

## Table Of Values For The Comparison Of 3 Different Lattices for RHIC

A. G. Ruggiero

January 1984

Collider Accelerator Department  
**Brookhaven National Laboratory**

**U.S. Department of Energy**

USDOE Office of Science (SC)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No.DE-AC02-76CH00016 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

## **DISCLAIMER**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

TABLES OF VALUES  
FOR THE COMPARISON  
OF 3 DIFFERENT LATTICES  
FOR RHIC

A. G. Ruggiero

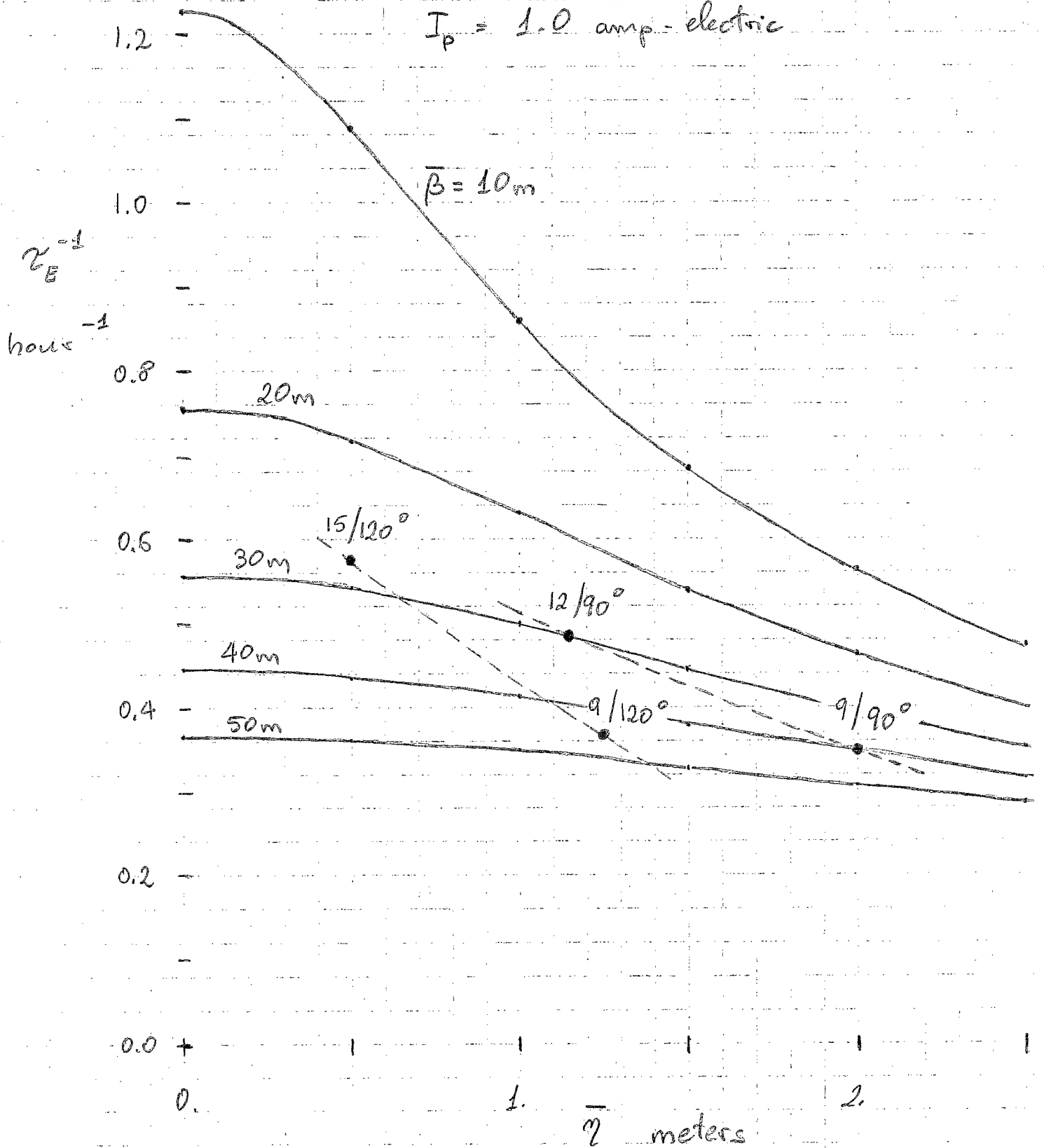
(BNL, January 16, 1984)

