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Booster Vacuum Requirements Due To Electron Capture And Loss

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RHIC-PG-16

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(BNL, December 8, 1983)

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12/7/83

Booster Vacuum Requirements: Atomic Charge Changing

$\sigma_{capture}$ & σ_{loss}

We find in general in the energy range of interest that atomic capture cross sections scale as β^{-6} and loss cross sections scale as β^{-2}. Around 10 MeV/A ($\beta = .145$) the cross sections are of equal magnitude for heavy ions such as Pb^{40+}; below that capture tends to dominate and above that loss dominates until the ion is bare. Then only capture remains, which is why we can store heavy ions at all in the collider.

The remaining beam after a path length l is $\eta = \frac{I}{I_0} = e^{-\sigma_T n_0 P l}$,
$n_0 = 3.27 \; 10^{16} \frac{\text{molecules}}{\text{cm}^3 \text{ torr}}$ at $20°C$ (warm), P in torr, $\sigma_T = \sigma_{capture} + \sigma_{loss}$
$l = \beta c t$, t the cycle time, or storage time.

For acceleration, the exponent becomes $\left(\frac{\int \beta\sigma_T dt}{\int dt}\right) n_0 P c \int dt$, as β and σ_T vary over the acceleration cycle through dependence on β. If we assume a constant field ramp, $\frac{dB}{dt} = K$, then we write

$$\frac{\int \beta \sigma_T \, dt}{\int dt} = \frac{\int \beta\sigma_T \left(\frac{dB}{dt}\right)^{-1} dB}{\int \left(\frac{dB}{dt}\right)^{-1} dB} = \frac{\int \beta\sigma_T \left(\frac{dt}{dB}\right)\left(\frac{dB}{d\beta}\right) d\beta}{\int \left(\frac{dt}{dB}\right)\left(\frac{dB}{d\beta}\right) d\beta}$$

Then use $B\rho = \frac{Am}{300\,q}\beta\gamma$ $\quad$ A is mass #, q is charge state, ρ in meters, B in Tesla and $m = 931.5$ MeV $= 1$ amu

Thus $\frac{dB}{d\beta} = \frac{Am}{300\,q\rho}\frac{d(\beta\gamma)}{d\beta}$ and our ratio becomes

$$\langle \beta\sigma \rangle_T = \frac{\int \beta\sigma_T \frac{d(\beta\gamma)}{d\beta} d\beta}{\int \frac{d(\beta\gamma)}{d\beta} d\beta} = \frac{\int \beta\sigma_T \gamma^3 d\beta}{(\beta\gamma)_{final} - (\beta\gamma)_{initial}}$$

, using $\frac{d(\beta\gamma)}{d\beta} = \gamma^3$.

Thus for capture we need the integral ($\sigma_c \propto \beta^{-6}$) $\int \frac{d\beta}{\beta^5 (1-\beta^2)^{3/2}}$

and for loss ($\sigma_L \propto \beta^{-2}$) $\int \frac{d\beta}{\beta (1-\beta^2)^{3/2}}$

From a table of integrals, we quickly find $\int \frac{d\beta}{\beta(1-\beta^2)^{3/2}} = \gamma + \log \frac{\beta\gamma}{\gamma+1}$

For the capture case, substitute $\beta = \sin\theta$, $(1-\beta^2)^{1/2} = \cos\theta$ to get $\int \frac{d\theta}{\sin^5\theta \cos^2\theta}$ which can be evaluated from

$$\int \frac{dx}{\sin^m x \cos^n x} = -\frac{1}{m-1} \frac{1}{\sin^{m-1}x \cos^{n-1}x} + \frac{m+n-2}{m-1} \int \frac{dx}{\sin^{m-2}x \cos^n x}$$, and

$$\int \frac{dx}{\sin x \cos^2 x} = \frac{1}{\cos x} + \log \tan \frac{x}{2} = \frac{1}{\cos x} + \log \frac{\sin x}{1+\cos x}$$, giving

$$\int \frac{d\beta}{\beta^5 (1-\beta^2)^{3/2}} = -\frac{1}{4} \frac{\gamma}{\beta^4} - \frac{5}{8} \frac{\gamma}{\beta^2} + \frac{15}{8} \left(\gamma + \log \frac{\beta\gamma}{\gamma+1} \right)$$

We crank out these factors for the following cases. Energies are based on 2 stage tandem operation, selection of most probable charge state at tandem exit via stripping, and assume acceleration thru the booster to a maximum $B\rho$ = 16.66 Tm (12 kG in booster magnets).

Two stage tandem operation is chosen based on discussions with H Wegner concerning 1) Voltage stability

2) Source access

3) Availability of 2 complete tandems with dual sources to insure continuous collider injection over weeks.

(More discussion on this issue will certainly occur.)

Ion	Tandem exit – 2 stage E/A	$\beta\gamma$	β	q	Booster exit E/A	$\beta\gamma$	β
$^{12}_{6}C$	8.75	.1374	.1361	6^+	1735.5	2.683	.9370
$^{32}_{16}S$	4.69	.1005	.1000	15^+	1589.7	2.515	.9292
$^{63}_{29}Cu$	2.86	.0784	.0782	22^+	1046.8	1.874	.8822
$^{127}_{53}I$	1.65	.0595	.0594	32^+	634.9	1.352	.8040
$^{197}_{79}Au$	1.14	.0495	.0494	37^+	390.9	1.0077	.7098

We evaluate the above integrals to get $\langle\beta\sigma\rangle_{capture}$ and $\langle\beta\sigma\rangle_{loss}$, in units of $\beta\sigma(\beta=1)$, which is given below.

