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Long Term Tracking with PATRIS: Part 3: Dynamic Aperture - How Stringent is the Concept?

J. Milutinovic

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Collider Accelerator Department

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Accelerator Development Department Accelerator Physics Division BROOKHAVEN NATIONAL LABORATORY Associated Universities, Inc. Upton, NY 11973

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Part 3: Dynamic Aperture —
How Stringent is this Concept?

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Abstract

We continue our investigations of properties of dynamic aperture, by exploring its finer structure in some detail. Using the RHICAGR lattice¹ as a representative case and PATRIS² as our computational tool, we track particles for 1000 turns. By launching 25000 particles uniformly distributed in the emittance interval $[0, 25] \pi \cdot \text{mm} \cdot \text{mrad}$, and recording the actual number of turns each particle makes, we observe a great richness of different particles' behaviors, displaying regions of stability interspersed by regions of instability, in a fairly wide emittance region extending far beyond what we used to call "the edge of the dynamic aperture." Our first pictorial association with this behavior was a sort of "foam", while one of our colleagues used the expression "bubbles" and another one mentioned "holes". We introduce a so-called "particle survival density", which is 1 in a totally stable region, 0 in a region with no stable subdomains, and between 0 and 1 in regions containing both domains of stability and instability. The central conclusion of our investigation is that this density does not drop to zero like a step function at the edge of the dynamic aperture. Instead of this, it goes to zero in a series of descending peaks and valleys, implying survival of a significant number of particles beyond the edge of the dynamic aperture. It has to be seen whether this behavior is preserved if number of turns is

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increased beyond 1000 and this will be one of the most important subjects of our further long-term tracking studies. If the picture we present in this note remains essentially unchanged, then one should somewhat relax the notion (or practical significance) of the edge of the dynamic aperture, since a significant proportion of particles launched in these fuzzy regions might survive in the actual accelerator.

1. Introduction

One of the central concepts in particle tracking and one of its main subjects of investigation is dynamic aperture. It is usually defined as the region of stability, i.e. the region in which a particle launched for tracking survives a certain number of specified turns. Depending upon a particular study in question, this number varies and is usually taken to be from several hundred up to a few thousand turns for general investigations of particle stability and more for special investigations. In the latter case, it can be in the range of 10^5 - 10^6 turns.

Particle "survival" means that its distance from the origin in the plane of (X,Y) transverse coordinates during the course of tracking does not exceed a certain predetermined value, which is by most investigators taken to be 1000 millimeters. This number might look somewhat arbitrary, but in all cases of practical interest a particle whose distance from the origin surpasses 1000 mm very rapidly goes out and reaches astronomical distances and must be eliminated from tracking, otherwise the computer code typically crashes because of the numerical overflow condition. In practice, of course, one can check this distance only at finite number of places in the lattice and these places are typically taken to be positions of nonlinear elements, i.e. sextupoles and higher multipoles within dipoles and quadrupoles.

By particle's belonging to a certain region (of stability or instability) we mean that its initial coordinates lie in this region, i.e. the particle is launched from that region. This means that even in the case of purely transverse motion (i.e. without synchrotron oscillations) the region is four-dimensional, i.e. determined by the four phase space variables X, X', Y, and Y'. In case of coupling, the dynamical problem is really four-dimensional and the region of stability is expected to be a 4-D domain in the phase space, containing

the origin. Practical limitations (computer resources and the investigator's own time) necessitate a reduction in the number of initial conditions that can be explored. The first sacrifice of generality is the typical step of taking X' = Y' = 0, which restricts the initial conditions to a two-dimensional region around the origin in the (X, Y) plane, or equivalently a two-dimensional subset (plus initial betatron phase condition imposed) in the (ϵ_X, ϵ_Y) emittance plane.

Practical limitations act then to further reduce generality of most investigations and most researchers then choose X = Y initial conditions, or sometimes $X \neq 0$, Y = 0 or X = 0, $Y \neq 0$ conditions. We believe that this is too restrictive and have conducted our investigation for the total of eight directions in the (ϵ_X, ϵ_Y) plane, as described in our first technical note⁵ in this series. We would here remind the reader that the perfect mid-plane symmetry of the error-free lattice we use reduced the number of directions, along which we actually have to track particles, from eight to five.