$$\gamma = 100$$

$$\text{Au: } A = 197 \quad Z = 79$$

$$\varepsilon_N = 10\pi \text{ mm mrad}, \quad \sigma_E/E = 0.5 \times 10^{-3}$$

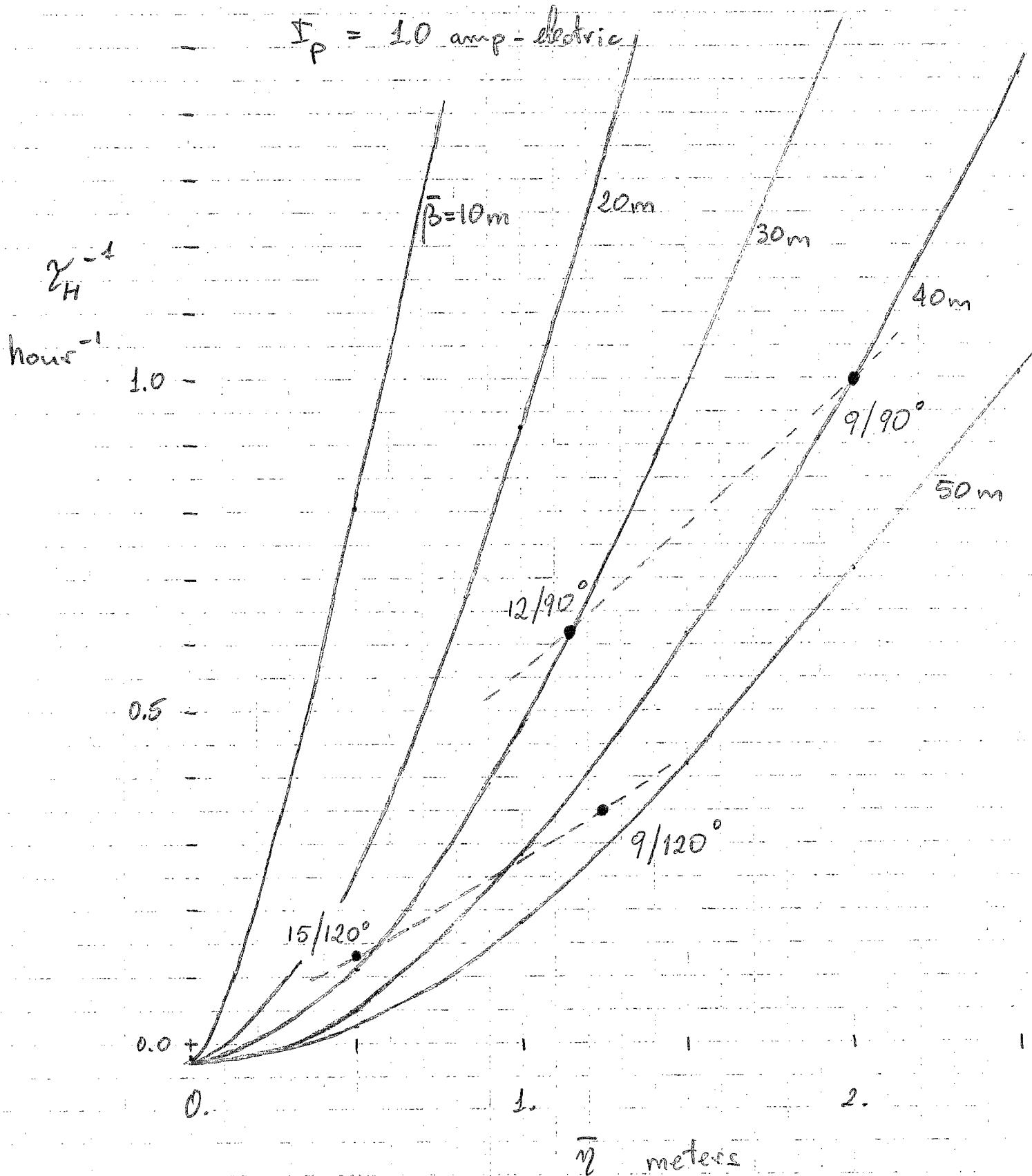
$$I_p = 1.0 \text{ amp-electric}$$

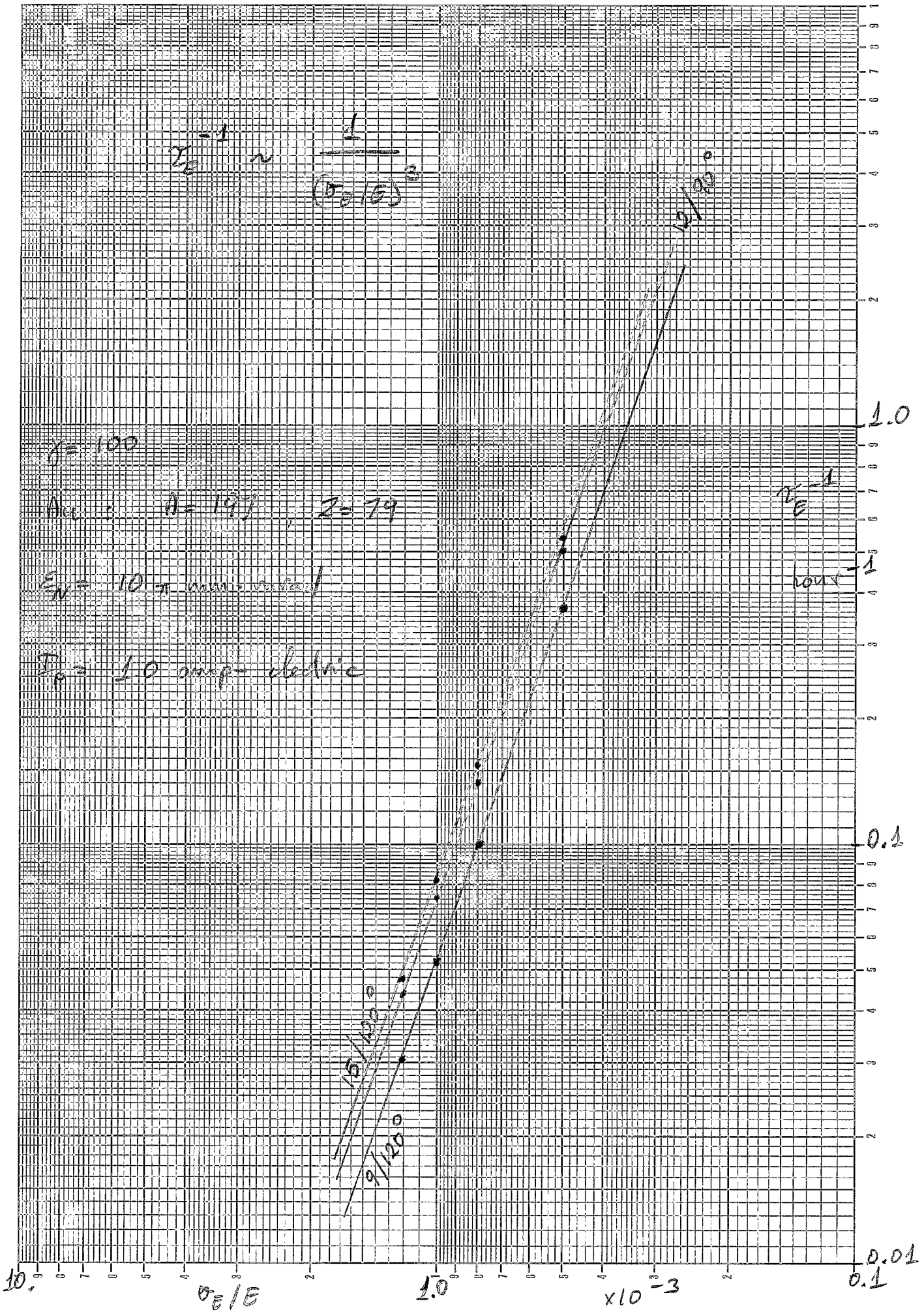


$\gamma = 100$       Au:     $A = 197$      $Z = 79$

$E_N = 10 \pi$  mm-mrad,     $\sigma_E / E = 0.5 \times 10^{-3}$

$I_P = 1.0$  amp-electric





$\sigma_E^{-1} \sim \frac{1}{E_N}$

$\gamma = 100$

AW: A = 197, Z = 79

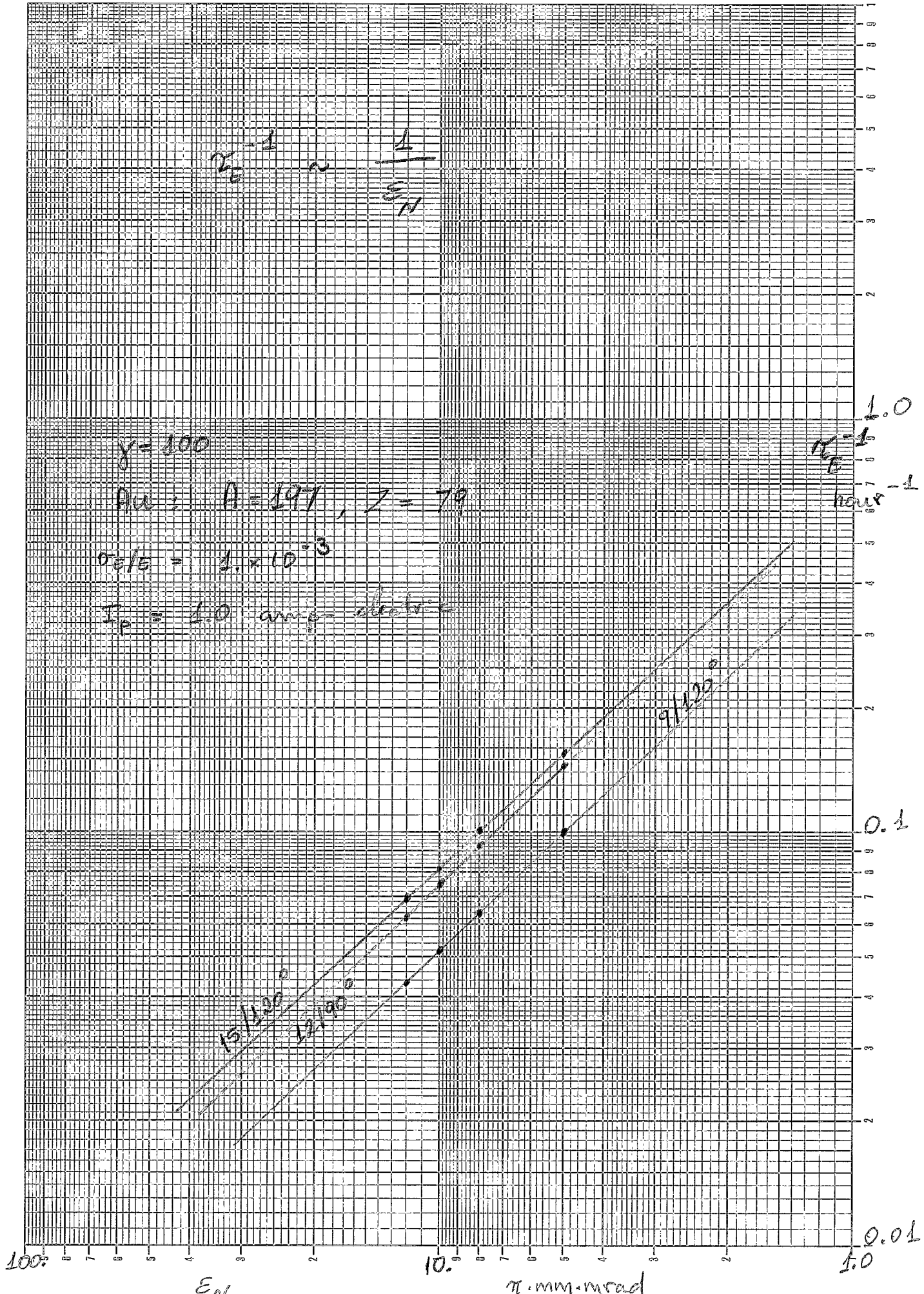
$\sigma_E/E = 1. \times 10^{-3}$

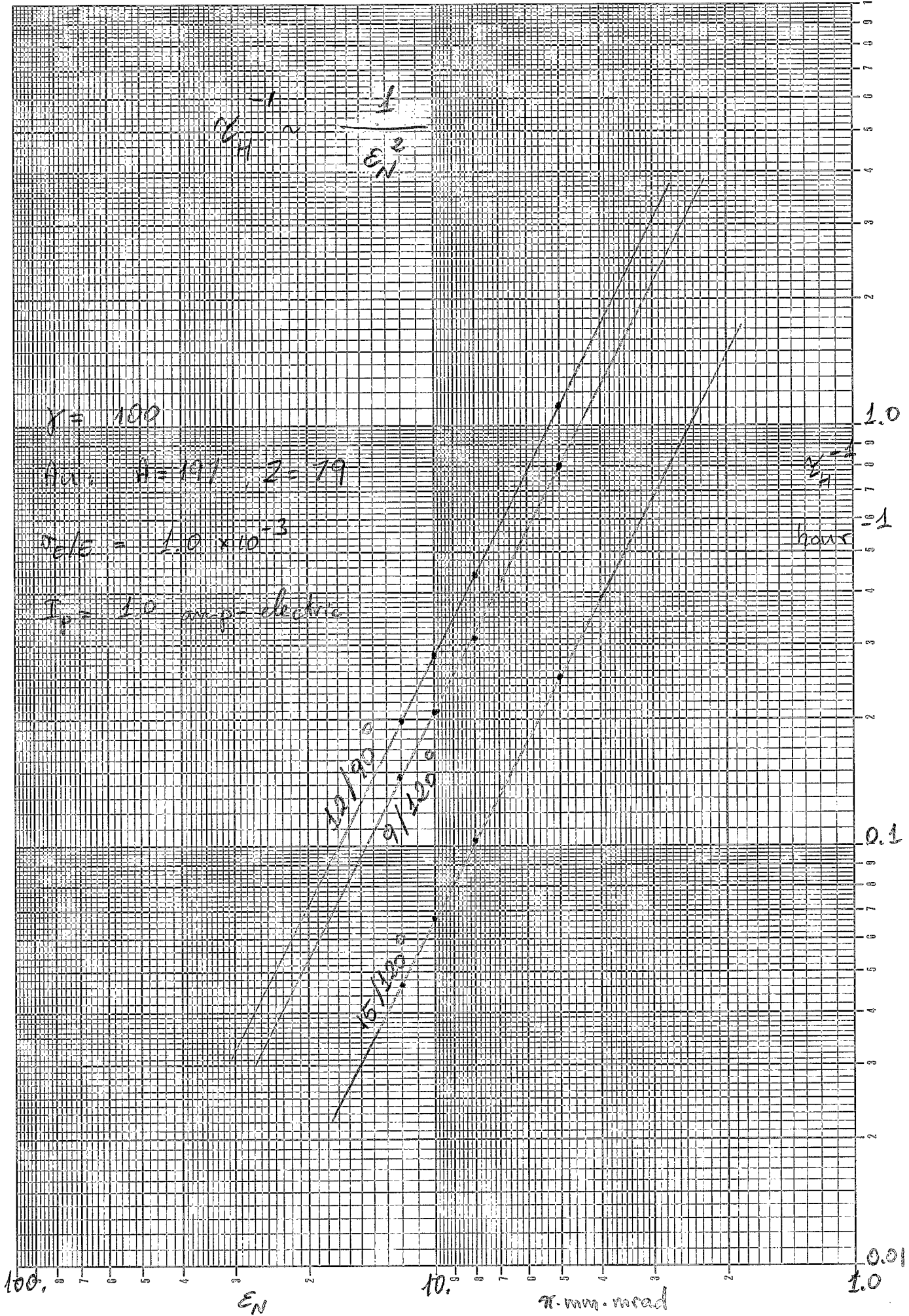
$I_p = 1.0$  amp-electron

$\sigma_E^{-1}$   
hour<sup>-1</sup>

0.1

0.01







$$\tau_H^{-1} \sim \frac{1}{\sigma_E/E}$$

$\gamma = 100$

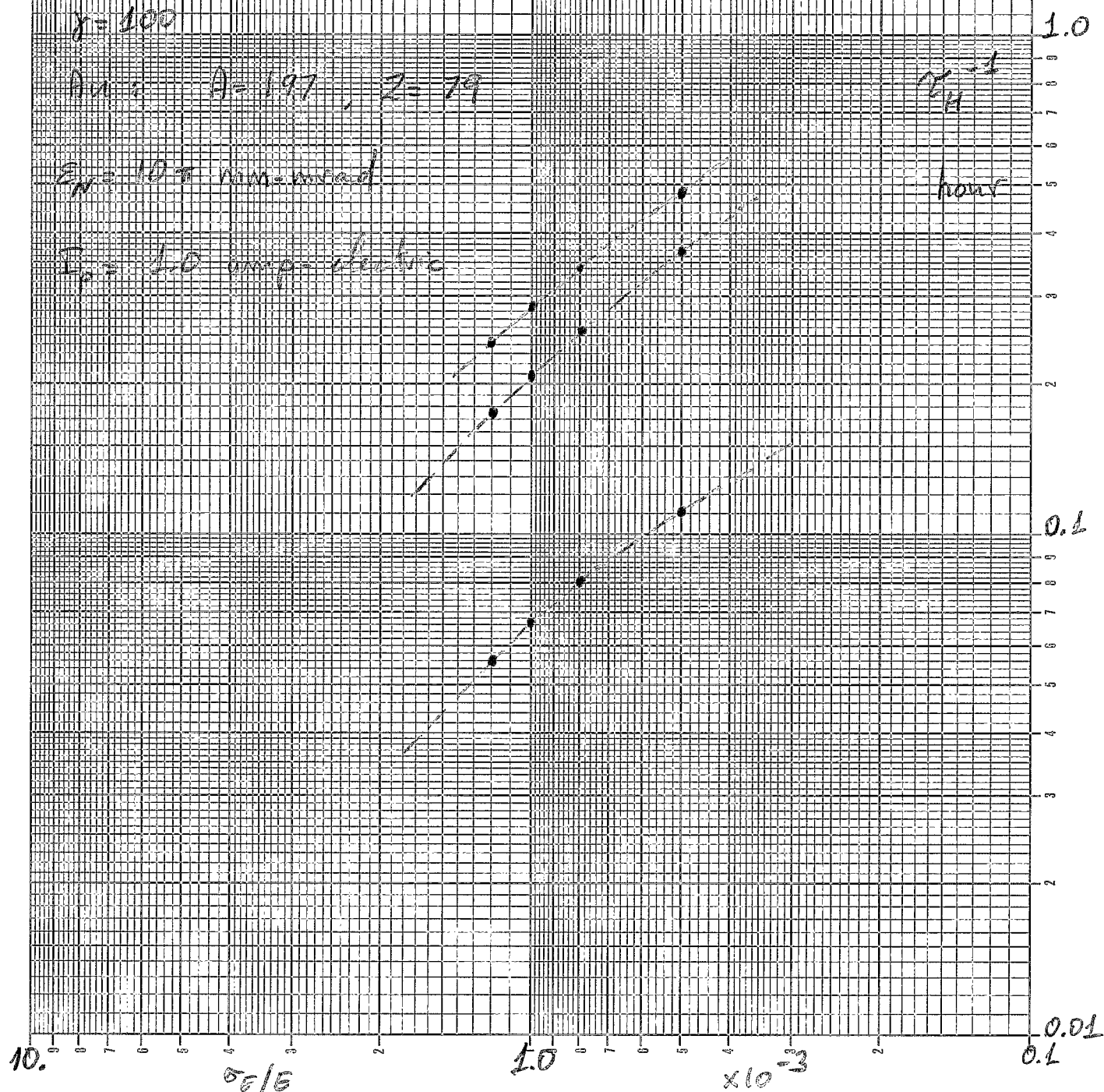