Ion	$\langle\beta\sigma\rangle_{capture}$	β^{-2} $\langle\beta\sigma\rangle_{loss}$	β^{-1} $\langle\beta\sigma\rangle_{loss}$
^{12}C	~~[illegible]~~ 304	1.64	1
^{32}S	1069	1.78	1
^{63}Cu	3794	2.14	1
^{127}I	15,705	2.72	1
^{197}Au	44,134	3.55	1

} see following pages for discussion of this column.

Now we need some cross sections; given what exists in the literature, we have to do some scaling with β, Z, q. Alonso & Gould find the following rules (J. Alonso & H. Gould, Phys Rev A26, 1134 (1982))

$$\sigma_{capture} \propto Z^{0} q^{3} \beta^{-6} \qquad\qquad \sigma_{loss} \propto Z^{2.5} q^{-4} \beta^{-2}$$

The exponents given are somewhat uncertain and involve some fitting error; we have chosen the nearest half integer, with probably the greatest uncertainty in the q^{-4} term for σ_{loss}. The β^{-6} dependence for $\sigma_{capture}$ may not be strong enough with velocity (β^{-12} is predicted in the high velocity limit: see R K Janev and P Hvelplund, Comments At. Mol. Phys 11, 75 (1981) and H D Betz, in Methods of

Experimental Physics: Atomic Physics, Accelerators, edited by P. Richard, Academic Press, New York, 1980, Vol 17, p. 73) The β^{-2} dependence is only true in the velocity limit where the ion velocity exceeds the electron velocity in the outer orbitals. For lower energies the dependence can reverse: σ_{loss} for a given charge state will exhibit a maximum near an ion velocity equal to or up to two times the electron orbital velocity, and will decrease at lower ion velocities. Alonso & Gould in fact find a β dependence closer to $\beta^{-1.5}$ for e.g. Xe^{27+} to Xe^{42+} between $\beta = .072$ and $.134$, and an even slower dependence β^{-1} for Pb^{36+} to Pb^{51+} for $\beta = .099$ to $.134$. At very high energies, Gould et al (LBL-16467) find that the relativistic Bethe formula works well for ionization loss of U^{90+} and U^{91+} at 437 MeV/A and 962 MeV/A. This formula has the dependence

$$\sigma \propto \beta^{-2} \ln (K\beta^2\gamma^2 - \beta^2)$$

A general "rule of thumb" that works well for ions in charge states lower than the equilibrium charge state corresponding to their velocity is that the β dependence approaches β^{-2}. (Note the Bethe formula eventually gives a relativistic rise due to the γ^2 term, but for the collider & the AGS the ions will be fully stripped, so $\sigma_{loss} \equiv 0$. However, this will affect ionization of residual gas in the collider vacuum by the beam and will have to be evaluated vis à vis clearing electrodes, pressure bump, tune depression, etc...)

For now, we will evaluate σ_{loss} for both β^{-2} and $\beta^{-1.5}$ dependences. The latter will give more constraints.

The integral for $\langle\beta\bar\sigma\rangle_{loss}$ for $\beta^{-1.5}$ dependence is (see page 3 for values)

$$\int \frac{d\beta}{\beta^{1/2}(1-\beta^2)^{3/2}} = \text{mess!}$$ square root of a cubic. Pessimistic upper limit is $\int \frac{d\beta}{(1-\beta^2)^{3/2}}$ which equals the denominator.

We use the following cross sections

Alonso & Gould:

Pb^{37+} $\beta = ~~.099~~ .134$ $\sigma_{capture} = 6.5 \times 10^{-18}$ cm²/molecule N_2
$\beta = .099$ $\sigma_{loss} = 20 \times 10^{-18}$ cm²/molecule N_2

Xe^{32+} $\beta = .134$ $\sigma_{capture} = 4.5 \times 10^{-18}$ cm²/molecule N_2
$\beta = .099$ $\sigma_{loss} = 12 \times 10^{-18}$ cm²/molecule N_2

Ar^{18+} $\beta = .134$ $\sigma_{capture} = 1.1 \times 10^{-18}$ cm²/molecule N_2
(Gould & Marrus, Phys Rev Lett 41, 1457 (1978))

N^{6+} $\beta = .030$ $\sigma_{capture} = 2 \times 10^{-16}$ cm²/molecule N_2
(Angert et al

Cl^{4+} $\beta = .035$ $\sigma_{loss} = 2.3 \times 10^{-17}$ cm²/molecule N_2
(HA Scott et al Phys Rev A 18, 2459 (1978) This is for single + multiple loss; multiple loss dominates here as Cl^{4+} is well below equilibrium q for $\beta = .035$)

C^{5+} $\beta = .067$ $\sigma_{loss} = 6.0 \cdot 10^{-19}$ cm²/atom He
N^{6+} $\beta = .062$ $\sigma_{loss} = 4.1\ 10^{-19}$ cm²/atom He
O^{7+} $\beta = .073$ $\sigma_{loss} = 2.5\ 10^{-19}$ cm²/atom He
F^{8+} $\beta = .073$ $\sigma_{loss} = 1.7\ 10^{-19}$ cm²/atom He