Another concern is density of selected points for tracking in the $(\epsilon_{\rm X}, \epsilon_{\rm y})$ plane, or equivalently the step size we use to move on the emittance axis from a representative point to the next one. Most investigators use a fraction of a millimeter to separate the initial points in the emittance plane. For $\beta \sim 50 {\rm m}$ this corresponds to $\Delta \epsilon \sim 0.05 - 0.10 \ \pi \cdot {\rm mm \cdot mrad}$. We followed these guidelines in our initial investigations and often used even a smaller step size, e.g. $\Delta \epsilon = 0.04 \ \pi \cdot {\rm mm \cdot mrad}$. However, even with this fairly small step size one can miss some of the fine details. We briefly mentioned some of these problems in our previous technical note⁶ (see a discussion on page 8). Therefore, we opted for a much smaller step size $\Delta \epsilon = 0.001 \ \pi \cdot {\rm mm \cdot mrad}$ which then revealed much of the fine structure which we discuss in this note.

2. A Brief Description of the Tracking Method

We decided to explore a wide range of emittances, i.e. the whole interval between 0 and 25 π ·mm·mrad on the $\epsilon_{\rm X}$ or $\epsilon_{\rm Y}$ axis. The spacing we choose was $\Delta\epsilon=0.001~\pi$ ·mm·mrad and that means a total of 25000 particles for each of the five directions along which we actually track particles. PATRIS was modified to track in each run N=5001 particles, which were uniformly spread over an interval of 5 π ·mm·mrad. In each interval the desired subdivision was achieved since $\epsilon_{\rm TOT}/(N-1)=5/5000=0.001$. Hence, for each direction

in the (ϵ_x, ϵ_y) plane the whole 25 π ·mm·mrad interval was covered by five long tracking runs.

Each particle was tracked for 1000 turns and the actual number of turns it made was recorded in a separate file. After tracking in the direction #1, we realized that 5001 particles for 1000 turns took approximately 1 hour of the CRAY-2 CPU time. We found that this was typical even in the emittance interval [10.0, 15.0] π ·mm·mrad where over 50% of the tracked particles failed to make full 1000 turns. This was true even though PATRIS was previously modified so that while tracking it always checks to see if a particle was lost sometime before and, if it was, the particle is not subject to the action of the lattice magnets. This was done by skipping the whole action of the magnet on the previously lost particle, in the loop running over the whole bunch of particles (5001 in this case) in the magnet. The particle was retained in the loop but was subject to no action. The rate of consumption of the CRAY time was deemed to be too expensive. Each direction in the $(\epsilon_{\rm X}, \epsilon_{\rm Y})$ plane would take 5 hours and the total could surpass 25 hours of CRAY time, which was not a negligible portion of our annual allocation.

Two attempts were made to reduce the amount of actual computing. A simpler approach was to check after each turn if a particle was lost, starting from the particle with the highest emittance, i.e. with the top value of the counter in the loop running over the bunch of the tracked particles in each magnet. If a particle was lost, the next particle was checked and so on and so forth until the first surviving particle, i.e. the surviving particle with the highest emittance was found. Then the total number of particles was redefined to be the counter of this outermost surviving particle, i.e. the whole loop was truncated. This means that by dismissing the particles beyond the outermost survivor in each turn, we gradually shrink the size of the loop as we track and we save a considerable amount of computer time, despite the fact that each loop still contains dummies, i.e. lost particles which keep the loop oversized and still engage some computer resources in each turn. A second, more sophisticated approach was to check which particles were lost in each turn and totally dismiss them by relabeling the loop variables so that they describe only the survivors. In this way the loop was not only truncated but also compressed and the number of particles - the loop size was shrinking much more rapidly than in the case of simple truncation. This also resulted in a considerable saving of computer time, but the first operating version with this feature was less efficient that the simple truncating method. The relabeling of loops after each turn took time and this method consumed 20 - 50% more time than the simple one. One of the possible advantages of the loop compression method is the fact that it is vectorizable, unlike the simple method which must contain IF-statements in each loop to work at all. But vectorization was not attempted at this time and might remain as one of the future options in PATRIS⁷. Therefore, we pursued with the simple truncation method and completed tracking for all five directions with less than 10 CRAY hours, instead of 25.