Amplitude  $A = 197$ ,  $Z = 79$

$E_H = 10 \pi$  mm-lead

$P_0 = 1.0$  amp-electric

$$\tau_H^{-1}$$

hours



	15/120°	12/90°	9/120°
$\tau_E^{-1}$	0.54 h <sup>-1</sup>	0.50 h <sup>-1</sup>	0.37 h <sup>-1</sup>
$\tau_H^{-1}$	0.11	0.48	0.37

$$\epsilon_N = 10 \pi \text{ mm} \cdot \text{mrad} \quad \sigma_E/E = 0.5 \times 10^{-3}$$

$$\text{Au: } A=197, Z=79 \quad \gamma=100 \quad I_p = 1.0 \text{ Amp-electric}$$

$$\text{Define } \delta = \sigma_E/E \quad \rightarrow \quad I_p = NeZ/\sqrt{2\pi} \sigma_E$$

$$\tau_E^{-1} = \frac{1}{\delta} \frac{d\delta}{dt}$$

$$\tau_H^{-1} = \frac{1}{\epsilon_N} \frac{d\epsilon_N}{dt}$$

$\tau_E^{-1} = \frac{A_E I_p}{\epsilon_N \delta^3}$	$\tau_H^{-1} = \frac{A_H I_p}{\epsilon_N^2 \delta}$
---	---

$$A_E = 1.25 \tau_E^{-1}$$

