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Intrabeam Scattering Computer Calculations And Other Performance Issues

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Intrabeam Scattering Computer Calculations

Smooth Lattice ($\eta' = \beta' = 0$) $\beta = 30\,m$ $\eta = 2.0\,m$
(line #54)

δ, rms relative energy spread

$\varepsilon = 2\pi\sigma^2/\beta$, rms betatron emittance

$\tau_\beta^{-1} = \dot{\varepsilon}/2\varepsilon$

$\tau_E^{-1} = \dot{\delta}/\delta$ $(\tau_E^{-1})_{Bunched} = 0.5 \times (\tau_E^{-1})_{Unbunched}$

I, average current for unbunched beam $= Ne f_{revol.}$
, peak current for bunched beam

For bunched beam

$$I = \frac{Ne\,\beta c}{\sqrt{2\pi}\,\sigma_\ell}$$

βc, beam velocity
N, no of particles in the bunch
σ_ℓ, rms bunch length

Case considered: Gold (Au) $A = 197$ $Z = 79$

The numbers in the next two tables are for

$I = 1\,mA$-particle

Emittances given are normalized
τ_β is for horizontal plane. The vertical emittance is stable or slightly damped

	ε_H	ε_V	δ	τ_ε^{-1}	τ_β^{-1}		case
	normalized						
	$\pi \cdot$mm$\cdot$mrad		10^{-3}	hour^{-1}	hour^{-1}		#
B	0.4	10.	0.04	42.952	2.3208		13
B			0.12	1.3797	1.8133		15
B			0.4	.0389	0.8406	$\gamma = 12$	18
U			0.4	.0778	0.8406		1
B			1.2	.0014	0.3007		2
B	2.0	2.0	0.04	24.3211	0.0812		17
B			0.1	2.643	0.0553	$\gamma = 12$	4
B			0.4	0.0184	0.0061		14
U			0.4	0.0367	0.0061		7
B	4.0	4.0	0.1	1.21	0.0127		5
U			0.2	0.3767	0.0079	$\gamma = 12$	9
U			0.4	0.0347	0.0029		8
U			0.8	0.0004	0.0001		10
B	0.4	10.	0.035	1.2813	0.6290	$\gamma = 100$	16
B			0.35	0.0028	0.1358		3
B	4.0	4.0	0.1	0.0663	0.0262		6
B			0.2	0.0123	0.0195	$\gamma = 100$	11
B			0.4	0.0019	0.0119		12

Eighteen (18) cases have been considered

For each case 100 lattice values (β, η) have been also considered

$$\begin{cases} \beta = 5 \text{ to } 50, \text{ step } 5 \text{ meters} \\ \eta = 0.5 \text{ to } 5., \text{ step } 0.5 \text{ meters} \end{cases}$$

Space Charge Calculation At Injection in the Booster

ε_N, normalized emittance

Actual emittance, $\varepsilon = 6\pi\,\sigma^2/\beta_{Lat.} = \varepsilon_N/(\beta\gamma)$

N_B, total number of particles injected into the Booster

Space charge limit Formula is

$$\frac{N_B}{\varepsilon_N} = (\beta\gamma^2)\,\frac{\pi B_f\,\Delta\nu}{2 r_0 F}\,\frac{A}{Q^2} \qquad (1)$$

Assumption : "round" beam, that is $\varepsilon_H = \varepsilon_V$

B_f, bunching factor $= 0.5$

$\Delta\nu$, betatron-tune depression $= 0.1$

$r_0 = 1.535 \times 10^{-18}$ m, classical proton radius

A, atomic mass $= 197$ for Gold

Q = charge status $= 34$, partially stripped ions

F, form factor ~ 1 for $\beta \ll 1$ and $\gamma \sim 1$

We take a Booster Acceptance of $40\,\pi$ mm-mrad and we assume we dilute the beam, by steering and multi-turn injection, until this aperture is full and the space charge limit is reached.

The following table explores the effect of the injection Kinetic energy (T) on the space charge limit.