3. Results of Tracking

As we have already mentioned, we record the number of turns each particle makes together with its initial emittance values in a separate file. The file for each direction in the $(\epsilon_{\rm X}, \epsilon_{\rm Y})$ plane contains 25000 lines, which are too many for any kind of reasonable, i.e. discernable plots. A quick inspection of these files revealed the fact that we knew from our previous work (and also known to some other researchers^{3,4}), namely that there is a big region of stability extending from the origin in the (ϵ_X, ϵ_Y) plane until some values of $(\epsilon_{x}, \epsilon_{y})$ where particles suddenly get lost. If we continue increasing the emittance values particles are still being lost until a new values of (ϵ_X, ϵ_y) are attained for which particle stability (for a fixed number of turns, being 1000 in our cases) is suddenly restored. One can then continue increasing the emittance values with no subsequent particle loss until another pair of (ϵ_X, ϵ_Y) values are reached and a particle loss starts again. One can still increase (ϵ_x, ϵ_y) and later pass into another narrow region of stability. In general, there are several regions of stability interspersed by regions of instability until a pair of (ϵ_x, ϵ_y) are reached beyond which no particle will survive the nominal number of turns (1000). The emittance range between the lost particles nearest the origin in the (ϵ_x, ϵ_y) plane and the most distant surviving particles can be quite considerable, especially in the directions #2 and #4. In the direction #4 we find the lost particles nearest the origin at $2.32 \pi \cdot \text{mm} \cdot \text{mrad}$ and took it as the edge of the dynamic aperture, while at the same time the most distant survivor was located at 9.2 π ·mm·mrad!

Before continuing, we would like to mention here that by inspecting the output files we also verified that the edge of the dynamic aperture found by our improved searching method and reported in our previous note⁶ in Table 1 was indeed correct. That note (see discussion on p. 8) also provides part of our motivation for the present study.

For further analysis, these output files had to be suitably processed. We decided to divide the emittance axis into subintervals (bins) and count the number of particles per bin which make more turns than certain specified thresholds. We chose the bin size to be 0.25 π ·mm·mrad and the thresholds to be at 200, 400, 600, 800 and 1000 turns. That meant the total of 100 bins in the whole emittance range between 0 and 25 π ·mm·mrad which we explore. We also introduce a so-called "particle surviving density" as the total number of particles per bin which survive 1000 turns divided by the total number of particles in the bin (i.e. 250 for our bin size and step size 0.001). The extremes of the emittance range produced trivial results. In the bins lying entirely inside the edge of the dynamic aperture, the total number of particles exceeding each threshold was always 250, since all of them survived, and particle surviving density was 1. At the other extreme all of the numbers were zero since none of the particles passed even the lowest threshold of 200 turns. In between these two extremes, numbers varied but not in a monotonic fashion, as we shall see later in this note.

Tables 1 through 5 contain the results of counting the surviving number of particles per bin (emittance interval). To save space each table is split into two halves and the second half is put beside the first one, on the same sheet of paper. Therefore, the right portion of the sheet contains just the continuation of the left portion. First column represents bins, i.e. the emittance intervals in which we count. The bin size is always 0.25π -mm·mrad, hence it initially contains 0.25/0.001 = 250 particles, uniformly spread over the bin. The next five columns represent the results of respective counting of the number of particles which survived 200, 400, 600, 800 and 1000 turns in the bin. Until a certain cutoff size of emittance is reached, all particles in the bin survive 1000 turns and all of the five columns contain 250. This is the region within the dynamic aperture. In column 7 we display the so-called particle surviving density, i.e. the fraction of the particles in the bin which survive 1000 turns. It is obtained simply by dividing the contents of the 6th column by 250. Within the dynamic aperture its value is obviously 1. Outside the dynamic aperture, it is clearly an average quantity and it depends on the bin size.