$$A_H = 50. \tau_H^{-1}$$

with  $\tau_E^{-1}$  and  $\tau_H^{-1}$  given in the top table, provided  $I_p$  is in Amp-electric,  $\epsilon_N$  in  $\pi \cdot \text{mm} \cdot \text{mrad}$  and  $\delta$  in %.

Sum of the Diffusion Rates  $(\chi^{-1} \approx 0)$

$$\chi^{-1} = \frac{A_E I_P}{E_N^2 \delta} \left( \frac{A_H}{A_E} + \frac{E_N}{\delta^2} \right) \quad \nabla$$

Integration for the beam dimensions

$$\dot{\delta} = \frac{A_E I_P}{E_N \delta^2} \quad \text{and} \quad \dot{E}_N = \frac{A_H I_P}{E_N \delta} \quad (1)$$

from which

$$S = 2 \frac{A_E}{A_H} E_N - \delta^2 = \text{invariant} \quad \nabla$$

The system (1) can be reduced down to one equation

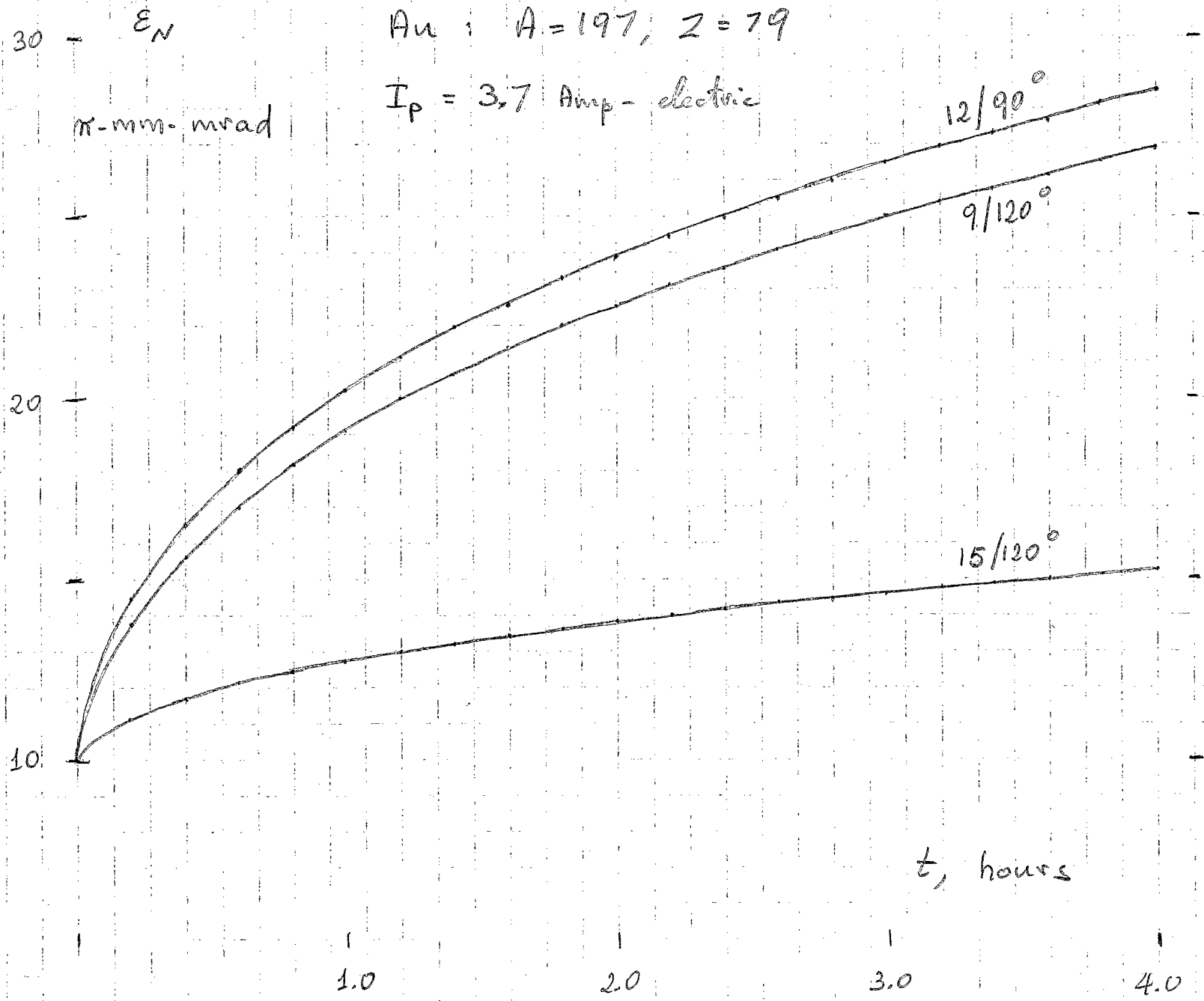
$$\dot{\delta} = \frac{2 A_E^2 I_P / A_H}{\delta^2 (\delta^2 + S)}$$