} all correspond to removal of last electron

TR Dillingham et al Phys Rev A24, 1237 (1981)

C^{6+}	$\beta = .067$	$\sigma_{capture}$ =	$8.4\ 10^{-20}$ cm²/atom He
N^{7+}	$\beta = .062$	$\sigma_{capture}$ =	$2.5\ 10^{-19}$ cm²/atom He
O^{8+}	$\beta = .073$	$\sigma_{capture}$ =	$8.6\ 10^{-20}$ cm²/atom He
F^{9+}	$\beta = .073$	$\sigma_{capture}$ =	$1.4\ 10^{-19}$ cm²/atom He

We have little data for multiple electron capture, but from the measurements of Knudsen et al (H Knudsen et al, Phys Rev A 23, 597 (1981)) already at $\beta = .05$ the double capture cross sections are an order of magnitude smaller than the single capture cross sections. As the statistical and systematic errors in the cross sections, especially for large q, are of order 10% or more, we safely neglect multiple capture.

Scaling with target charge is naively expected to go as Z_{Target}. Theoretical treatment in the Bohr-Lindhard model (N Bohr, K Dan Vidensk Selsk Mat Fys Medd 18, #8 (1948); N Bohr & J Lindhard, ibid 28, #7 (1954); see also Knudsen above) gives a dependence

$$\sigma_{capture} \propto Z_T^{2/3}\, I^{-1}$$, I the target atom ionization potential. However, from plots of I vs atomic number for noble gases, $I \propto Z^{-1/3}$, so again $\sigma_{capture} \propto Z_T$. However, for hydrogen, molecular effects lead to a ratio $\sigma_c(H_2)/\sigma_c(H)$ with limiting value of nearly 4, not the factor of two expected naively. This is based on data near $\beta = 0.1$; whether such effects are still important for $\gamma \geq 2$ has not been investigated experimentally.

We use the following values, for our 'standard' heavy ions, for $\sigma(\beta=1)$ in the charge states obtained by stripping at the tandem exit in 2 stage operation at 15 MV :

Ion	($\beta=1$) $\sigma_{capture}$	β^{-2} scaling $\sigma_{loss}(\beta=1)$	β^{-1} scaling $\sigma_{loss}(\beta=1)$
$^{12}C^{6+}$	4.8×10^{-27} cm^2	$\equiv 0$	$\equiv 0$ (bare)
$^{32}S^{15+}$	2.6×10^{-25} cm^2	4.4×10^{-21} cm^2	6×10^{-20} cm^2
$^{63}Cu^{22+}$	6.1×10^{-25} cm^2	4.3×10^{-21} cm^2	6×16^{-20} cm^2
$^{127}I^{32+}$	1.9×10^{-24} cm^2	8.0×10^{-21} cm^2	8.1×10^{-20} cm^2
$^{197}Au^{37+}$	2.7×10^{-24} cm^2	1.3×10^{-20} cm^2	1.3×10^{-19} cm^2

These are all cross sections <u>per hydrogen atom</u>. The factor of 2 in going from H_2 to H mentioned above (in addition to the expected factor of 2) is ignored at this level of discussion when we are trying to find which decade of pressure we must obtain.

Using the $\frac{\langle\beta\sigma\rangle}{\sigma(\beta=1)}$ values from page 3, we get the following $\langle\beta\sigma\rangle_{Total}$ table

Ion	β^{-2} scaling $\langle\beta\sigma\rangle_{TOT}$	β^{-1} scaling $\langle\beta\sigma\rangle_{TOT}$	β^{-1} scaling $\langle\beta\sigma\rangle_{TOT}+\sigma_{geom}$
$^{12}C^{6+}$	1.5×10^{-24} cm^2	1.5×10^{-24} cm^2	2.6×10^{-24} cm^2
$^{32}S^{15+}$	8.1×10^{-21} cm^2	4.0×10^{-20} cm^2	⟶
$^{63}Cu^{22+}$	1.2×10^{-20} cm^2	6.2×10^{-20} cm^2	⟶
$^{127}I^{32+}$	5.2×10^{-20} cm^2	1.1×10^{-19} cm^2	⟶
$^{197}Au^{37+}$	1.7×10^{-19} cm^2	2.5×10^{-19} cm^2	⟶

The third column adds the nuclear geometric cross section, assuming 90% H_2, 10% CO_2 and $\sigma_{nuclear}=\sigma_{geometric}=\pi(R_1+R_2)^2$, to the results of β^{-1} scaling of σ_{loss}. Only carbon is affected here.

On the following graphs we plot $\eta = \frac{I}{I_0} = e^{-\langle\beta\sigma\rangle_T c n_0 P t f}$, the survival fraction of the beam, We assume room temperature vacuum with a 90% H_2 + 10% CO_2 mixture of residual gas. Here c is speed of light, $n_0(22°C) = 3.27 \times 10^{16} \frac{\text{molecules}}{\text{cm}^3\ \text{torr}}$, P is in torr, t is the acceleration time and $f = (0.9 \times 2 \times 1 + 0.1 \times 1 \times 6 + 0.1 \times 2 \times 8) = 4$ accounts for the gas composition and variation of σ with Z_{target}.

We take P from 10^{-11} to 10^{-6} torr, t = .2, .5 and 1 second; one graph is made for each ion. Beam-gas scattering is included for carbon. We use the β^{-1} scaling for σ_{loss}, as it gives the more restrictive pressure values by ~ ×2 to ×8. Pure H_2 relaxes requirements by ~2 (maybe - see above).

