



BNL-101562-2014-TECH

RHIC/PG/19;BNL-101562-2013-IR

# Regular Cell And Dispersion Killer For The 120 degree Phase Advance Case

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December 1983

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**U.S. Department of Energy**

USDOE Office of Science (SC)

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REGULAR CELL AND DISPERSION KILLER  
FOR THE 120° PHASSE ADVANCE CASE

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(BNL, December 9, 1983)



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## Lattice for a Regular Cell

(Thin lenses approximation)

$$\left\{ \begin{array}{l} L, \quad \text{length of half-cell} \\ \theta, \quad \text{bending for half-cell} \\ \varphi, \quad \text{phase advance per half-cell} \end{array} \right.$$

$$\frac{B'_{12}}{B\varphi} = \frac{2 \sin \varphi}{L}$$

$$\left\{ \begin{array}{l} \beta_{\max} = \frac{L}{\sin \varphi} \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2} \\ \beta_{\min} = \frac{L}{\sin \varphi} \left( \frac{1 - \sin \varphi}{1 + \sin \varphi} \right)^{1/2} \end{array} \right.$$

$$\left\{ \begin{array}{l} \eta_{\max} = \frac{L\theta}{\sin \varphi} \left( \frac{1}{\sin \varphi} + \frac{1}{2} \right) \\ \eta_{\min} = \frac{L\theta}{\sin \varphi} \left( \frac{1}{\sin \varphi} - \frac{1}{2} \right) \end{array} \right.$$

$$\left\{ \begin{aligned} \bar{\beta} &= \frac{\beta_{max} + \beta_{min}}{2} = \frac{2L}{\sin 2\varphi} \\ \bar{\eta} &= \frac{\eta_{max} + \eta_{min}}{2} = \frac{L\theta}{\sin^2 \varphi} \end{aligned} \right.$$

$$\delta_T = \sqrt{R / \bar{\eta}} \quad R, \text{ average radius}$$

For  $\varphi = 60^\circ$

$$\frac{B'_{l_2}}{B\varphi} = \frac{1.732}{L}$$

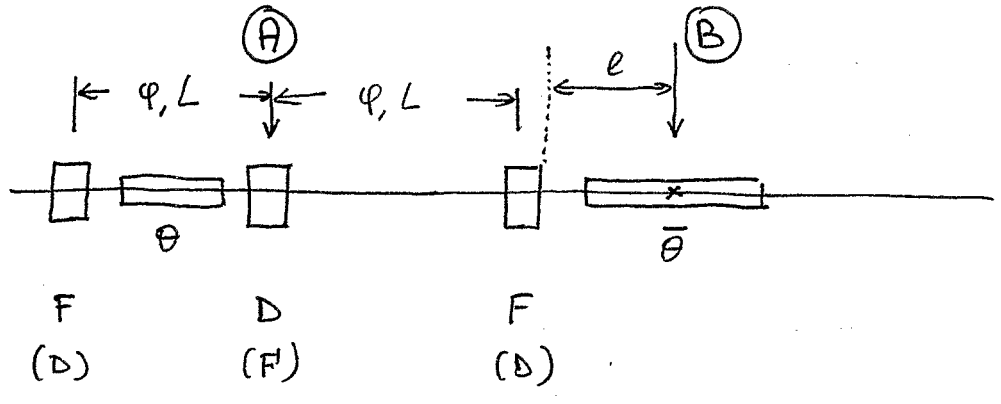
$$\left\{ \begin{aligned} \beta_{max} &= 4.3094 L \\ \beta_{min} &= 0.3094 L \end{aligned} \right.$$

$$\bar{\beta} = 2.3094 L$$

$$\left\{ \begin{aligned} \eta_{max} &= 1.9107 L\theta \\ \eta_{min} &= 0.7560 L\theta \end{aligned} \right.$$

$$\bar{\eta} = 1.3333 L\theta$$

## Dispersion Killer (thin lenses)



Transfer matrix from A to B

$$M = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ P/2 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} (1 - lP)(1 + L\frac{P}{2}) + \frac{lP}{2} & L(1 - lP) + l \\ P/2 - P(1 + \frac{LP}{2}) & 1 - LP \end{bmatrix}$$

$$P = \frac{B'l_2}{B_f} \quad , \quad \frac{LP}{2} = \sin\phi$$



$$\text{At (A)} \quad \eta = \eta_A, \quad \eta' = 0$$

$$\text{At (B)} \quad \eta = 0, \quad \eta' = 0$$

For the condition  $\eta_B = 0$

$$\left(1 - \frac{l}{p}\right) \left(1 + L \frac{p}{2}\right) + \frac{l}{2} = 0$$

from which

$$\frac{l}{L} = \frac{1 + \sin \varphi}{\sin \varphi \cdot (1 + 2 \sin \varphi)}$$

For  $\varphi = 60^\circ$

$$l = 0.788575 L$$

$$\eta_B' = \left[ \frac{p}{2} - p \left( 1 + \frac{Lp}{2} \right) \right] \eta_A$$

$$= - \frac{\sin \varphi}{L} (1 + 2 \sin \varphi) \eta_A$$

$$\eta_A = \frac{L \theta}{\sin \varphi} \left( \frac{1}{\sin \varphi} \pm \frac{1}{2} \right)$$

upper sign if QF at (A)  
 lower sign if QD at (A)

$$\eta_B' = - \theta \frac{(1 + 2 \sin \varphi) (2 \pm \sin \varphi)}{2 \sin \varphi}$$

For  $\varphi = 60^\circ$

$$\eta_B' = - \theta \times \begin{cases} 4.520726 & \text{with QF at (A)} \\ 1.788675 & \text{with QD at (A)} \end{cases}$$

Clearly the solution with QD at (A) is preferred.

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$\bar{\theta}$ , bending angle of the dispersion corrector dipole

$$\bar{\theta} = 1.788675 * \theta$$