This can be easily integrated analytically  $\nabla$

$$\gamma = 100$$

$$\text{Au} : A = 197, Z = 79$$

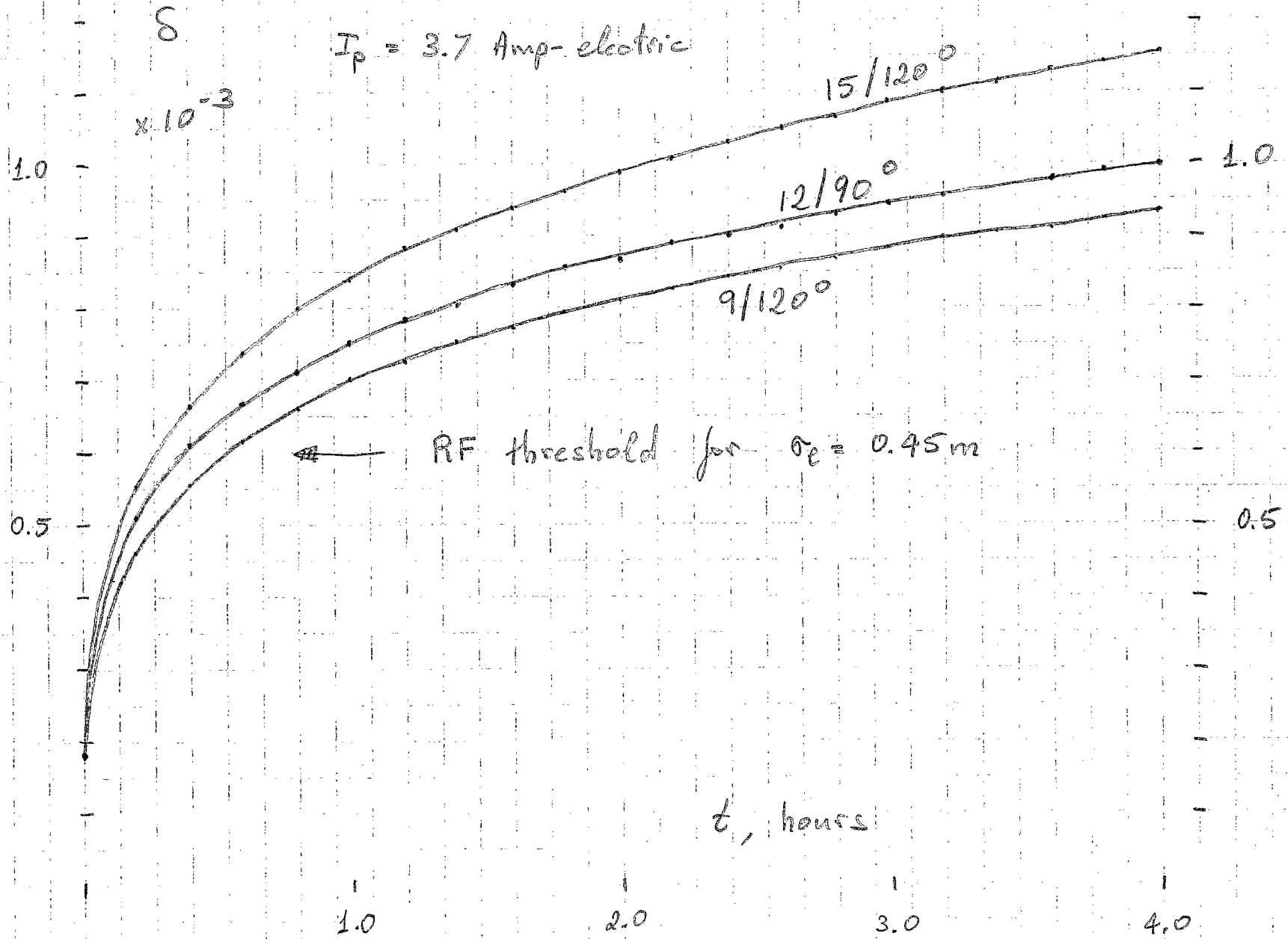
$$I_p = 3.7 \text{ Amp-electric}$$



$$\gamma = 100$$

Au:  $A = 197$ ,  $Z = 79$

$I_p = 3.7$  Amp-electric

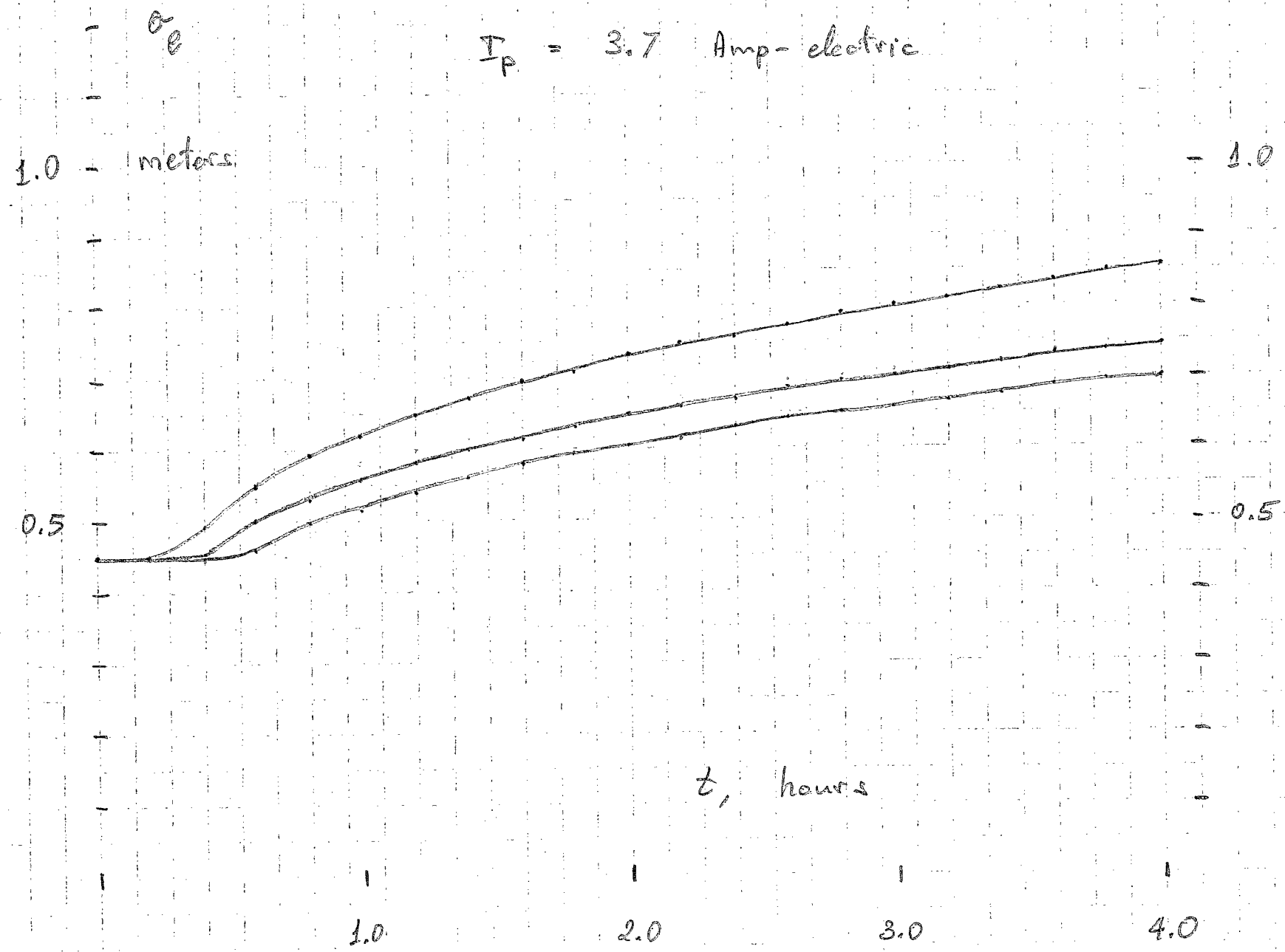


rms bunch length

$$\gamma = 100$$

$$Au : A = 197, Z = 79$$

$$I_P = 3.7 \text{ Amp-electric}$$





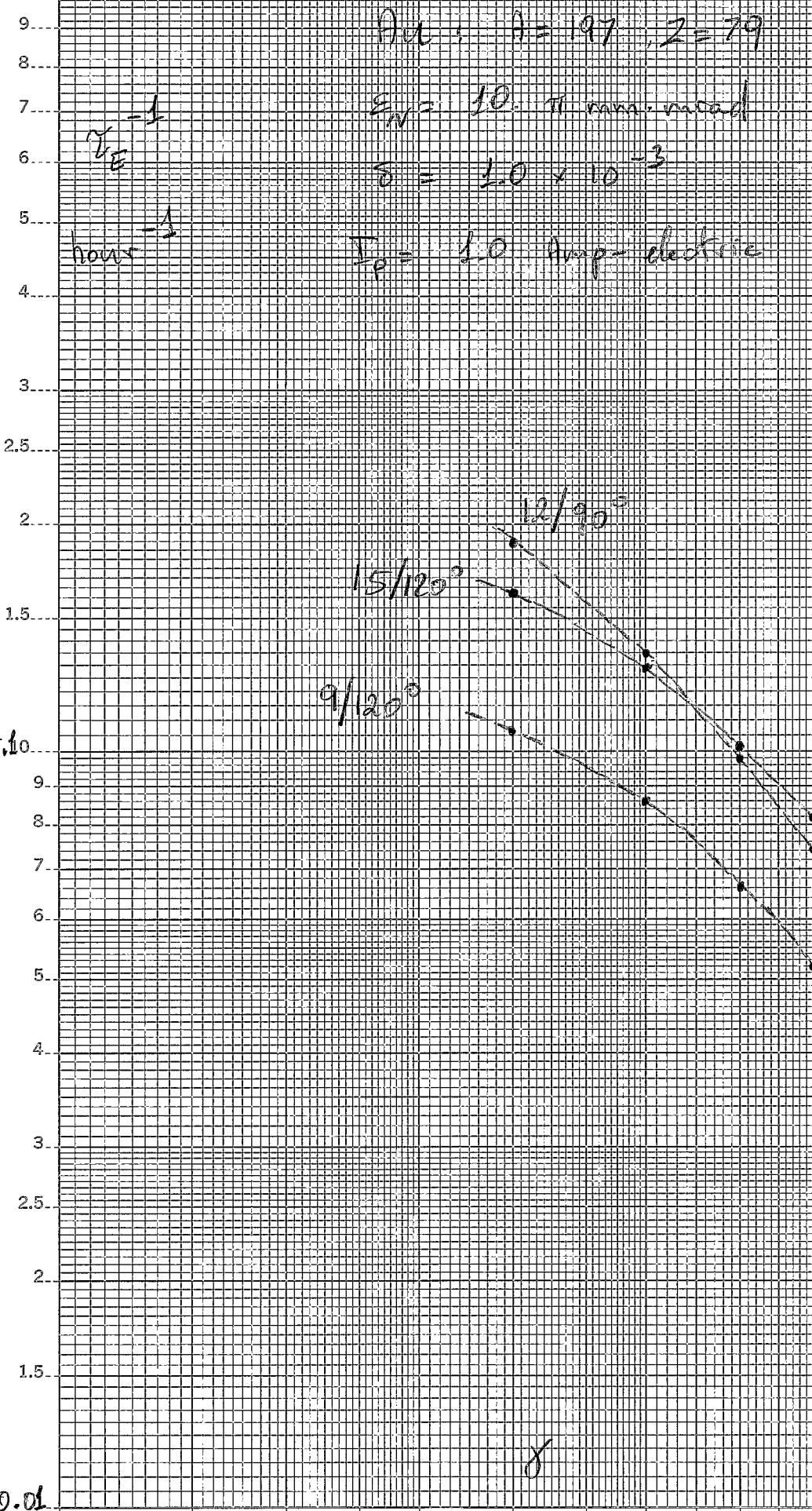
	2 hours - Operation Mode			4 hours - Operation Mode		
	9/120°	12/90°	15/120°	9/120°	12/90°	15/120°