Once the threshold in the emittance values is reached, the particles start getting lost and columns 2 through 6 will tell us how many survived beyond certain number of turns. Obviously, numbers in columns 2 through 6 in each row must either be constant or decrease monotonically. However, numbers in each column do not decrease monotonically with the emittance increase and the same is true for the density in column 7. In certain regions as we increase the emittance we have more particles survive a specified number of turns and the particle surviving density starts increasing. It is also worthwhile to mention again that surviving density depends on the bin size and on the step size. For a fixed step size (0.001), an appropriate reduction of the bin size will create bins lying outside the narrow definition of dynamic aperture in which all of the particles survive and the density is 1. On the other hand, probing these bins with a step size smaller than 0.001 or with over 1000 turns might discover lost particles and again reduce the density below its value 1.

The results from Tables 1 through 5 are plotted in Figures 1 through 5. In these figures numbers associated with certain emittance ϵ refer to the bin $[\epsilon - 0.25, \epsilon]$. The numbers on the graphs are those from the columns 2 through 6. The density function would produce a curve whose shape would match the data referring to 1000 turns in these figures.

We notice that in all cases all particles survive until a threshold (the edge of the dynamic aperture in the narrow sense) in the emittance plane is reached and after that the density of surviving particles starts dropping. This drop continues until a local minimum is reached and then the density starts increasing until a peak is attained. In a bin around this peak a significant proportion of particles survive 1000 turns. After this peak, there are other lower peaks, where fewer particles survive 1000 turns, but most of them still make more turns than particles in the preceding dip.

Figure 6 displays the number of particles which survive a certain number of turns versus the number of turns. On the horizontal axis we plot the thresholds of 200, 400, 600, 800 and 1000 turns, while the number of particles surpassing each threshold is on the vertical axis. This diagram refers to the direction #1 and is a cross section obtained by drawing vertical lines at various emittances (bins ending at 12.50, 12.75, 13.00 and 13.25 π ·mm·mrad, respectively) in Figure 1 and reading off the numbers at the intersections of this vertical line and the interpolating curves of Figure 1, referring to the cases of 200, 400, 600, 800 and 1000 turns, respectively. Figure 6 shows clearly an example of how

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the definition of "dynamic aperture" can change with the number of revolutions being surveyed.

As before, the five directions in the emittance plane fall into three categories of similar cases. Case #1 and #5 form one group, case #2 and #4 another one, while case #3 stands alone. Inspection of primary (i.e. unprocessed) output files, Tables 1 through 5 and Figures 1 through 5 confirm this. By comparing case #5 with #1 and case #4 with #2, one can observe that for X < 0 initial conditions the onset of instability comes slightly closer to the origin in the (ϵ_X, ϵ_Y) plane, but once the first region of instability is reached the amplitude growth is somewhat less rapid. Therefore, even if a particle is unstable it will have a tendency to reach higher thresholds in the selected set of 200, 400, 600 and 800 turns. The figures indicate that more particles reach the appropriate thresholds for X < 0 than for X > 0 initial conditions.

We also mentioned that more efforts were needed to pinpoint the edge of the dynamic aperture in case #4 and in our previous technical note⁶ we substantially revised the result reported in the first note⁵ (Table 1 of the second note). The reason why case #4 was tougher than case #2 comes from the fact that the stable regions outside the edge of the dynamic aperture were thicker in case #4 and this resulted in a higher probability to miss the edge of the dynamic aperture during the searching procedure. This property, however, is not seen from Tables 2 and 4 or Figures 2 and 4. The reason is that the typical width of an external stable region is $\sim 0.010 - 0.015 \,\pi\cdot\text{mm}\cdot\text{mrad}$, which is small when compared with the bin size and so these subtle differences between case #2 and #4 become averaged out.

