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# Intrabeam Scattering Computer Calculations And Other Performance Issues

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INTRABEAM SCATTERING COMPUTER CALCULATIONS  
AND  
OTHER PERFORMANCE ISSUES

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# Intrabeam Scattering Computer Calculations

Smooth lattice ( $\eta' = \beta' = 0$ )       $\beta = 30\text{m}$        $\eta = 2.0\text{m}$   
 (line #54)

$\delta$ , rms relative energy spread

$\epsilon = 2\pi\sigma^2/\beta$ , rms betatron emittance

$$\tau_{\beta}^{-1} = \dot{\epsilon}/2\epsilon$$

$$\tau_E^{-1} = \dot{\delta}/\delta \quad (\tau_E^{-1})_{\text{bunched}} = 0.5 \times (\tau_E^{-1})_{\text{unbunched}}$$

$I$ , average current for unbunched beams =  $Ne f_{\text{revol}}$   
 $I$ , peak current for bunched beams

For bunched beams

$$I = \frac{Ne\beta c}{\sqrt{2\pi}\sigma_e}$$

$\beta c$ , beam velocity

$N$ , no. of particles in the bunch

$\sigma_e$ , rms bunch length

Case considered Gold (Au)       $A = 197$        $Z = 79$

The numbers in the next two tables are for

$$I = 1\text{mA-particle}$$

Emittances given are normalized

$\tau_{\beta}^{-1}$  is for horizontal plane. The vertical emittance is stable or slightly damped

	$E_H$	$E_V$	$\delta$	$\tau_E^{-1}$	$\tau_\beta^{-1}$		case #
	normalized						
	$\pi \cdot \text{mm} \cdot \text{mrad}$		$10^{-3}$	hour <sup>-1</sup>	hour <sup>-1</sup>		#
B	0.4	10.	0.04	42.952	2.3208	$\gamma = 12$	13
B			0.12	1.3797	1.8133		15
B			0.4	.0389	0.8406		18
U			0.4	.0778	0.8406		1
B			1.2	.0014	0.3007		2
B	2.0	2.0	0.04	24.3211	0.0812	$\gamma = 12$	17
B			0.1	2.643	0.0553		4
B			0.4	0.0184	0.0061		14
U			0.4	0.0367	0.0061		7
B	4.0	4.0	0.1	1.21	0.0127	$\gamma = 12$	5
U			0.2	0.3767	0.0079		9
U			0.4	0.0347	0.0029		8
U			0.8	0.0004	0.0001		10
B	0.4	10.	0.035	1.2813	0.6290	$\gamma = 100$	16
B			0.35	0.0028	0.1358		3
B	4.0	4.0	0.1	0.0653	0.0262	$\gamma = 100$	6
B			0.2	0.0123	0.0195		11
B			0.4	0.0019	0.0119		12

Eighteen (18) cases have been considered

For each case 100 lattice values ( $\beta, \eta$ ) have been also considered

$\beta = 5$  to  $50$ , step  $5$  meters  
 $\eta = 0.5$  to  $5$ , step  $0.5$  meters

## Space Charge Calculation At Injection in the Booster

$\epsilon_N$ , normalized emittance

Actual emittance,  $\epsilon = 6\pi \sigma^2 / \beta_{lat} = \epsilon_N / (\beta\gamma)$

$N_B$ , total number of particles injected into the Booster

Space charge limit Formula is

$$\frac{N_B}{\epsilon_N} = (\beta\gamma)^2 \frac{\pi B_f \Delta\nu}{2\epsilon_0 F} \frac{A}{Q^2} \quad (1)$$

Assumption: "round" beam, that is  $\epsilon_H = \epsilon_V$

$B_f$ , bunching factor = 0.5

$\Delta\nu$ , betatron-tune depression = 0.1

$\epsilon_0 = 1,535 \times 10^{-18}$  m, classical proton radius

$A$ , atomic mass = 197 for Gold

$Q$ , charge state = 34, partially stripped ions

$F$ , form factor  $\sim 1$  for  $\beta \ll 1$  and  $\gamma \sim 1$

We take a Booster Acceptance of  $40\pi$  mm-mrad and we assume we dilute the beam, by steering and multi-turn injection, until this aperture is full and the space charge limit is reached.

The following table explores the effect of the injected kinetic energy (T) on the space charge limit.

Procedure:

- i Assign Kinetic injection energy,  $T$
- ii Calculate relativistic factors  $\beta$  and  $\gamma$
- iii Calculate phase-space density at the limit  $N_B/E_N$ , from (1)
- iv Calculate Normalized Emittance from

$$E_N = (\beta\gamma) (40 \pi \text{ mm.mrad})$$

- v Calculate number of particles  $N_B$  that can be injected into the Booster
- vi Assume 3 bunches and a 50% efficiency for full stripping between Booster and AGS  
The number of particles/bunch at transfer to the Collider is

$$N = N_B / (2 \times 3)$$

- vii Calculate revolution period  $\tau = C/\beta c$   
 $C$ , Booster circumference = 201.75 m