As usual, gold dominates matters. For 90% survival of the beam, we need:

(Pressures in torr)

Ion	1 second	0.5 second	0.2 second
C^{6+} (bare)	1.1 10^{-5}	2.2 10^{-5}	5.5 10^{-5}
S^{15+}	4.5 10^{-10}	9.0 10^{-10}	2.3 10^{-9}
Cu^{22+}	4.4 10^{-10}	8.8 10^{-10}	2.2 10^{-9}
I^{32+}	2.4 10^{-10}	4.8 10^{-10}	1.2 10^{-9}
Au^{37+}	1.1 10^{-10}	2.2 10^{-10}	5.5 10^{-10}

The linac would help the situation with, eg Au^{37+}. For 5 MeV/A ejection $\beta\gamma = .1038$, $\beta = .1032$, $\langle\beta\sigma\rangle_{capture}/\sigma(\beta=1) = 2002$, $\langle\beta\sigma\rangle_{TOT} = 1.35 \times 10^{-19}\ cm^2$

This gives ($\eta = 0.9$)

linac + Au^{37+}	2.0 10^{-10}	4.0 10^{-10}	1.0 10^{-9}

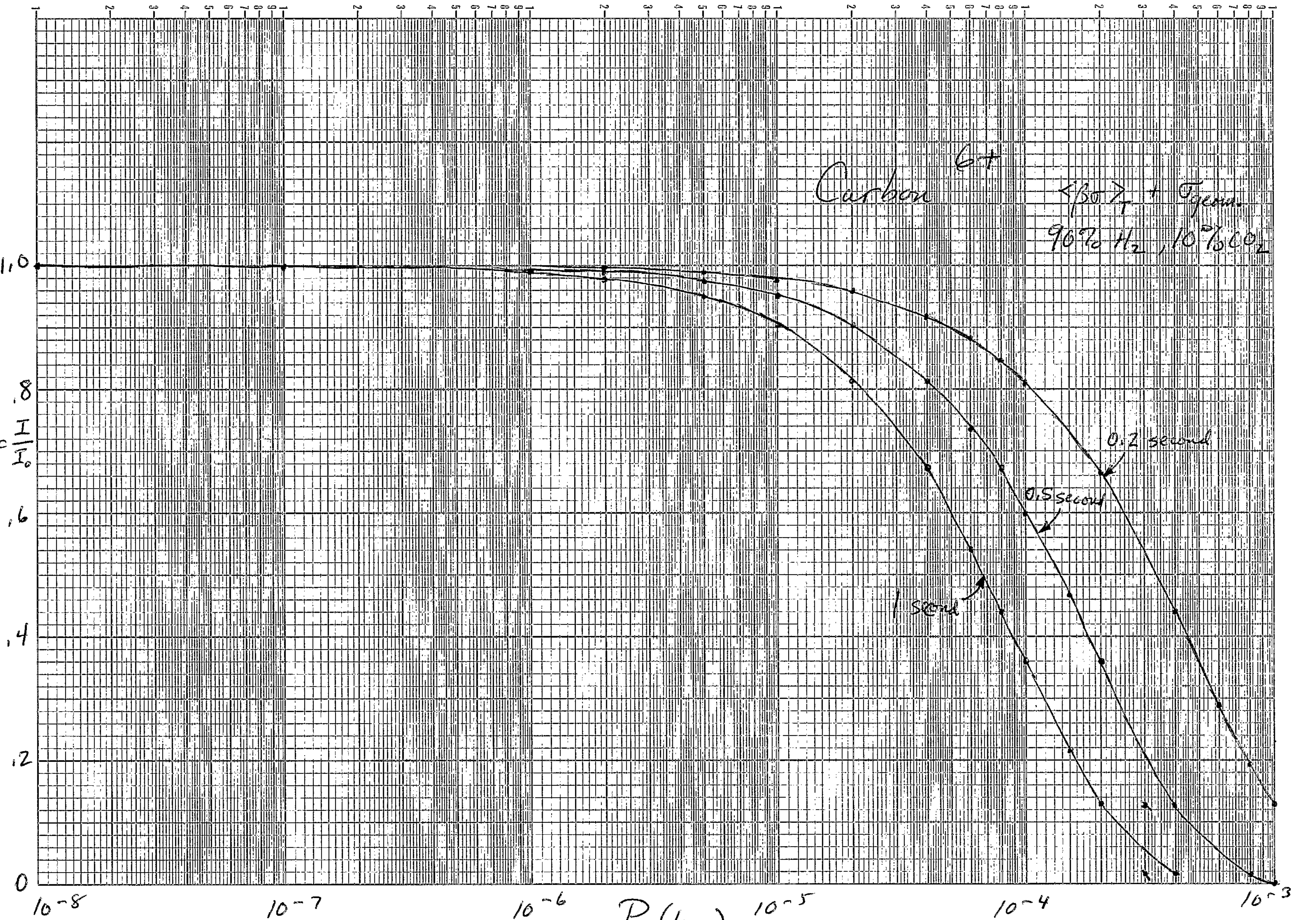
We also added curves for fully stripped S^{16+} and Cu^{29+}. (β=.14 at linac exit)

For 99% survival, decrease these values by 10.48. We conclude that 10^{-10} torr could be acceptable, but the designed 10^{-11} torr ensures nearly complete survival.