4. Conclusion

This detailed investigation of fine structure of dynamic aperture has answered some questions and opened up new ones. The most pressing question and a definite subject of our future investigations is what happens when the number of attempted tracking turns is increased beyond N = 1000. For example, what happens with the edge of the dynamic aperture? Does it shrink toward the origin in the emittance plane, and how much if it does? Next, what happens with a stable region lying outside the edge of the dynamic aperture? If a bunch of particles, spaced by 0.001 intervals on the emittance axis, all survive 1000

Conclusion 9

turns, what happens if we go to 10^4 , 10^5 or 10^6 turns? What happens if we probe this external region of stability with finer divisions, at each $0.0001 \, \pi \cdot \text{mm·mrad}$? And if the central stable region shrinks as N grows and then stops shrinking at some "magic" N, a minimum for a realistic determination of the dynamic aperture, what happens with all the external stable (for $N=1000 \, \text{turns}$) regions? Will any survive or will they all die out and leave the central region as a clean and unequivocal dynamic aperture? We hope to be able to catch a glimpse of what is going on here.

There are other interesting questions. We discovered these external stable regions in all eight directions in the (ϵ_x, ϵ_y) plane. If we introduce more directions and probe the dynamic aperture in between these eight directions, we can also expect to find similar regions. By continuity, these external regions of stability must be two-dimensional in the emittance plane. This is obvious, since if we are in a stable region on a particular direction and rotate this direction slightly we will find stable particles on the rotated direction, provided the rotation is not too big. Now the question is whether these external stable regions are connected with the central stable region via some connecting stable bands meandering in the (ϵ_x, ϵ_y) plane or not. If not, are the external stable regions bands of stability encircling the central region or isolated islands? Or, are both bands and islands present? Our feeling is that at least those external regions which are closest to the central region do form bands, but this expectation is difficult to verify numerically. Another question is what happens if the condition X' = Y' = 0 is lifted. Would the whole 4-D region of the dynamic aperture still have isolated external regions of stability or would they now all be connected to the central region? All these questions are academically interesting, but are beyond the scope of any practical investigations.

And finally, if we drastically increase the density of test particles per emittance interval, would the central region become porous, i.e. would it start displaying small holes down to the origin in the $(\epsilon_{\rm X}, \epsilon_{\rm y})$ plane? An affirmative answer would shed interesting light on the true nature of dynamic aperture, but in practical terms could not be significant. After all, in a well designed accelerator an overwhelming majority of particles in a good beam do indeed survive.

10 Comments

5. Comments

At this point we would like to reiterate the purpose of our research. For instance, somebody may be wondering why we are taking RHICAGR as the reference lattice. First of all, we want to point out that our goal is not to determine the dynamic aperture or the stability limit of a particular storage ring project; this is definitely not our task; rather our aim is to learn the physics and the effects that are causing the limitations. We are convinced that these effects are the same in every collider except that there may be variations in magnitude. The features of behavior we have already disclosed in this and previous notes are common to every project. Our aim is a systematic approach and a critical review of the methods being commonly used to estimate the dynamic aperture. Everyone else can then follow our approach and apply it more specifically to the project of their interest. For example, we count already three researchers doing tracking for the RHIC project and therefore our involvement in that direction is not necessary.

Moreover, when we initiated our research, the RHIC lattice had not been finalized yet and at that time we needed something close enough to reality to use for our program. We adopted RHICAGR since it was already available and also for the following reasons. RHICAGR has a higher degree of symmetry which makes it easy to represent as input for any computer code for tracking; it is also easier to examine and understand any particular feature in a closer investigation. So far we have only analyzed the effects of sextupoles added to the lattice for the chromaticity correction. These effects appear to be stronger when compared to those in RHIC91 and RHIC92; but we have used these effects intentionally to make our results more apparent. Very soon we shall include other ingredients in our research, like multipole field errors, gradient errors, closed orbit deviations and synchrotron oscillations. Our study will then consider the theme of long term behavior.

6. References

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- 7. As a matter of fact, after the completion of this work, we designed a vectorized version of the "loop compression method" for PATRIS. This first vectorized version can certainly be further optimized, but even without further optimization it was about 30% more economical than the "loop truncation method", used to complete this work.

