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Luminosity Formulae

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LUMINOSITY FORMULAE

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~~LUMINOSITY CALCULATIONS~~

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A. Both Colliding Beams are bunched

$$L = N_1 N_2 \, f_{encounter} \, F$$

$$F = \frac{2}{(2\pi)^{3/2} (\sigma_{\ell_1}^2 + \sigma_{\ell_2}^2)^{1/2}} \int_{-\infty}^{+\infty} \frac{ds}{(\sigma_{x_1}^2 + \sigma_{x_2}^2)^{1/2} \, (\sigma_{z_1}^2 + \sigma_{z_2}^2)^{1/2}} \times$$

$$\times \exp\left\{-2s^2 \left(\frac{1}{\sigma_{\ell_1}^2 + \sigma_{\ell_2}^2} + \frac{\alpha^2/4}{\sigma_{x_1}^2 + \sigma_{x_2}^2}\right)\right\} \qquad (1)$$

$$= \int_{-\infty}^{+\infty} G(s) \, ds$$

$\sigma^2 = \frac{\varepsilon \, \beta(s)}{6\pi}$ H and V, zero dispersion

ε emittance for 95% of beam with bi-gaussian distribution

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

N_1, N_2, number of particles per bunch

σ_ℓ, rms bunch length

α, total crossing angle assumed in the x-plane.

If both beams are made of B bunches each

$$f_{encounter} = B \cdot f_{revolution}$$

B. One Bunched Beam colliding with an Unbunched Beam

$$L = N_U N_B f_{encounter} \; F'$$

$$F' = \frac{1}{2\pi^2 R} \int_{-\infty}^{+\infty} \frac{ds}{\left(\sigma_{xU}^2 + \sigma_{xB}^2\right)^{1/2} \left(\sigma_{zU}^2 + \sigma_{zB}^2\right)^{1/2}} \times$$

$$\times \exp\left\{ - \frac{\alpha^2 s^2}{2\left(\sigma_{xU}^2 + \sigma_{xB}^2\right)} \right\} \qquad (2)$$

$$= \int_{-\infty}^{+\infty} G(s)\, ds$$

$2\pi R$, circumference of reference orbit ~~assumed~~ ~~to~~ ~~be~~ assumed to be the same for both beams

N_U, total number of particles in the unbunched beam

N_B, no. of particles per bunch in the bunched beam

B, no. of bunches in the bunched beam

$$f_{encounter} = B \; f_{revolution}$$

Extreme Cases

A. Both Colliding Beams are bunched

Long Bunches : $\sigma_\ell >> 2\sigma_x/\alpha$

σ_x and σ_z are constant over interaction region

Define effective cross-sections and length

$$\overline{\sigma_x}^2 = \frac{\sigma_{x1}^2 + \sigma_{x2}^2}{2}$$

$$\overline{\sigma_z}^2 = \frac{\sigma_{z1}^2 + \sigma_{z2}^2}{2}$$

$$\overline{\sigma_\ell}^2 = \frac{\sigma_{\ell 1}^2 + \sigma_{\ell 2}^2}{2}$$

We have

$$L = \frac{N_1 N_2 \, B \, f_{revolution}}{2\pi \, \alpha \, \overline{\sigma_\ell} \, \overline{\sigma_z}} \qquad (3)$$

having assumed that both beams have the same number of bunches B.

B. One Bunched Beam colliding with an Unbunched Beam

σ_x and σ_z are constant over interaction region

Define effective cross-sections

$$\bar{\sigma}_x^2 = \frac{\sigma_{xU}^2 + \sigma_{xB}^2}{2}$$

$$\bar{\sigma}_z^2 = \frac{\sigma_{zU}^2 + \sigma_{zB}^2}{2}$$

We have

$$L = \frac{N_U N_B \, B \, f_{revolution}}{2\pi \, \alpha \, (\sqrt{\pi} R) \, \bar{\sigma}_z} \qquad (4)$$

Comment

For crossing at a large angle and for long bunches or coasting beams the equations shown above are correct as long

$$\sqrt{2}\ \frac{\overline{\sigma_x}}{\alpha} \ll \ell \qquad (5)$$

where ℓ is half of the length of the vacuum chamber shared by both beams unshielded from each other.

Eq. (3) and (4) are derived assuming eq. (5) holds.

If relation (5) does not hold the integration limits in eq. (1) and (2) are to be replaced by $-\ell$ and $+\ell$ respectively instead of $-\infty$ and $+\infty$.

C. Short Bunches Colliding Head-on ~~[illegible]~~

Extreme Case from eq. (1) with $\alpha = 0$ -

$\overline{\sigma_x}$ and $\overline{\sigma_z}$ are constant over interaction region -

Assume $\overline{\sigma_\ell} \ll \ell$

Then we have with the usual notation

$$L = \frac{N_1 N_2 B f_{revolution}}{4\pi \overline{\sigma_x} \overline{\sigma_z}}$$

having again assumed that both beams have the same number of bunches B -

References

Lloyd Smith, PEP-Note-20, April 27, 1972

A.G. Ruggiero, FN-271, December 1974