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Luminosity Estimates For The Lighter Ions: Equal Mass Collisions C+C, S+S, Cu+Cu, I+I

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RHIC-PG-22

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(see RHIC - PG-10)

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RHIC-PG-

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Luminosity for Light Beams : Equal Mass Collisions

We can have $3 \times (6 \times 10^8) = 1.8 \times 10^9$ ions/bunch for $^{197}Au^{79+}$ ions in the collider, according to Messrs. Ruggiero, Parzen & Lee. Beyond this, it evidently becomes difficult to handle intrabeam scattering for the bunch lengths we are considering. As intra beam scattering is a small angle Coulomb scattering problem, it must scale as $(Z^4/A^2)^{-1}$. We can thus handle the following # ions/bunch for our other ions, assuming this scaling:

Ion	Ions / bunch in collider
$^{197}Au^{79+}$	1.8×10^9
$^{127}I^{53+}$	3.7×10^9
$^{63}Cu^{29+}$	1.0×10^{10}
$^{32}S^{16+}$	2.8×10^{10}
$^{12}C^{6+}$	2.0×10^{11}
$^{1}H^{1+}$	1.8×10^{12}

Compare these to the values calculated for a tune depression $\Delta\nu = 0.1$ in booster.
(use $N_B/\epsilon_N = \beta\gamma^2 \dfrac{\pi B_f}{2 r_0} \dfrac{\Delta\nu}{F} \dfrac{A}{Q^2}$; set $B_f = 0.5$, $F \sim 1$, $\Delta\nu = 0.1$; $r_0 = 1.535 \times 10^{-18}$ m)

Ion	T/A	β	γ	N_B/ϵ_N ($\pi^{-1} m^{-1}$)	ϵ_N π mm mrad ($\epsilon_N = \beta\gamma \cdot 40\pi$ mm mrad)	N_B
$^{197}Au^{37+}$	5	.1032	1.00537	7.68×10^{14}	4.15	3.19×10^9
$^{127}I^{32+}$	7.8	.1286	1.00837	8.30×10^{14}	5.19	4.31×10^9
$^{63}Cu^{22+}$	11.6	.1564	1.01245	1.07×10^{15}	6.33	6.77×10^9
$^{32}S^{15+}$	14.1	.1720	1.01514	1.29×10^{15}	6.98	9.00×10^9
$^{12}C^{6+}$	16.5	.1858	1.01771	3.28×10^{15}	7.56	2.48×10^{10}

We took charge states for 2 stage tandem operation at 15 MV and assumed addition of the Heidelberg linac (see Fig 5, M Grieser et al IEEE NS- , (1983)) Before calculating the number of turns that would have to be injected into the booster from the tandem to reach these currents, we first estimate tandem performance for all ions in the standard set.

The tandem ion source can inject 200 μA (particle, as it makes 1^- ions for injection) for all our beams. Roy Middleton (U. Penn) has seen instances of 800 μA operation, but conditions for this are not yet reproducible, or even understood. Thus, we design for now with the 200 μA value.

We calculate based on 2 stage tandem operation @ 15 MV.

Ion	Terminal Charge State	% Abundance	High Energy End Final Energy	Exit Charge State	% Abundance	(200 μA injected) Current
^{12}C	6+	21.8	105	6+	100%	32.7
^{32}S	9+	34.3	150	15+	32.4	16.7
^{63}Cu	11+	26.6	180	22+	28.4	11.3
^{127}I	13+	20.8	210	32+	19.0	5.9
^{197}Au	14+	17.8	225	37+	16.0	4.3

The values in the last column are particle μA and include 75% transmission through the tandem. We can now calculate the number of turns that must be injected to reach the booster tune depression limit (assuming we have the linac in place)

Ion	Rotation τ (μsec)	N_B/turn	no. turns	Tandem Pulse length (μsec)
^{12}C	3.62	7.4 10^8	33.5	121.3
^{32}S	3.91	4.1 10^8	22.0	86.0
^{63}Cu	4.30	3.0 10^8	22.6	97.2
^{127}I	5.23	1.9 10^8	22.7	118.7
^{197}Au	6.52	1.5 10^8	21.3	138.9