Average luminosity $\times 10^{27} \text{ cm}^{-2} \text{ s}^{-1}$	0.52	0.55	0.80	0.47	0.44	0.73
Normalized Emittance $\pi \cdot \text{mm} \cdot \text{mrad}$	22.5	23.8	13.8	26.9	28.6	15.3
Rms Energy Spread $\times 10^{-3}$	0.81	0.87	0.98	0.94	1.00	1.16

Beam-Beam Tune Shift is the same for all cases

$$\Delta \nu_{\text{BB}} = 0.0026 \quad (\text{initial})$$



10



46 7080

LOGARITHMIC 2 X 1 CYCLES  
KEUFFEL & ESSER CO. MADE IN U.S.A.



0.01

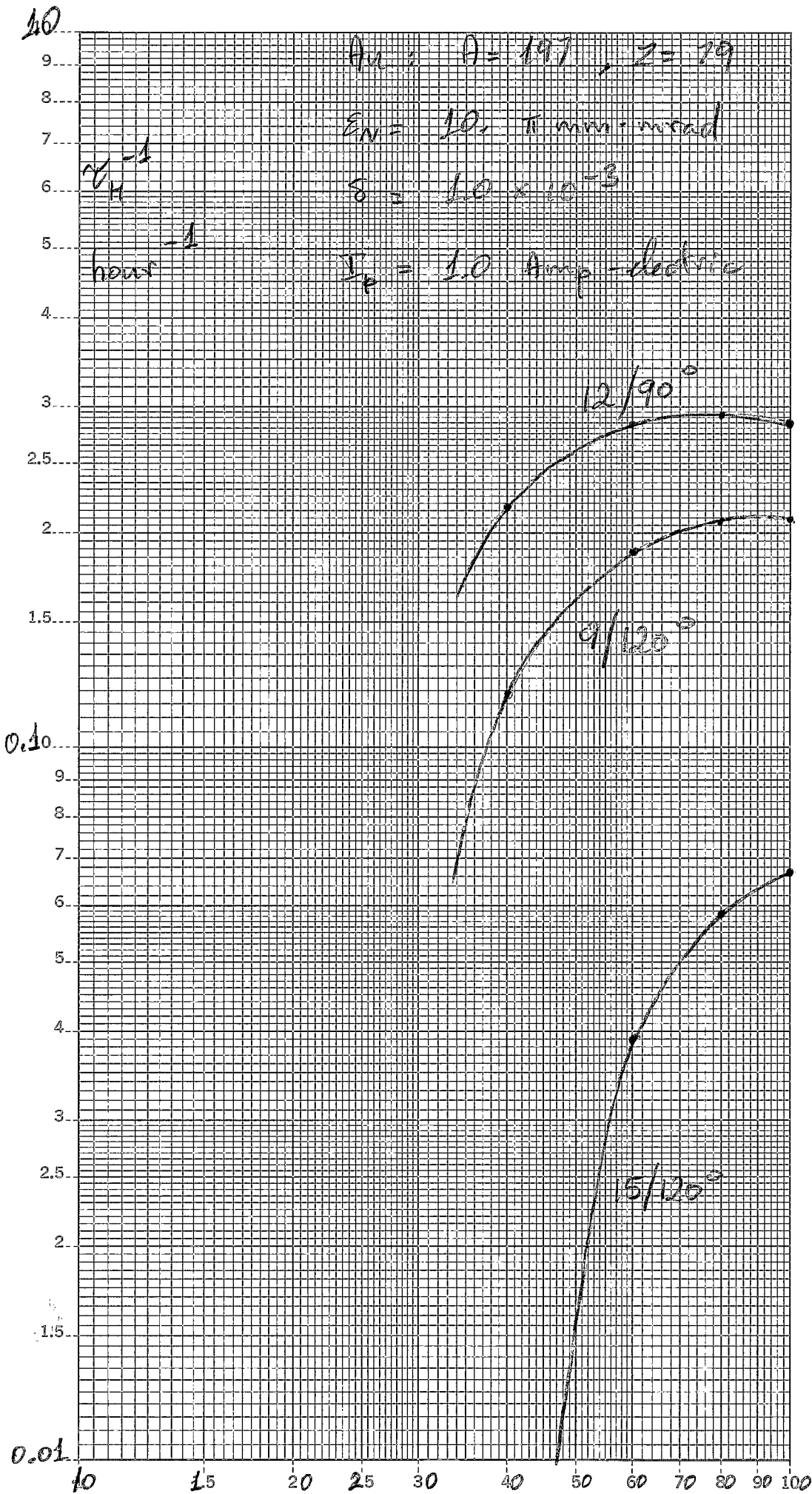
10 15 20 25 30 40 50 60 70 80 90 100

$\delta$

46 7080

LOGARITHMIC 2 X 1 CYCLES  
KEUFFEL & ESSER CO. MADE IN U.S.A.