- viii Assume a pulse from the Tandem 250 psec and a d.c. current of 4.4  $\mu\text{Amp}$  - particle.  
Calculate no. of particles per turn

$$N_B/\text{turn} = (4.4 \mu\text{Amp-particle}) \times \tau$$

- ix Calculate required number of turns at injection to get at the total  $N_B$

$$\text{no. of turns} = N_B / (N/\text{turn})$$

- x Calculate required pulse length = (no. of turns)  $\times \tau$

T	$\beta$	$\delta$	$N_B/E_N$	$E_N$	$N_B$
MeV			$\pi^{-1} m^{-1}$	$\pi mm \cdot mrad$	
1	.04613	1.001066	$4 \times 10^{14}$	1.85	$7.4 \times 10^8$
1.5	.05648	1.001599	$5 \times 10^{14}$	2.26	$1.13 \times 10^9$
2	.06519	1.002132	$5.7 \times 10^{14}$	2.61	$1.5 \times 10^9$
3	.07978	1.003197	$7 \times 10^{14}$	3.20	$2.24 \times 10^9$
4	.09204	1.004263	$8.1 \times 10^{14}$	3.70	$3 \times 10^9$
5	.10283	1.005329	$9.1 \times 10^{14}$	4.15	$3.74 \times 10^9$

T	$\tau$	N	$N_B/\text{turn}$	no. turns	Pulse length
MeV	$\mu\text{sec}$	$\times 10^8$	$\times 10^8$		$\mu\text{s}$
1	14.6	1.23	4	1.85	27
1.5	11.9	1.88	3.3	3.4	40.5
2	10.3	2.48	2.83	5.3	54.6
3	8.43	3.73	2.32	9.7	81.8
4	7.3	5.0	2.0	15	109.5
5	6.54	6.24	1.8	21	137



## Performance

Long Bunches, High Intensity  
Crossing at an Angle

$$L = N_c^2 B f_{rev} / 4\pi \left(\frac{d}{2} \sigma_e\right) \sigma_v$$

$B$ , no. of bunches = 3

$N_c$ , no. of particles / Bunch in colliding mode =  $6 \times 10^{10}$

$f_{rev}$ , revolution frequency = 78,1973 KHz

$d$ , total crossing angle = 4 mrad

$\sigma_e$ , rms bunch length = 50 ns

$\sigma_v$ , rms beam height = 0.037 mm

$$L = 1.8 \times 10^{27} \text{ cm}^{-2} \text{ s}^{-1}$$

# Longitudinal stability of the Long Bunches

$$|Z/n| = \frac{E |\eta|}{e I_p} \left( 2 \frac{\sigma_E}{E} \right)^2 \frac{A}{Z^2}$$

$$I_p, \text{ peak current} = \frac{N_e \beta_c}{c} / (\sigma_z \sqrt{2\pi})$$

$$N_e, \text{ no. of particles / bunch} = 6 \times 10^{10}$$

$$\sigma_z, \text{ rms bunch length} = 50 \text{ m} \quad (\sigma_z = 0.17 \text{ } \mu\text{sec})$$

$$I_p = 0.023 \text{ Amp-particle} \quad (\beta \approx 1)$$

For Gold (Au)  $A = 197$  and  $Z = 79$

Assume a coupling impedance of  $|Z/n| = 10 \text{ ohms}$

$E$ , energy per nucleon  $\approx 100 \text{ GeV/A}$

$$\eta = \gamma^{-2} - \gamma_T^{-2}, \quad \gamma_T = \text{transition energy / rest energy}$$

$\sigma_E/E$ , rms energy spread at stability (could be larger)

$$B, \text{ bunch area} = 6\pi \sigma_E \cdot \sigma_E$$

Assuming 3 bunches per beam the total area of each beam is

$$3B$$

The following table explores the dependence with  $\gamma_T$  of the threshold spread  $\sigma_E/E$  at 100 GeV/A and the unbunched beam rms spread at 12 GeV/A using the formula

$$3B = 4 T_{\text{total}} \sigma_E, \quad T_{\text{total}} = 12.788 \text{ } \mu\text{sec}$$

$\delta_T$	$ z $	$\sigma_E/E$ threshold @ 100 GeV/A	B eV/A-sec	$\sigma_E/E$ unbunched @ 12 GeV/A
10	.0099	.43 $\times 10^{-4}$	13.8	$675 \times 10^{-4}$
20	.0024	.87	27.9	$1.4 \times 10^{-4}$
30	$1.011 \times 10^{-3}$	1.34	42.9	$2.0 \times 10^{-4}$
50	$3 \times 10^{-4}$	2.5	80.1	$3.9 \times 10^{-4}$
80	$5.625 \times 10^{-5}$	5.7	182.7	$8.9 \times 10^{-4}$

In the following table, T is the kinetic energy at injection into the Booster, N is the number of ions per bunch at transfer between AGS and Collider.