All of these pulse lengths are well under the "standard" 250 μsec pulses being tried on the tandem. The number of turns to be injected is large ($\sim 20-30$) but this largely arises from assuming we can handle 1.8×10^9 Au ions/bunch in the collider from the standpoint of intrabeam scattering. If we have to drop back to 6×10^8 ions/bunch, then all the values for number of turns decrease by $\times 3$ to be in the range of $\sim 7-11$ turns, which should be feasible. We are using as booster acceptance of 40π mm mrad, and early emittance measurements on the tandem only yielded values of (unnormalized) $\epsilon = 1.8\pi$ mm mr for 100 MeV ^{16}O
$= 0.51\pi$ mm mr for 140 MeV ^{32}S
(See P Thieberger, M McKeown + H Wegner IEEE NS-30 #4, p2746 (1983))

The linac would only improve this, assuming no dilution. The number of turns to be injected would have to be then less than $\frac{40}{1.8} = 22.2$ for ^{16}O and less than $\frac{40}{0.51} = 78.4$ for ^{32}S. However, if we take into account the increase in $\beta\gamma$ through the linac, $\frac{(\beta\gamma)_{out}}{(\beta\gamma)_{in}} = 1.13$ for ^{16}O and $\frac{(\beta\gamma)_{out}}{(\beta\gamma)_{in}} = 1.8$ for ^{32}S, these values become 25.1 turns for ^{16}O and 141.1 turns for ^{32}S, which are in the range needed. Clearly, plenty of attention has to be paid to careful injection stacking into the booster to be sure the betatron phase space is loaded as efficiently as possible. One should also remark that the emittance measurements for the tandem beams are "1st pass" numbers and may well improve with further experience in setting source parameters.

To get an idea of luminosity that might be obtained with lighter beams, we calculate for the common magnet case on page 3 of note RHIC-PG-10. We use formulae (same ion in each beam)

$$L = \frac{N^2 B\ f_{rev}}{4\pi\ \sigma_V^* \sigma_H^*\ f}, \qquad \Delta\nu_{\text{Beam beam}} = \frac{N r_0\ \beta_V^* Z^2}{4\pi\ \sigma_V^* \sigma_H^* f\ \gamma A}$$

N = # ions/bunch, f_{rev} = 78.194 kHz. ϵ_N = 10π mm-mrad, $\beta^* = 1$m, $\sigma^*_{V \text{ and } H} = \sqrt{\frac{\epsilon_N/\beta\gamma \cdot \beta^*}{6\pi}}$ = 0.013 cm, $f = \sqrt{1 + \frac{\alpha\,\sigma_\ell}{2\sigma_H^*}}$ is 1 for $\alpha = 0$, head on collisions. Using N from 1st page, $B = 57$ bunches, we find

Ion	N	L (cm⁻² sec⁻¹)	$\Delta\nu_{BB}$
^{12}C	$2\ 10^{11}$	$8.4\ 10^{31}$	.040
^{32}S	$2.8\ 10^{10}$	$1.6\ 10^{30}$	.017
^{63}Cu	$1.0\ 10^{10}$	$2.1\ 10^{29}$	.0089
^{127}I	$3.7\ 10^{9}$	$2.9\ 10^{28}$	.0055
^{197}Au	$1.8\ 10^{9}$	$6.8\ 10^{27}$	.0038

$(\alpha = 0^\circ)$

where $\gamma = 108.35$ (100 GeV/A, 1amu = 931.5 MeV)

So even though the luminosities look stupendous, the beam beam tune shifts are too large. We expected this trouble because we scaled the ions/bunch from intrabeam scattering by Z^4/A^2 (Coulomb law Z, A dependence) while $\Delta\nu_{BB}$ only scales as Z^2/A.