KE



$B_g = 839.5$ T.m	$15/120^\circ$	$12/90^\circ$	$9/120^\circ$
<b>Lattice:</b>			
Cell length, m	23.7	29.6	39.5
$\beta_{max}$ , m	49.8	49.8	82.8
$\eta_{max}$ , m	0.70	1.54	1.82 (1.97)
$\gamma_T$	39.0	25.1	26.9 (23.6)
$\nu_{H,V}$ approx.	50. (30.0)	38. (18.0)	38. (17.8)
Chromaticity	69. (49.3)	43. (22.8)	49. (28.8)
<b>Dipoles:</b>			
Field, T	3.3	3.05	5.0
Length, m	2.91	10.70	4.36
Sagitta, cm	3.1	5.2	1.4
Radius of Curv., m	254.4	275.4	158.3
Number	180	144	216
<b>Quadrupoles:</b>			
Gradient, KG/cm	6.8	4.4	5.4
Length, m	1.90	1.90	1.40
Number	180	144	108

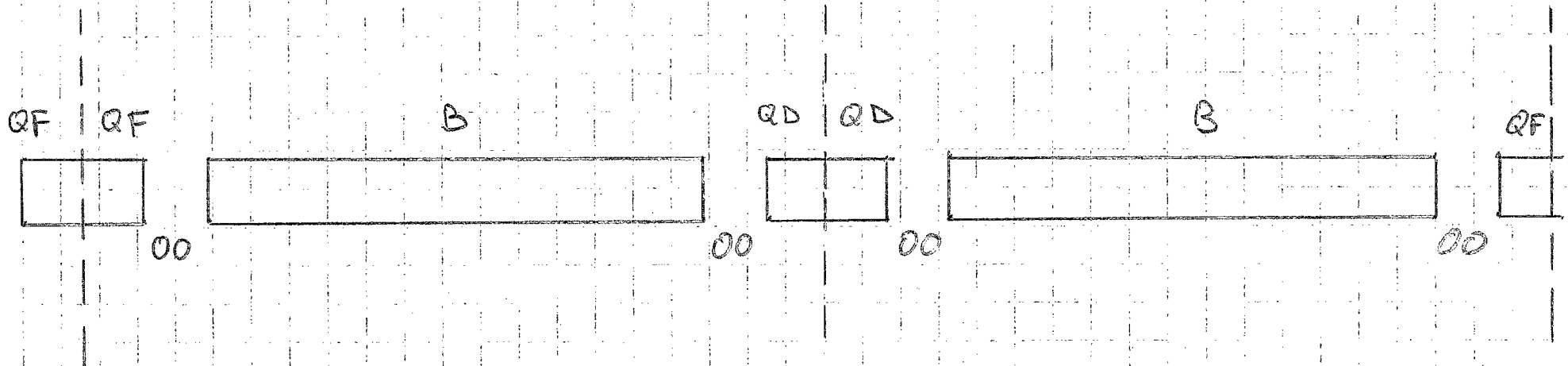
## Comments to the previous Table:

1. The assumed rigidity  $BP = 839.5 \text{ T}\cdot\text{m}$  is equivalent to  $100 \text{ GeV}/A$  for gold.
2. The transition energy value  $\gamma_T$  has been obtained by multiplying the contribution of the arcs by a factor 1.34 to take into account the long-straight sections.
3. The numbers in brackets for tune and chromaticity are the exact contribution from the regular cells. A factor of 20 has been added to them to take into account the expected contribution from the long straight sections.
4. There are 24 extra special dipoles for the dispersion killing insertions. There are also 24 more special dipoles to bring the two beams to collide either head-on or at an angle.
5. The quadrupoles in the long straight sections have not been included in the list.

15/120° Lattice

B, dipole: length = 7.906637 m  
Field = 3.3 Tesla

00, drift: length = 1.02109 m



QF, QD, half-quad: length = 0.95 m

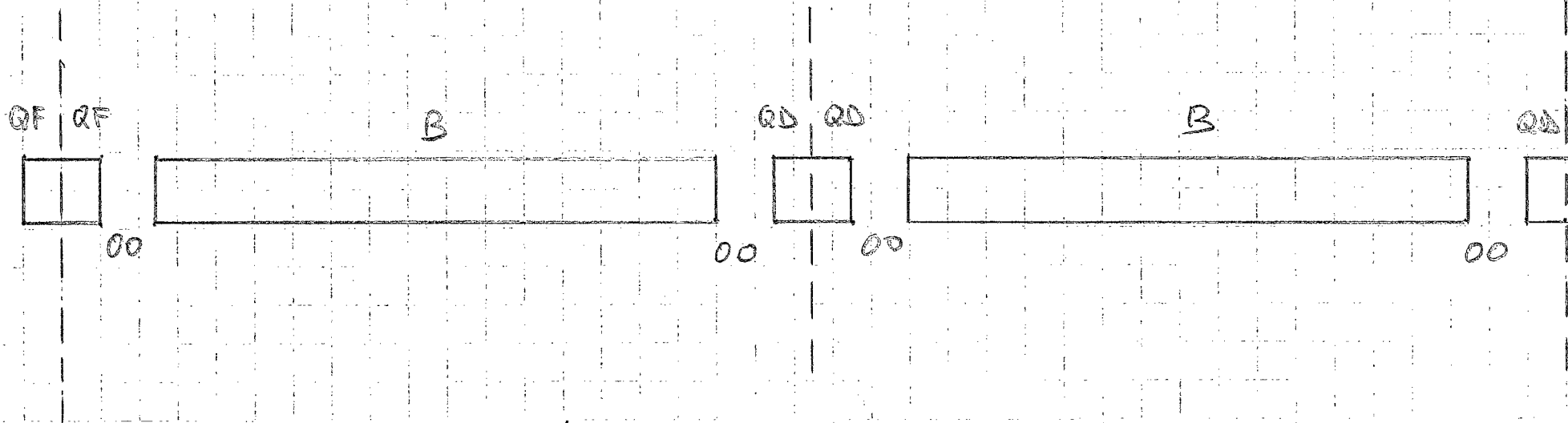
$B'/B_p = 0.081449 \text{ m}^{-2}$  for QF  
 $-0.081386$  for QD