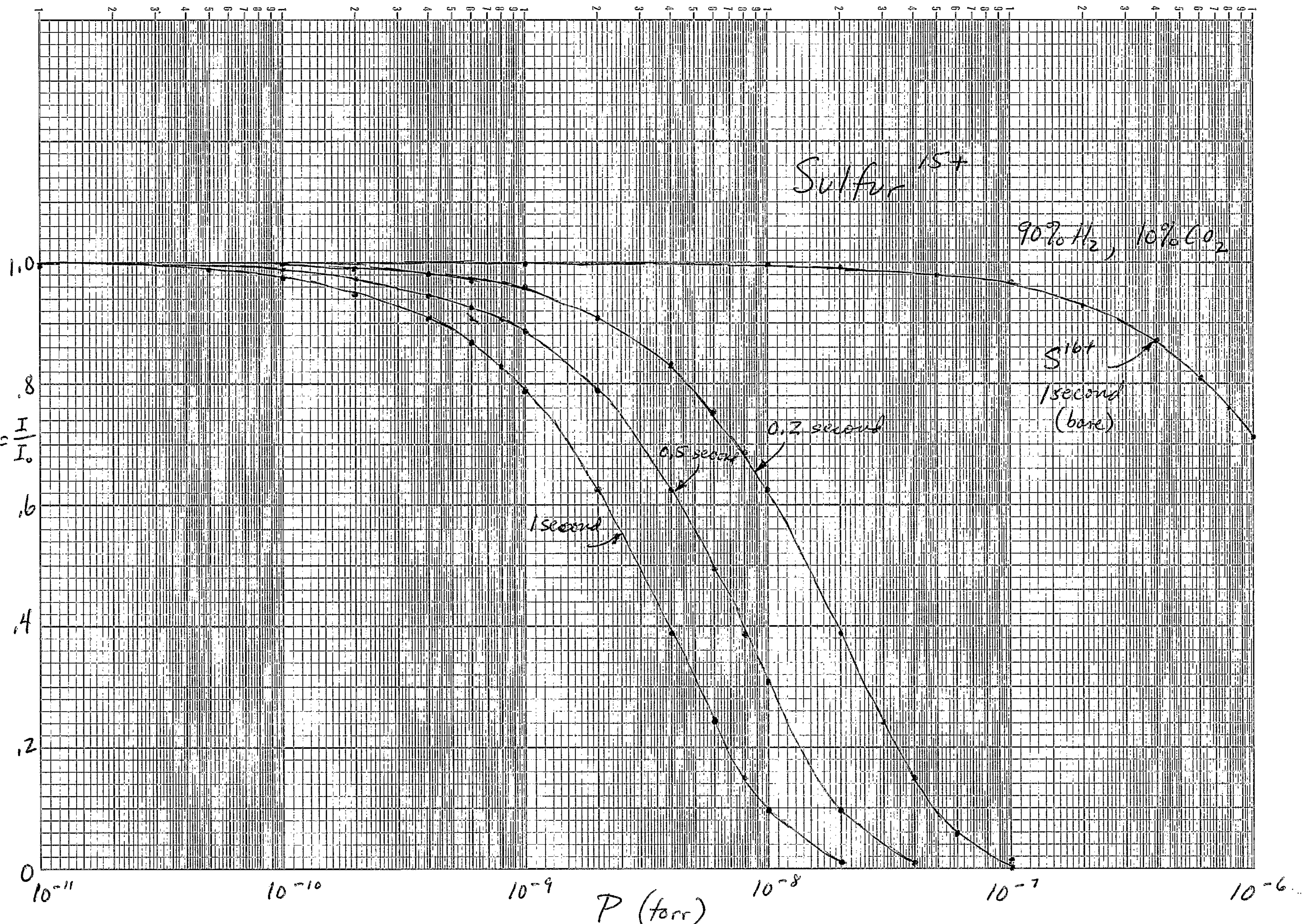
We have made rather conservative estimates in our scaling of the existing data on charge capture and loss. This was done first because it is misery to retrofit a vacuum system for better performance, and second because we have a sufficiently large number of other potential beam loss mechanisms that we should eliminate all those possible to preserve our small numbers of particles.

The dominant cross section, especially if we include the linac after the tandem, is by far the loss cross section. This can be ameliorated by stripping to as high a charge state as possible and thus asking for as much tandem voltage as can be held reliably. For ions lighter than Cu, stripping at the linac exit can be nearly completely efficient, resulting in a nearly complete elimination of vacuum problems – compare the curves for S^{15+} and S^{16+}, or Cu^{22+} and Cu^{29+}.