A more sensible set of values is obtained by taking N determined from the booster space charge limit. Again using the common magnet scenario, we get the following

Ion	N	L ($cm^{-2} sec^{-1}$)	$\Delta\nu_{BB}$ ($\alpha = 0°$)
^{12}C	$2.5\ 10^{10}$	$1.3\ 10^{30}$	.0050
^{32}S	$9.0\ 10^{9}$	$1.7\ 10^{29}$	.0048
^{63}Cu	$6.8\ 10^{9}$	$9.7\ 10^{28}$	.0060
^{127}I	3.4×10^{9}	$2.4\ 10^{28}$	.0050
^{197}Au	1.6×10^{9}	$5.4\ 10^{27}$	.0033

(We used stripping efficiencies of 50% for Au, 80% for I and 100% for Cu & S out of the booster into the AGS. C is already fully stripped in the booster)

These are getting closer. If one insists on $\Delta\nu_{BB} \leq 0.003$, then the following table for N, L, $N_{booster}$ and # turns injected into the booster results.

($\alpha = 0°$)

Ion	$\Delta\nu_{BB}$	N	L	$N_{booster}$	# turns
^{12}C	.003	$1.50\ 10^{10}$	$4.7\ 10^{29}$	$1.50\ 10^{10}$	20.1
^{32}S	.003	$5.63\ 10^{9}$	$6.6\ 10^{28}$	$5.63\ 10^{9}$	13.8
^{63}Cu	.003	$3.37\ 10^{9}$	$2.4\ 10^{28}$	$3.37\ 10^{9}$	11.2
^{127}I	.003	$2.04\ 10^{9}$	$8.7\ 10^{27}$	$2.55\ 10^{9}$	13.4
^{197}Au	.003	$1.42\ 10^{9}$	$4.2\ 10^{27}$	$2.84\ 10^{9}$	19.0

(The number of turns for ^{12}C has always been anomalously large due to selecting charge state 6^+ in the tandem terminal. If 5^+ were selected, which has a 64% abundance (foil stripper) compared to 6^+ abundance of 21.8%, the number of turns injected into the booster could be relaxed by almost 3 — not exactly 3, as the linac exit energy would drop slightly as the injection energy from the tandem would drop from 105 MeV to 90 MeV (15 MV tandem).)

These luminosities for lighter ions are probably still large enough to fry eggs or detectors, so it is not clear whether great effort needs be expended to increase them. Input from experimenters will help clarify this, as the number of secondary particles in a C+C collision is much smaller than for Au+Au, but the scaling with mass is not clear.

We can calculate another table based on the septum magnet solution in RHIC-PG-10 page 3. Now much shorter bunches are use, $\sigma_\ell = 15$ cm for better L, but the #ions/bunch then must decrease. Following PG-10, use 6×10^8 /bunch for Au. Now use $\beta^* = 2$ m, $\gamma = 108.35$ (100 GeV/A), $\epsilon_N = 10\pi$ mm mrad, $\sigma_V{}^* = \sigma_H{}^* = 0.0175$ cm, $\rho = \frac{\alpha \sigma_\ell}{2\sigma_H{}^*} = 0.857$, $f = \sqrt{1+\rho^2} = 1.317$, $f_{rev} = 78.194$ kHz and $B = 171$ (ie find a fast kicker magnet). For N, use the values from the booster or the scaled (by Z^4/A^2) value based on intrabeam scattering for Au in the collider. For Au, I and Cu, the intrabeam scattering limit is smaller; for S and C the booster limit is smaller.

$\alpha = 2$ mrad — septum magnet

Ion	N /bunch	L (cm^{-2} sec^{-1})	$\Delta\nu_{BB}$
^{12}C	$2.5\ 10^{10}$	1.6×10^{30}	.0042 a little too big
^{32}S	$9.0\ 10^{9}$	2.1×10^{29}	.0040 a little too big
^{63}Cu	$3.3\ 10^{9}$	2.9×10^{28}	.0025
^{127}I	$1.2\ 10^{9}$	3.8×10^{27}	.0015
^{197}Au	6×10^{8}	9.5×10^{26}	.0011

The values for C and S are still a little large for $\Delta\nu_{BB}$.
Asking for $\Delta\nu_{BB} \le .003$ gives

Ion	N	L	$\Delta\nu_{BB}$
C	1.78 10^{10}	8.2 10^{29}	.003
S	6.75 10^{9}	1.2 10^{29}	.003

still "OK" in terms of L.