Procedure:

i Assign Kinetic injection energy, T

ii Calculate relativistic factors β and γ

iii Calculate phase-space density at the limit N_B/ε_N, from (1)

iv Calculate Normalized Emittance from

$$\varepsilon_N = (\beta\gamma)\,(40\,\pi\ \text{mm}\cdot\text{mrad})$$

v Calculate number of particles N_B that can be injected into the Booster

vi Assume 3 bunches and a 50% efficiency for full stripping between Booster and AGS

The number of particles / bunch at transfer to the Collider is

$$N = N_B / (2 \times 3)$$

vii Calculate revolution period $\tau = C / \beta c$

C, Booster circumference $= 201.75$ m

viii Assume a pulse from the Tandem 250 μsec and a d.c. current of 4.4 μAmp-particle

Calculate no. of particles per turn

$$N_B/\text{turn} = (4.4\ \mu\text{Amp-part.}) \times \tau$$

ix Calculate required numbers of turns at injection to get at the total N_B

$$\text{no. of turns} = N_B / (N/\text{turn})$$

x Calculate required pulse length $= (\text{no. of turns}) \times \tau$

T	β	γ	N_B/ε_N	ε_N	N_B
MeV			$\pi^{-1}\,m^{-1}$	π mm·mrad	
1	.04613	1.001066	4×10^{14}	1.85	7.4×10^{8}
1.5	.05648	1.001599	5×10^{14}	2.26	1.13×10^{9}
2.	.06519	1.002132	5.7×10^{14}	2.61	1.5×10^{9}
3.	.07978	1.003197	7×10^{14}	3.20	2.24×10^{9}
4.	.09204	1.004263	8.1×10^{14}	3.70	3×10^{9}
5.	.10283	1.005329	9.1×10^{14}	4.115	3.74×10^{9}

T	τ	N	N_B/turn	no. turns	Pulse length
MeV	μsec	$\times10^{8}$	$\times10^{8}$		μs
1.	14.6	1.23	4.	1.85	27.
1.5	11.9	1.88	3.3	3.4	40.5
2.	10.3	2.48	2.83	5.3	54.6
3.	8.43	3.73	2.32	9.7	81.8
4.	7.3	5.0	2.0	15	109.5
5.	6.54	6.24	1.8	21	137.

Performance

Long Bunches, High Intensity
Crossing at an Angle

$$L = N_c^2 B f_{rev} \Big/ 4\pi \left(\frac{\alpha}{2} \sigma_e\right) \sigma_v$$

B, no. of bunches = 3

N_c, no of particles / Bunch in colliding mode = 6×10

f_{rev}, revolution frequency = 78.1973 KHz

α, total crossing angle = 4 mrad

σ_e, rms bunch length = 50 m

σ_v, rms beam height = 0.037 mm

$$L = 1.8 \times 10^{27} \text{ cm}^{-2} \text{ s}^{-1}$$

Longitudinal Stability of the Long Bunches

$$\left|\frac{Z}{n}\right| = \frac{E\,|\eta|}{e\,I_p}\left(2\,\frac{\sigma_E}{E}\right)^2 \frac{A}{Z^2}$$

I_p, peak current $= N_c e \beta c \,/\, (\sigma_\ell \sqrt{2\pi})$

N_c, no. of particles / bunch $= 6 \times 10^{10}$

σ_ℓ, rms bunch length $= 50$ m $\quad (\sigma_t = 0.17\ \mu\text{sec})$

$$I_p = 0.023 \text{ Amp-particle} \quad (\beta \sim 1)$$

For Gold (Au) $\quad A = 197$ and $Z = 79$

Assume a coupling impedance of $|Z/n| = 10$ ohm

E, energy per nucleon ~ 100 GeV / A

$\eta = \gamma^{-2} - \gamma_T^{-2}$, γ_T = transition energy / rest energy

σ_E / E, rms energy spread at stability (could be larger)

B, bunch area $= 6\pi\, \sigma_E \cdot \sigma_t$

Assuming 3 bunches per beam the total area of each beam is

$$3B$$

The following table explores the dependence with γ_T of the threshold spread σ_E/E at 100 GeV/A and the unbunched beam rms spread at 12 GeV/A using the formula $\quad 3B = 4\, T_{revol}\, \sigma_E$, $T_{rev} = 12.788\ \mu$sec

γ_T	$\lvert\eta\rvert$	σ_E/E threshold @ 100 GeV/A	B eV/A-sec	σ_E/E unbunched @ 12 GeV/A
10	.0099	$.43 \times 10^{-4}$	13.8	$.675 \times 10^{-4}$
20	.0024	.87	27.9	1.4×10^{-4}
30	1.011×10^{-3}	1.34	42.9	2.0×10^{-4}
50	3×10^{-4}	2.5	80.1	3.9×10^{-4}
80	5.625×10^{-5}	5.7	182.7	8.9×10^{-4}

In the following table, T is the kinetic energy at injection into the Booster, N is the number of ions per bunch at transfer between AGS and Collider.

We assume: there are 3 bunches in the Booster, only these 3 bunches are accelerated in the AGS at the time, so that there are only 3 bunches transferred to the Collider every AGS pulse.

n, number of bunches from the AGS required to get $3 \times 6 \times 10^{10} = 1.8 \times 10^{11}$ ions per beam

The number of AGS pulses are therefore $n/3$

The Collider (each ring) is filled with 19 AGS pulses in a 'box-car' fashion ($h = 57$) and n_s RF stacking pulses.