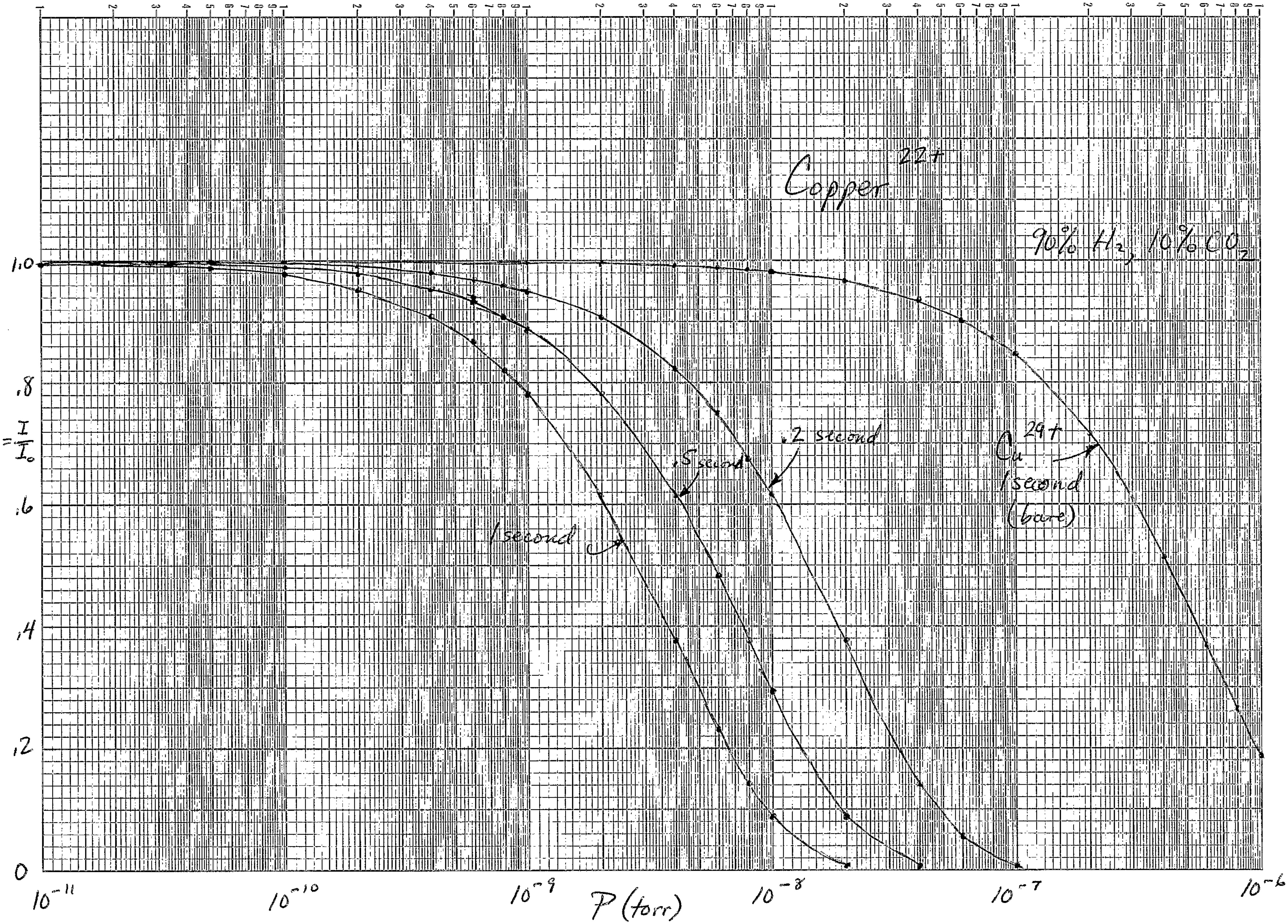
The charge changing data on which this note is based are sparse. Large numbers of atomic-physics-type measurements exist; unfortunately they cluster strongly below 1 MeV/A, as this is the energy range of interest to the controlled fusion physicists who need the data for impurity studies in plasmas. There are recent LBL and ORNL results that can be added to this note when they become available. It would also help to have measurements in the energy range spanned by the Bevalac (and our accumulator/booster.)



Carbon 6+
$\langle \beta\sigma \rangle_T + \sigma_{geom}$
90% H_2, 10% CO_2
0.2 second
0.5 second
1 second
1.0
.8
.6
.4
.2
0
$\frac{I}{I_0}$
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}



Sulfur 15+
90% H2, 10% CO2
S16+
1 second (base)
0.2 second
0.5 second
1 second
1.0
.8
.6
.4
.2
0
I/I0
10^-11
10^-10
10^-9
10^-8
10^-7
10^-6
P (torr)



Copper 22+
90% H2, 10% CO2
1.0
.8
.6
.4
.2
0
η = I/I0
.2 second
.5 second
1 second
Cu 29+
1 second
(bare)
10^-11
10^-10
10^-9
10^-8
10^-7
10^-6
P (torr)