Total Cell Length = 23.5976 m

12/90° Lattice

B, dipole : length = 10.7 m  
Field = 3.05 Tesla

00, drift : length = 1.1055 m



QF, QD, half-quad : length = 0.95 m

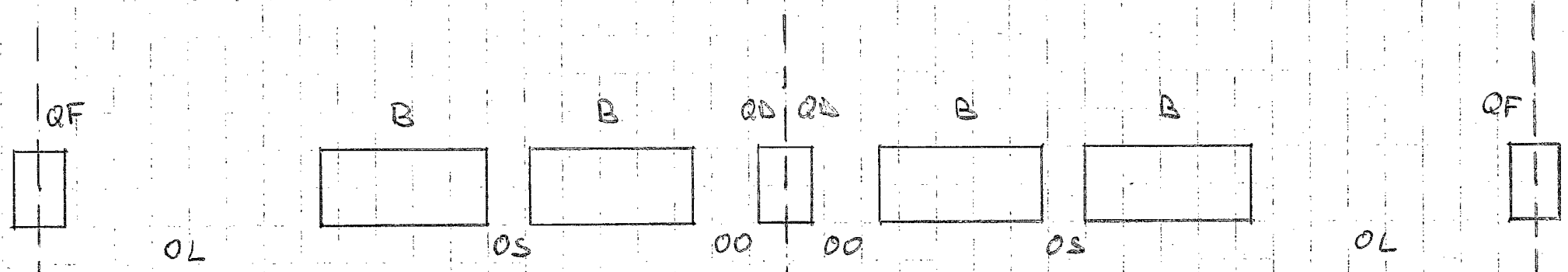
$B'/B\rho = 0.052588 \text{ m}^{-2}$  for QF  
 $-0.052513$  for QD

Total Cell length = 29.6221 m

# 9/120° Lattice

Drifts: OL length = 6.66 m  
 OS = 1.15  
 OO = 1.8058

B, dipole : length = 4.36 m  
 Field = 5.0 Tesla



QF, QD, half-quad : length = 0.70 m

$$B'/B\rho = \begin{matrix} 0.063823 \text{ m}^{-2} & \text{for QF} \\ -0.063613 & \text{for QD} \end{matrix}$$

Total Cell Length = 23.6976 m

	15/120°	12/90°	9/120°	
$\sigma_T$	39.0	25.1	26.9	23.6
Accel. Period	30 sec	30 sec	5 min	
Energy Gain /turn	38.1 keV/A	38.1	3.81	
Peak Voltage for Acceleration	1.0 MV	1.0	0.5	
$\rho_s$	5.45°	5.45°	1.09°	
Bucket Area :				
inject.	0.66 eV/A-sec	0.72	0.52	0.50
top energy	5.7	3.5	2.7	2.3
Bucket Height:				
inject.	± 0.24 %	0.26	0.19	0.18
top energy	0.24	0.15	0.11	0.10
Peak Voltage in Storage Mode at 100 GeV/A	1.0 MV	2.6	2.3	3.0
$\nu_s$ @ 100 GeV/A	0.0005	0.0013	0.0013	

The Bucket Area and Height given in this Table are for Stationary Bucket (no Acceleration)



	15/120°	12/190°	9/120°	
			A	B
$\delta_T$	39.0	25.1	26.9	23.6
$S = 0.2 \text{ eV/A-sec}$				
character. time	31.8 msec	17.7	52.5	44.1
bunch length	$\pm 0.40 \text{ nsec}$	0.54	0.44	0.48
bunch spread	$\pm 0.51 \%$	0.59	0.67	0.70
$S = 0.5 \text{ eV/A-sec}$				
character. time	31.8 msec	17.7	52.5	44.1
bunch length	$\pm 0.63 \text{ nsec}$	0.85	0.70	0.76
bunch spread	$\pm 0.80 \%$	0.93	1.06	1.10
Betatron Emittance @ $\delta = \delta_T$ 95% of beam	0.39 $\pi \cdot \text{mm} \cdot \text{mrad}$	1.14 $\pi \cdot \text{mm} \cdot \text{mrad}$	1.00 $\pi \cdot \text{mm} \cdot \text{mrad}$	1.14 $\pi \cdot \text{mm} \cdot \text{mrad}$

Shown here are the beam bunch dimensions at transition energy crossing -

# BEAM DIMENSIONS

	15/120°	12/90°	9/120°	
			A	B
Betatron Emittance;	$\epsilon_N = 15.3$	28.6	26.9	
(95%) injection transition top energy	1.23 $\pi$ mm-mrad 0.39 0.14	2.29 1.14 0.26	1.00 0.25	1.14
$\delta$ , rms $\times 10^{-3}$ ;				
injection transition top energy	1.16 3.27 1.16	1.00 3.80 1.00	4.33 0.94 0.94	4.49
$\beta_{max}$ , m $\eta_{max}$ , m	49.8 0.70	49.8 1.54	1.82 82.8	1.97
$6 \times \sigma_v$ : mm				
injection transition top energy	$\pm 19.2$ 10.8 6.5	$\pm 26.2$ 18.5 8.8	22.3 32.8 11.1	23.8
$6 \times \sigma_H$ : mm				
injection transition top energy	$\pm 19.8$ 17.5 8.1	$\pm 27.8$ 39.7 12.8	34.4 52.3 15.1	34.6 58.2 15.7

Assume a 2/3 - rule for the good field region as compared to the coil i.d.

Lattice

Coil i.d.

15/120°

6.0 cm

12/90°

12.0 cm

9/120° : A  
B

15.0 cm

17.5 cm

# SEXTUPOLES

9/120°

$B_g = 839.5 \text{ T-m}$

15/120°

12/90°

A

B

$\eta_F$	0.70	1.55	1.82	1.97
$\eta_D$	0.29	0.75	0.65	0.80
$\beta_{max}$	49.8	49.8	82.8	
$\beta_{min}$	3.8	8.8	6.3	
no. of sextupoles	2x90	2x72	2x54	
natural chromat.	69	43	49	
length, $l_s, m$	1.9	1.9	1.7	
$B''l_s/B_g, m^{-2}$				
F	0.276	0.097	0.076	0.070
D	0.718	0.235	0.228	0.185
Bore Radius, cm	3.0	6.0 (3.81)	(6.35)	
Pole Tip Field at QD, KG	1.43	1.88 (0.76)	(2.27)	(1.84)

$$|Z/n| \approx 3 \frac{E/n}{e I_p} \left( \frac{\sigma_E}{E} \right)^2 \frac{A}{Z}$$

$$I_p = 3.7 \text{ Amp-electric}$$

$$\sigma_E/E = 0.6 \times 10^{-3}$$

$$\text{Gold : } A = 197, \quad Z = 79$$

$$E = 100.9308 \text{ GeV/A}$$

Lattice

$|Z/n|$

15/120°

41.9 ohm

12/90°

110.2

9/120° A:

95.5

B:

125.6

# Closed Orbit Analysis

For 98% confidential level

$$\hat{y} = 2.1 \sqrt{M \bar{\beta} \beta_{\max}} \langle \psi \rangle$$

	Horizontal	Vertical
Dipole	$\theta \langle \Delta B/B \rangle$	$\theta \langle \Delta \theta_{\text{rot}} \rangle$
Quadrupole	$\kappa l_q \langle \Delta y \rangle$	$\kappa l_q \langle \Delta y \rangle$

$\theta$ , bending angle

$$\langle \Delta B/B \rangle, \text{ rms field error} = 10^{-4}$$

$$\langle \Delta \theta_{\text{rot}} \rangle, \text{ rms roll angle} = 0.1 \text{ mrad}$$

$$\kappa = B'/B\rho$$

$l_q$ , quadrupole length

$$\langle \Delta y \rangle, \text{ rms lateral displacement} = 0.1 \text{ mm}$$

# Max. Expectation of Closed Orbit Distortion

	15/120°	12/90°	9/120°
$\bar{\beta}$	26.8	29.3	44.6
$\beta_{max}$ m	49.8	49.8	82.8
M:			
dipoles	180	144	216
quads	180	144	108
$\theta$ msad	31.1	38.9	25.9
$Kla$ m <sup>-1</sup>	0.1545	0.0999	0.0892
$\langle y \rangle$ :    mm			
dipole	1.6	1.8	2.4
quad	8.0	4.8	5.8
$\langle y \rangle_{total} =$			
$= \sqrt{\langle y^2 \rangle_0 + \langle y^2 \rangle_a}$	8.2	5.2	6.2