We assume: there are 3 bunches in the Booster, only these 3 bunches are accelerated in the AGS at the time, so that there are only 3 bunches transferred to the Collider every AGS pulse.

n, number of bunches from the AGS required to get  $3 \times 6 \times 10^{10} = 1.8 \times 10^{11}$  ions per beam

The number of AGS pulses are therefore  $n/3$

The Collider (each ring) is filled with 19 AGS pulses in a 'box-car' fashion ( $h=57$ ) and  $n_s$  RF stacking pulses

The filling time is calculated assuming two minutes for a RF stacking cycle = (1 min AGS + 1 min RF)

T	N	n	no. of AGS pulses	filling sequence	Filling Time
MeV	$\times 10^8$				
1	1.23	1461	487	19 BoxCar * 77 RF stacking	2h 34 min
1.5	1.88	957	319	51	1h 42 min
2	2.48	726	242	39	1h 18 min
3	3.73	483	161	26	52 min
4	5.0	360	120	19	38 min
5	6.24	289	97	16	32 min

The average current at injection ( $\gamma \sim 12$ ) is

$$I = 2.25 \text{ nA - particle}$$

and the peak current at top energy ( $\gamma \sim 100$ ) is also the same

Take  $\gamma_1 = 10$

$\sigma_E/E$  @ 100 GeV/A  $.43 \times 10^{-4}$

$\sigma_E/E$  @ 12 GeV/A  $.675 \times 10^{-4}$

$E_N$	$\gamma$	$t_E$	$t_B$
$\pi$ .mm.mrad		hours	hours
2	12	0.03	6.
4.	12.	0.09	20.
2	100.	-	-
4.	100.	1.5	11

Individual Bunch Area at transfer from the AGS to the Collider

$S = 0.014 \text{ eV/A-sec} / 0.072$

having included a 50% dilution factor for transfer, RF stacking and Acceleration.

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Take  $\gamma_T = 20$

$\sigma_E/E$  @ 100 GeV/A  $\cdot 87 \times 10^{-4}$

$\sigma_E/E$  @ 12 GeV/A  $1.4 \times 10^{-4}$

$E_N$ π-mm-mrad	$\gamma$	$t_E$ hours	$t_B$ hours
2	12	0.12	9.
4.	12.	0.3	40.
2	100.	-	-
4.	100	5.6	15.

Individual Bunch Area at transfer from the AGS to the Collider

$S = 0.0229 \text{ eV/A-sec} / 0.14$

having included a 50% dilution factor for transfer RF stacking and Acceleration,

~~XXXXXXXXXXXXXXXXXXXXXXXXXXXX~~

Take  $\gamma_T = 30$

$\sigma_E/E$	@	100 GeV/A	$1.34 \times 10^{-4}$
$\sigma_E/E$	@	12 GeV/A	$2.0 \times 10^{-4}$

$E_N$	$\delta$	$t_E$	$t_p$
mm-sec		hours	hours
2	12	0.5	25
4	12	1.2	56
2	100	-	-
4	100	0	20

Individual Bunch Area at transfer from the AGS to the Collider

$$S = 0.044 \quad / \quad 0.22 \quad \text{eV/A-sec}$$

having included a 50% dilution factor for transfer, RF stacking and acceleration.

~~Individual Bunch Area at transfer from the AGS to the Collider~~

Take  $\gamma_T = 50$

$\sigma_E / E$  @ 100 GeV/A  $2.5 \times 10^{-4}$

$\sigma_E / E$  @ 12 GeV/A  $3.9 \times 10^{-4}$

$E_N$	$\gamma$	$t_c$	$t_p$
mm-rad		hours	hours
2	12	12	74
4	12	13	150
2	100	-	-
4	100	6	30

Individual Bunch Area at transfer from the AGS to the Collider

$S = 0.002 / 0.4 \text{ eV/A} \cdot \text{sec}$

having included a 50% dilution factor for transfer, RF stacking and Acceleration



Take

$\gamma_T = 80$

$\sigma_E/E$  @ 100 GeV/A

$5.7 \times 10^{-4}$

$\sigma_E/E$  @ 12 GeV/A

$8.9 \times 10^{-4}$

$E_N$	$\gamma$	$t_E$	$t_p$
Turn-around		hours	hours
2	12	2100	2100
4	12	1000	4500
2	100	-	-
4	100	200	50

Individual Beam Area at transfer from the AGS to the Collider

$S = 0.19 / 0.95 \text{ eV/A-sec}$

having included a 50% dilution factor for transfer, RF stacking and acceleration.

The luminosity of  $1.8 \times 10^{27} \text{ cm}^{-2} \text{ s}^{-1}$  can be obtained if

$$\sigma_v = 0.037 \text{ mm}$$

at the interaction region

This corresponds to a  $\beta_v^* = 2 \text{ m}$  with a normalized emittance  $E_N = 0.4 \pi \cdot \text{mm} \cdot \text{mrad}$

We have now

$E_N$	$\beta_v^*$
0.4 $\pi \cdot \text{mm} \cdot \text{mrad}$	2 m
2.0	0.4 m
4.0	0.2 m

or with  $\beta_v^* = 2 \text{ m}$

$E_N$	$L$
0.4 $\pi \cdot \text{mm} \cdot \text{mrad}$	$1.8 \times 10^{27} \text{ cm}^{-2} \text{ s}^{-1}$
2.0	0.8
4.0	0.57