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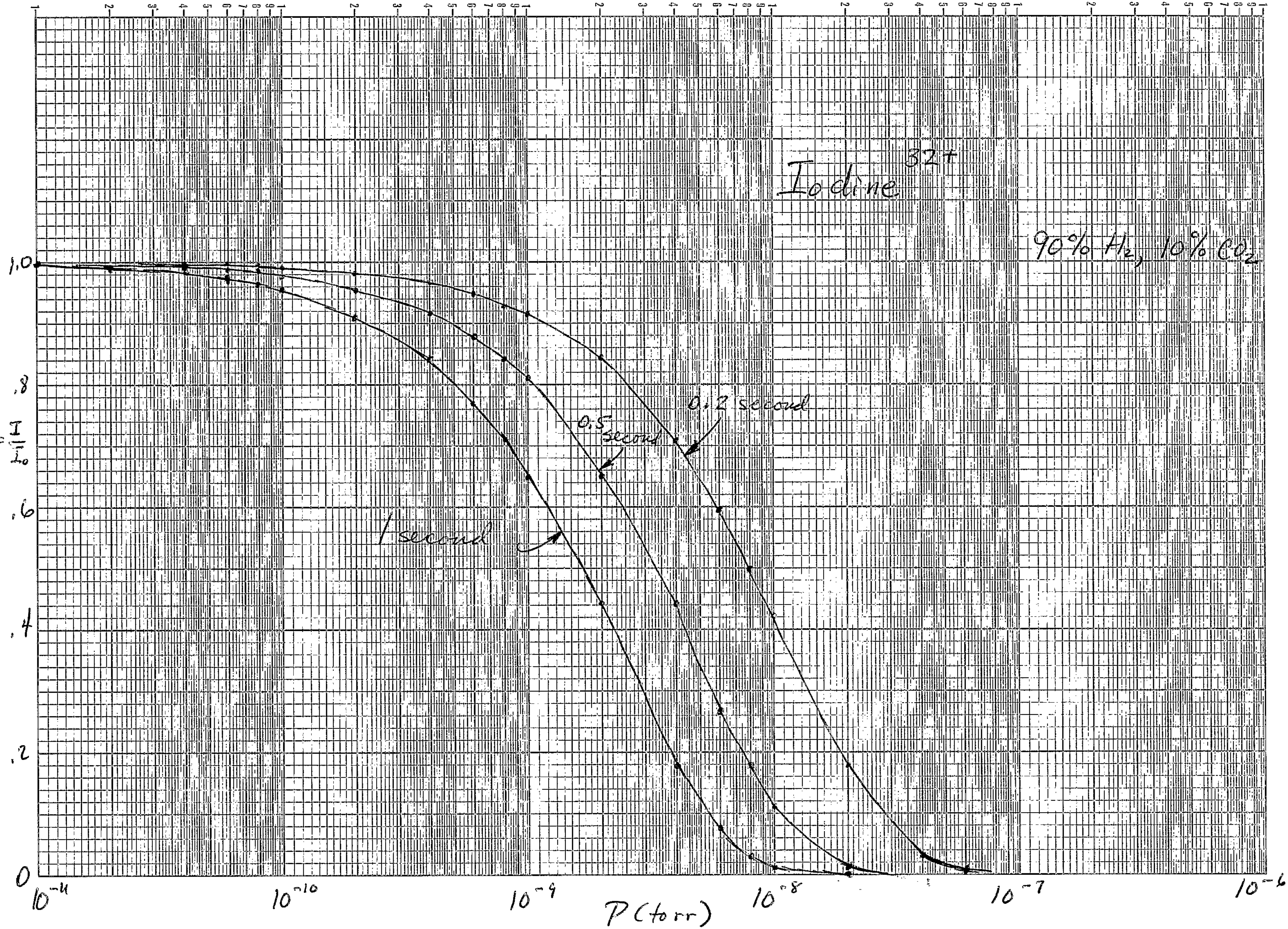

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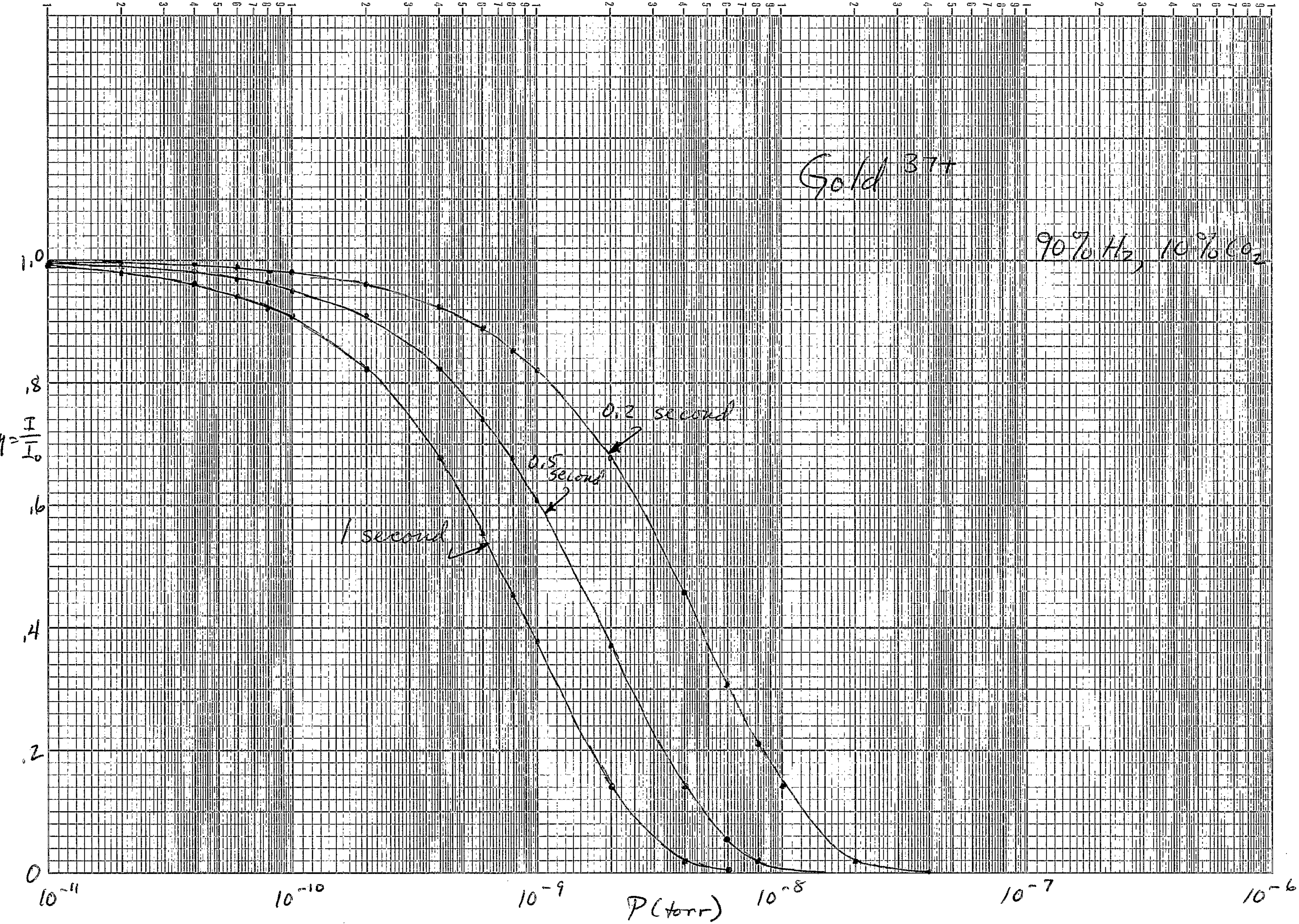

