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Crude Estimate Of <1> vs. Lattice Choice =Loss Rates Only=

G. R. Young

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Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

DISCLAIMER

RHIC-PG-25

CRUDE ESTIMATE OF $\langle l \rangle$

vs Lattice Choice

= Loss Rates Only =

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We make some "first try" comparisons of luminosity lifetime for some of the different lattices suggested. (see RHIC-PG-10 and RHIC-PG-13).

To greatly simplify this first attempt, we do the following

<u>Fix</u> the following parameters

$N = 1.87 \times 10^9$/bunch

$\gamma = 100$

Au

$B = 57$ bunches

$\epsilon_N = 10\pi$ mm-mrad (H & V)

(AR's "6σ emittances")

so $\sigma^* = \sqrt{\dfrac{\frac{\epsilon_N}{\beta\gamma}\cdot\beta^*}{6\pi}}$

$f_{rev} = 78{,}197$ Hz

Pick three crossing angles: $\alpha = 0, 2, 10$ mrad.

Fix $\sigma_L = 90$ cm (probably too large, but all lattices will be penalized the same factor if σ_L is decreased)

Perhaps this emittance choice is unfair to the stronger focussing lattice, but for this comparison it was felt to be best to concentrate just on the different values of diffusion rates resulting from changing $\bar{\beta}$, $\bar{\eta}$.

Clearly, more effort can be expended here. I suggest at least the following

1) more discussion with experimenters on how ~~long~~ a fill should last. Tom Ludlam's comments on operator tuning and experimenter calibration after each fill at the ISR need to be considered

2) Consideration of smaller emittances $\epsilon_N \sim 4\pi$ mm m rad ? The diffusion rates are not yet catastrophic at this point.

3) Performing a calculation which takes into account the decrease in loss rates with time. The present calculation only uses the initial loss rates, and so is unecessarily pessimistic in that sense

4) Arguing expense for stronger focussing vs expense for faster kickers to load more bunches into the collider vs expense for a larger booster (or higher peak field) to get better final stripping efficiency for Au out of the booster,

Luminosity

$$L = \frac{N^2 B\, f_{rev}}{4\pi\, \sigma_V^* \sigma_H^*\, f}$$

$$f = \sqrt{1+\rho^2}$$

$$\rho = \frac{\alpha\, \sigma_L}{2\, \sigma_H^*}$$

Use $N = 1.87 \times 10^9$ /bunch ($\gamma = 108.35$ 100 GeV/A Au)

$\sigma_L = 90\,cm$ booster limited and IBS limited

$$\sigma^*_{V,H} = \sqrt{\frac{\frac{\epsilon_{N_{H,V}}}{\beta\gamma}\, \beta^*_{H,V}}{6\pi}}$$

$B = 57$

$f_{rev} = 78,197$ Hz

Compare cases for $\alpha =$ 0 mrad head on

2 mrad smallangle, A vs A'

10 mrad small diamond

Compare L_0 and $\langle L \rangle$ for 1 hour, using

$$L = L_0\, e^{-t/\tau_L}$$

τ_L^{-1} = luminosity loss rate

Effect of different lattices $\bar{\beta}, \bar{\eta}$ $\gamma_t \sim \sqrt{R/\bar{\eta}}$

$R \sim 610.17$ meters

$\gamma_t \lesssim 25$ GeV avoids transition for protons $\therefore$ Unbunched protons not limited by passing thru γ_t.

Luminosity decay time

(RHIC-PG-21)

$$L = \frac{N^2 B f_{rev}}{4\pi \sigma_V^* \sigma_H^* f}$$

Calculus $\longrightarrow$ $$-\frac{1}{L}\frac{dL}{dt} = \frac{1}{\sigma_V^*}\frac{d\sigma_V^*}{dt} + \frac{1}{\sigma_H^*}\frac{d\sigma_H^*}{dt} + \frac{1}{f}\frac{df}{dt}$$

where $$\frac{1}{f}\frac{df}{dt} = \frac{\rho^2}{f^2}\left[\frac{1}{\sigma_L}\frac{d\sigma_L}{dt} - \frac{1}{\sigma_H^*}\frac{d\sigma_H^*}{dt}\right]$$

Assume ① bunch length $\propto$ energy spread

so $$\frac{1}{\sigma_L}\frac{d\sigma_L}{dt} = \tau_E^{-1}$$

② full H, V coupling (seconds, ie $\approx 10^5$ revolutions)

so $$\frac{1}{\sigma_V^*}\frac{d\sigma_V^*}{dt} = \frac{1}{\sigma_H^*}\frac{d\sigma_H^*}{dt} = \left(\tau_H^{-1} + \tau_V^{-1}\right)$$

Then

$$-\frac{1}{L}\frac{dL}{dt} \equiv \tau_L^{-1} = \frac{\rho^2}{f^2}\tau_E^{-1} + \left(2 - \frac{\rho^2}{f^2}\right)\left(\tau_H^{-1} + \tau_V^{-1}\right)$$

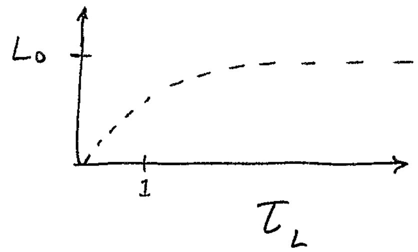
Headon, $\rho = 0$, $f = 1$

$$\tau_L^{-1} = 2\left(\tau_H^{-1} + \tau_V^{-1}\right)$$

Time average Luminosity

$$-\frac{1}{L}\frac{dL}{dt} = \tau_L^{-1} \quad \text{or} \quad L = L_0\, e^{-t/\tau_L}$$