The filling time is calculated assuming two minutes for a RF stacking cycle = (1 min AGS + 1 min RF)

T	N	n	no. of AGS pulses	filling sequence	Filling Time
MeV	$\times 10^8$				
1	1.23	1461	487	19 BoxCar × 77 RF stacking	2h 34 min
1.5	1.88	957	319	51	1h 42 min
2	2.48	726	242	39	1h 18 min
3	3.73	483	161	26	52 min
4.	5.0	360	120	19	38 min
5.	6.24	289	97	16	32 min

The average current at injection $(\gamma \sim 12)$ is

$$I = 225 \text{ mA-particle}$$

and the peak current at top energy $(\gamma \sim 100)$ is also the same

Take $\gamma_T = 10$

σ_E/E @ 100 GeV/A $\quad .43 \times 10^{-4}$

σ_E/E @ 12 GeV/A $\quad .675 \times 10^{-4}$

ε_N	γ	t_E	t_β
π·mm·mrad		hours	hours
2.	12	0.03	6.
4.	12	0.09	20.
2	100.	–	–
4.	100.	1.5	11

Individual Bunch Area at transfer from the AGS to the Collider

$S = 0.014$ eV/A-sec / 0.072

having included a 50% dilution factor for transfer, RF stacking and Acceleration.

~~[illegible] and [illegible]~~ ~~[illegible]~~

Take $\gamma_T = 20$

σ_E/E	@ 100 GeV/A	$.87 \times 10^{-4}$
σ_E/E	@ 12 GeV/A	1.4×10^{-4}

$\mathcal{E}_N$	γ	t_E	t_β
π·mm·mrad		hours	hours
2	12.	0.12	9.
4.	12.	0.3	40.
2.	100.	–	–
4.	100	5.6	15.

Individual Bunch Area at transfer from the AGS to the Collider

$$S = 0.0\cancel{2}4 \text{ eV/A-sec} \quad / \quad 0.14$$

having included a 50% dilution factor for transfer RF stacking and Acceleration,

Take $\gamma_T = 30$

σ_E/E @ 100 GeV/A 1.34×10^{-4}

σ_E/E @ 12 GeV/A 2.0×10^{-4}

ε_N	γ	t_E	t_β
πmm·mrad		hours	hours
2	12.	0.5	25
4.	12	1.2	56
2	100.	–	–
4.	100.	∞	20.

Individual Bunch Area at transfer from the AGS to the Collider

$$S = 0.044 \;/\; 0.22 \quad eV/A\text{-sec}$$

having included a 50% dilution factor for transfer, RF stacking and Acceleration.

Take $\gamma_T = 50$

σ_E / E @ 100 GeV/A $\quad 2.5 \times 10^{-4}$

σ_E / E @ 12 GeV/A $\quad 3.9 \times 10^{-4}$

ε_N	γ	t_E	t_β
π mm·mrad		hours	hours
2.	12.	12.	74.
4.	12	13.	160.
2.	100.	–	–
4.	100.	6.	30.

Individual Bunch Area at transfer from the AGS to the Collider

$S = 0.082 \;/\; 0.4$ eV/A - sec

having included a 50% dilution factor for transfer, RF stacking and Acceleration

Take $\gamma_T \doteq 80$

σ_E / E	@ 100 GeV/A	5.7×10^{-4}
σ_E / E	@ 12 GeV/A	8.9×10^{-4}

ε_N	γ	t_E	t_β
π mm-mrad		hours	hours
2.	12.	$\gtrsim 100$	$\gtrsim 100$
4.	12.	1,000	4,500
2.	100.	–	–
4.	100	200	50

Individual Bunch Area at transfer from the AGS to the Collider

$$S = 0.19 \; / \; 0.95 \quad eV/A-sec$$

having included ~~a~~ a 50% dilution factor for transfer, RF stacking and Acceleration.

The luminosity of $1.8 \times 10^{27}\ cm^{-2} s^{-1}$ can be obtained if

$$\sigma_v = 0.037\ mm$$

at the interaction region

This corresponds to a $\beta_v^* = 2m$ with a normalized emittance $\varepsilon_N = 0.4\ \pi \cdot mm \cdot mrad$

We have now

ε_N	β_v^*
0.4 π·mm·mrad	2m
2.0	0.4m
4.0	0.2m

or with $\beta_v^* = 2m$

ε_N	L
0.4 π·mm·mrad	$1.8 \times 10^{27}\ cm^{-2} s^{-1}$
2.0	0.8
4.0	0.57