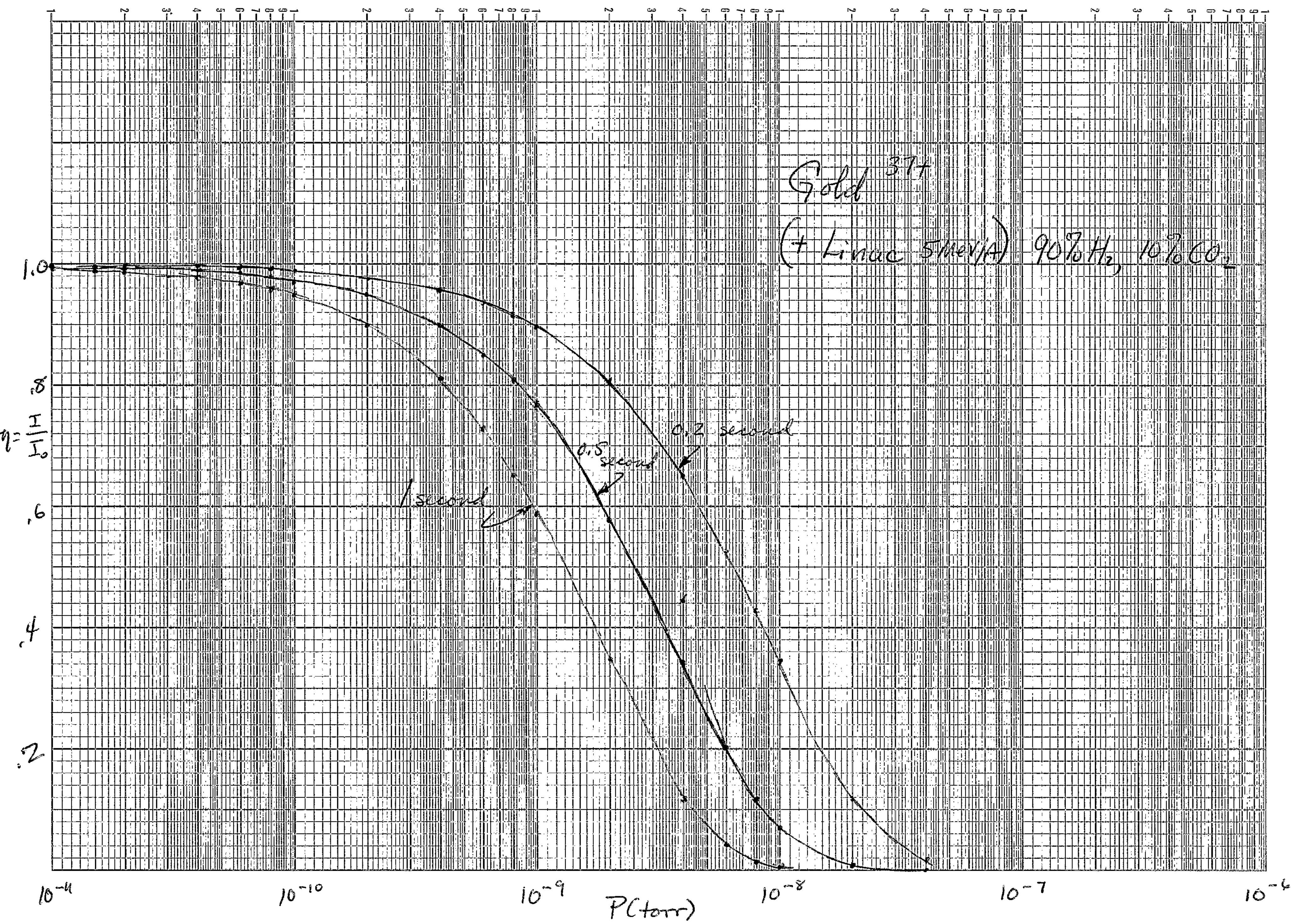

Gold 37+
(+ Linac 5 MeV/A) 90% H2, 10% CO2
1.0
.8
.6
.4
.2
η = I/I0
0.2 second
0.5 second
1 second
10^-11
10^-10
10^-9
10^-8
10^-7
10^-6
P(torr)