Then $\langle L \rangle_{1\,hour} = \dfrac{\int_0^1 L_0 e^{-t/\tau_L} dt}{\int_0^1 dt} = L_0 \tau_L (1 - e^{-1/\tau_L})$ $\quad \tau_L$ [hours]



τ_L = .1 hr	$\langle L \rangle$ = .1 L_0
1 hr	.63 L_0
5 hr	.91 L_0

3 cases $\gamma = 100$ (See RHIC-PG-10, RHIC-PG-13) For now, $\epsilon_N = 10\pi$ mm mrad (H & V)

$\bar{\beta}$	30 m	45 m	40 m
$\bar{\eta}$	0.5 m	1.5 m	2 m
$\sim\gamma_t$	35	20.2	17.5
τ_E^{-1} (h^{-1})	.0816	.0502	.0483
τ_H^{-1} (h^{-1})	.0669	.2875	.5681
τ_V^{-1} (h^{-1})	-.0147	-.0135	-.0116
$\alpha = 0$ $\quad \tau_L^{-1}$ (h^{-1})	.1044	.548	1.113
$\alpha = 2$ $\quad \tau_L^{-1}$ (h^{-1})	.1326	.333	.625
$\alpha = 10$ $\quad \tau_L^{-1}$ (h^{-1})	.1337	.3246	.6058

$\beta^*_{H,V}$ = 2m

$\sigma^*_{H,V}$ = .01826 cm

σ_L = 90 cm

α = 2 mrad $\quad \rho = 4.93$

$f = 5.03$

$\frac{\rho^2}{f^2} = .96$

α = 10 mrad $\quad \rho = 24.64$

$f = 24.66$

$\frac{\rho^2}{f^2} = .998$

		γ_t	35	20.2	17.5
$\alpha = 0$		L_0	$3.7 \; 10^{27}$	$3.7 \; 10^{27}$	$3.7 \; 10^{27}$
		$\langle L \rangle$	$3.5 \; 10^{27}$	$2.8 \; 10^{27}$	$2.2 \; 10^{27}$
$\alpha = 2$ mrad		L_0	$7.4 \; 10^{26}$	$7.4 \; 10^{26}$	$7.4 \; 10^{26}$
		$\langle L \rangle$	$6.9 \; 10^{26}$	$6.3 \; 10^{26}$	$5.5 \; 10^{26}$
$\alpha = 10$ mrad		L_0	$1.5 \; 10^{26}$	$1.5 \; 10^{26}$	$1.5 \; 10^{26}$
		$\langle L \rangle$	$1.4 \; 10^{26}$	$1.3 \; 10^{26}$	$1.1 \; 10^{26}$

$\boxed{\text{1 hour run}}$ with collider assumed

$\rightarrow$ For 1 hour runs, little difference

	$\sim\gamma_t$	35	20.2	17.5
$\alpha=0$	L_0	3.7 10^{27}	3.7 10^{27}	3.7 10^{27}
	$\langle L\rangle$	2.9 10^{27}	1.3 10^{27}	6.6 10^{26}
$\alpha=2$ mrad	L_0	7.4 10^{26}	7.4 10^{26}	7.4 10^{26}
	$\langle L\rangle$	5.4 10^{26}	3.6 10^{26}	2.3 10^{26}
$\alpha=10$ mrad	L_0	1.5 10^{26}	1.5 10^{26}	1.5 10^{26}
	$\langle L\rangle$	1.1 10^{26}	7.4 10^{25}	4.7 10^{25}

$\boxed{\text{5 hour run}}$ with collider assumed

marked dependence, esp. for head on

What happens if we blow up the emittance by $\times 2$ to $\epsilon_{N_{H,V}} = 20\pi$ mm mrad?

Compare Head-on, 10 mrad cases

Rule of thumb (see RHIC-PG-13) (leave $\frac{\sigma_E}{E}$ at 10^{-3})

$$\tau_E^{-1} \sim \frac{1}{\epsilon_N}$$

$$\tau_H^{-1} \sim \frac{1}{\epsilon_N^2}$$

$\tau_Y^{-1} \; ? \; \frac{1}{\epsilon_N}$ (small, anyway)

$\sigma^*_{H,V} = .0258$ cm

$\sigma_L = 90$ cm

$\alpha = 10$ mrad

$\rho = 17.44$

$f = 17.47$

$\frac{\rho^2}{f^2} = .996$

$\sim \gamma_t$	35	20.2	17.5
τ_E^{-1}	.0408	.0251	.0242
τ_H^{-1}	.0167	.0719	.1420
τ_Y^{-1}	$-.0074$	$-.0068$	$-.0058$
$\alpha = 0$ τ_L^{-1}	.0186	.1302	.2724
$\alpha = 10$ mrad τ_L^{-1}	.04998	.0902	.1608

		$\sim\gamma_t$	35		20.2		17.5	
$\alpha=0$		L_0	1.9	10^{27}	1.9	10^{27}	1.9	10^{27}
		$\langle L\rangle_{1\,hour}$	1.88	10^{27}	1.78	10^{27}	1.66	10^{27}
		$\langle L\rangle_{5\,hour}$	1.81	10^{27}	1.40	10^{27}	1.04	10^{27}
$\alpha=10\,mrad$		L_0	1.1	10^{26}	1.1	10^{26}	1.1	10^{26}
		$\langle L\rangle_{1\,hour}$	1.07	10^{26}	1.05	10^{26}	1.02	10^{26}
		$\langle L\rangle_{5\,hour}$	9.7	10^{25}	8.8	10^{25}	7.6	10^{25}

At 1 hour, ~ no difference
At 5 hours, ~ 40% difference