BNL-101562-2014-TECH

RHIC/PG/19;BNL-101562-2013-IR

Regular Cell And Dispersion Killer For The 120 degree Phase Advance Case

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December 1983

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USDOE Office of Science (SC)

DISCLAIMER

RHIC-PG-19

REGULAR CELL AND DISPERSION KILLER
FOR THE 120° PHASSE ADVANCE CASE

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(BNL, December 9, 1983)

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RHIC-PG-

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Lattice for a Regular Cell

(Thin lenses approximation)

$$\begin{cases} L, & \text{length of half-cell} \\ \theta, & \text{bending for half-cell} \\ \varphi, & \text{phase advance per half-cell} \end{cases}$$

$$\frac{B' l_Q}{B\rho} = \frac{2\sin\varphi}{L}$$

$$\begin{cases} \beta_{max} = \dfrac{L}{\sin\varphi}\left(\dfrac{1+\sin\varphi}{1-\sin\varphi}\right)^{1/2} \\[2ex] \beta_{min} = \dfrac{L}{\sin\varphi}\left(\dfrac{1-\sin\varphi}{1+\sin\varphi}\right)^{1/2} \end{cases}$$

$$\begin{cases} \eta_{max} = \dfrac{L\theta}{\sin\varphi}\left(\dfrac{1}{\sin\varphi}+\dfrac{1}{2}\right) \\[2ex] \eta_{min} = \dfrac{L\theta}{\sin\varphi}\left(\dfrac{1}{\sin\varphi}-\dfrac{1}{2}\right) \end{cases}$$

$$\begin{cases} \bar{\beta} = \dfrac{\beta_{max} + \beta_{min}}{2} = \dfrac{2L}{\sin 2\varphi} \\ \\ \bar{\eta} = \dfrac{\eta_{max} + \eta_{min}}{2} = \dfrac{L\theta}{\sin^2\varphi} \end{cases}$$

$$\gamma_T = \sqrt{R/\bar{\eta}}$$

R, average radius

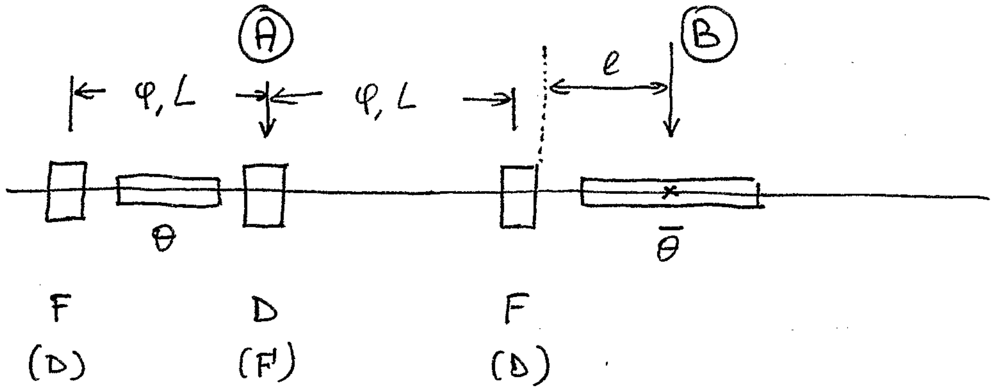
For $\varphi = 60°$

$$\frac{B' l_a}{B\rho} = \frac{1.732}{L}$$

$$\begin{cases} \beta_{max} = 4.3094 \; L \\ \beta_{min} = 0.3094 \; L \end{cases} \qquad \bar{\beta} = 2.3094 \; L$$

$$\begin{cases} \eta_{max} = 1.9107 \; L\theta \\ \eta_{min} = 0.7560 \; L\theta \end{cases} \qquad \bar{\eta} = 1.3333 \; L\theta$$

Dispersion Killer (thin lenses)



Transfer matrix from A to B

$$M = \begin{pmatrix} 1 & \ell \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -p & 1 \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ p/2 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} (1-\ell p)(1+L\frac{p}{2}) + \frac{\ell p}{2} & L(1-\ell p)+\ell \\ p/2 - p(1+\frac{Lp}{2}) & 1-Lp \end{bmatrix}$$

$$p = \frac{B' \ell_Q}{B\rho} \quad , \quad \frac{Lp}{2} = \sin\varphi$$

At A $\quad \eta = \eta_A \quad , \quad \eta' = 0$

At B $\quad \eta = 0 \quad , \quad \eta' = 0$

For the condition $\eta_B = 0$

$$(1 - \ell p)\left(1 + L\frac{p}{2}\right) + \frac{\ell p}{2} = 0$$

from which

$$\frac{\ell}{L} = \frac{1 + \sin\varphi}{\sin\varphi \cdot (1 + 2\sin\varphi)}$$

For $\varphi = 60°$

$$\ell = 0.788675 \; L$$

$$\eta_B' = \left[\frac{\rho}{2} - \rho\left(1 + \frac{L\rho}{2}\right)\right]\eta_A$$

$$= -\frac{\sin\varphi}{L}\,(1 + 2\sin\varphi)\,\eta_A$$

$$\eta_A = \frac{L\theta}{\sin\varphi}\left(\frac{1}{\sin\varphi} \pm \frac{1}{2}\right)$$

upper sign if QF at A
lower sign if QD at A

$$\eta_B' = -\theta\,\frac{(1+2\sin\varphi)(2 \pm \sin\varphi)}{2\sin\varphi}$$

For $\varphi = 60^\circ$

$$\eta_B' = -\theta \times \begin{cases} 4.520726 & \text{with QF at A} \\ 1.788675 & \text{with QD at A} \end{cases}$$

Clearly the solution with QD at A is preferred.

$\bar{\theta}$, bending angle of the dispersion corrector dipole

$$\bar{\theta} = 1.788675 \times \theta$$