma_{tr}^2 = \gamma_1 \gamma_2 \frac{(\Delta_1/\Delta_2)^2 \gamma_1 + \gamma_2}{\gamma_1 + (\Delta_1/\Delta_2)^2 \gamma_2}$$

For  $\gamma_1=12$  and  $\gamma_2=108$  follows the optimized transition energy ( $\Delta_1/\Delta_2 \approx 1.4$ ):

$$\gamma_{tr} = 36 \times 0.6 \approx 22$$