So, we conclude, based on the suggestions in RHIC-PG-10 for the bunched beams scenario, no RF stacking, that

a) for head on collision the beam-beam tune shift will catch us before the intrabeam scattering for the light ions, and we can relax the requested booster performance for light ions from the space charge limit

b) for small α crossing (2 mrad), intrabeam scattering limits heavy beam (Cu, I, Au) performance and beam beam tune shift still limits light ion performance

c) quite healthy luminosities are possible for C+C and S+S ($>10^{29}$ cm^{-2} sec^{-1}) and $L > 10^{28}$ cm^{-2} sec^{-1} is possible for Cu + Cu and I + I

It would be nice, once we settle on method of calculating intrabeam scattering, to make calculations for C, S, Cu and I to determine what Z and A scaling we should use.

To get some idea of luminosity at low energy, we calculate for 5 GeV/A ($\gamma = 6.368$) equal mass collisions. Now $\beta = .98759$, so for circumference 3833.8 m, $f_{rev} = 77.226$ kHz. Still use $\epsilon_N = 10\pi$ mm mrad, H and V.

For bunched beams head on, use $\beta^* = 1$ m, $\sigma^*_{V,H} = \sqrt{\frac{\epsilon_N/\beta\gamma \; \beta^*}{6\pi}} = .051$ cm

$f = 1$, $B = 57$ bunches. Using N/bunch from the table on page 5, we get

Ion	N/bunch	L ($cm^{-2}s^{-1}$)	$\Delta\nu_{BB}$
^{12}C	$1.50\ 10^{10}$	$3.0\ 10^{28}$	.0033
^{32}S	$5.63\ 10^{9}$	$4.3\ 10^{27}$	.0033
^{63}Cu	$3.37\ 10^{9}$	$1.5\ 10^{27}$	.0033
^{127}I	$2.04\ 10^{9}$	$5.6\ 10^{26}$	.0033
^{197}Au	$1.42\ 10^{9}$	$2.7\ 10^{26}$	.0033

$\alpha = 0$
5 GeV/A

For crossing at $\alpha = 2$ mrad, $\rho = \frac{\alpha \sigma_\ell}{2\sigma^*_H} = .294$, $f = \sqrt{1+\rho^2} = 1.042$, $B = 171$

$\beta^* = 2$ m so $\sigma^*_{H,V} = .0728$ cm

Ion	N/bunch	L ($cm^{-2}s^{-1}$)	$\Delta\nu_{BB}$
^{12}C	$1.78\ 10^{10}$	$6.0\ 10^{28}$	.0037
^{32}S	$6.75\ 10^{9}$	$8.7\ 10^{27}$	.0037
^{63}Cu	$3.3\ 10^{9}$	$2.1\ 10^{27}$	.0031
^{127}I	$1.2\ 10^{9}$	$2.7\ 10^{26}$	.0018
^{197}Au	$6.\ 10^{8}$	$6.9\ 10^{25}$	.0013

$\alpha = 2$ mrad
5 GeV/A

It might seem odd that the crossed beam luminosity is higher than the head on luminosity for C and S, but this is partly an artifact of the conditions assumed. For fixed Z, A and γ, if $\beta^*_H = \beta^*_V$ then the β^* dependence of $\Delta\nu_{BB}$ in the expression on page 4 divides out, and if one is limited by $\Delta\nu_{BB}$ then $N \propto f\, \Delta\nu_{BB}$ where $f = \sqrt{1+\rho^2}$. (Here, γ, Z, A, and ϵ_N are fixed.) Then

$$L \propto \frac{N^2 B}{f} \propto \frac{f B\, \Delta\nu_{BB}}{\beta^*}$$

. Since we have $f = 1$ for head on and 1.042 for $\alpha = 2$ mrad @ 5 GeV/A; $B = 57$ head on, but $B = 171$ for $\alpha = 2$ mrad; and $\beta^* = 1$ m head on, 2 m crossing then $L_{crossing} \approx 1.56\, L_{head\ on}$ evaluated at the same $\Delta\nu_{BB}$.

Fixing $\Delta\nu_{BB} = .003$ for ^{12}C, we get $L = 2.48\ 10^{28}$ head on and $L = 3.94 \times 10^{28}$ crossing, or a ratio of $1.59 : 1$.

For heavy beams the intrabeam scattering takes over and head on yields the highest L; the head on case also has a larger beam beam tune shift.