Table 1. Emittance intervals and number of particles which survive more than a specified number of turns. Case #1

• • • • • • • • • • • • • • • • • • • •														
EMITTANCE INTERVAL			RTICLE	S SURV	IVING OVER	SURV.	EMITTANCE INTERVAL (Pi.mm.mrad)	NUMBE 200	R OF P 400	ARTICL 600	ES SUR. 800	VIVING 1000	OVER turns	SURV.
(Pi.mm.mrad)	200	400	600	800	1000 turns			050	040	246	242	239		0.956
Γ 0.00 - 0.25]	250	250	250	250	250	1.000	[12.50 - 12.75]	250	249 239	246 229	221	212		0.848
0.25 - 0.50	250	250	250	250	250	1.000	12.75 - 13.00	249 216	178	150	125	105		0.420
0.50 - 0.75	250	250	250	250	250	1.000	[13.00 - 13.25]	250	247	242	238	234		0.936
0.75 - 1.00	250	250	250	250	250	1.000	[13.25 - 13.50]	206	164	146	138	128		0.512
1.00 - 1.25]	250	250	250	250	250	1.000	[13.50 - 13.75] [13.75 - 14.00]	43	21	15	11	10		0.040
1.25 - 1.50]	250	250	250	250	250	1.000	14.00 - 14.25	41	18	10	4	3		0.012
[1.50 - 1.75]	250	250	250	250	250	1.000	14.25 - 14.50	14	3	1	1	1		0.004
[1.75 - 2.00]	250	250	250	250	250	1.000 1.000	14.50 - 14.75	40	12	5	2	2		0.008
[2.00 - 2.25]	250	250	250	250	250	1.000	14.75 - 15.00	34	12	5	4	4		0.016
[2.25 - 2.50]	250	250	250	250	250 250	1.000	15.00 - 15.25	15	4	0	0	0		0.000
[2.50 - 2.75]	250	250	250	250 250	250	1.000	15.25 - 15.50]	1	1	1	1	1		0.004
[2.75 - 3.00]	250	250	250 250	250	250	1.000	15.50 - 15.75	0	0	0	0	0		0.000
[3.00 - 3.25]	250	250 250	250	250	250	1.000	15.75 - 16.00	0	0	0	0	0		0.000
[3.25 - 3.50]	250	250 250	250	250	250	1.000	[16.00 - 16.25]	2	0	0	0	0		0.000
[3.50 - 3.75]	250 250	250	250	250	250	1.000	16.25 - 16.50]	4	1	1	0	0		0.000
[3.75 - 4.00]	250 250	250	250	250	250	1.000	16.50 - 16.75]	6	2	2	2	2		0.008
[4.00 - 4.25]	250	250	250	250	250	1.000	[16.75 - 17.00]	19	8	4	2	2		0.008
[4.25 - 4.50]	250	250	250	250	250	1.000	[17.00 - 17.25]	5	3	3	3	2		0.008
[4.50 - 4.75] 4.75 - 5.00	250	250	250	250	250	1.000	[17.25 - 17.50]	0	0	0	0	0		0.000 0.000
	250	250	250	250	250	1.000	[17.50 - 17.75]	0	Ø	0	0	0		0.000
	250	250	250	250	250	1.000	[17.75 - 18.00]	1	1	1	1	0 0		0.000
[5.25 - 5.50] [5.50 - 5.75]	250	250	250	250	250	1.000	[18.00 - 18.25]	0	0	0	0	0		0.000
5.75 - 6.00	250	250	250	250	250	1.000	[18.25 - 18.50]	0	0	0	0 0	ø		0.000
6.00 - 6.25	250	250	250	250	250	1.000	[18.50 - 18.75]	0	0	0 0	ø	ő		0.000
6.25 - 6.50	250	250	250	250	250	1.000	[18.75 - 19.00]	0	0 0	0	ő	ő		0.000
6.50 - 6.75	250	250	250	250	250	1.000	[19.00 - 19.25]	0	0	0	ő	ø		0.000
6.75 - 7.00	250	250	250	250	250	1.000	19.25 - 19.50	0 0	Ø	ø	ő	ĕ		0.000
7.00 - 7.25	250	250	250	250	250	1.000	[19.50 - 19.75]	9	ø	ő	ø	ă		0.000
7.25 - 7.50	250	250	250	250	250	1.000	[19.75 - 20.00]	ø	ø	ø	ŏ	ŏ		0.000
7.50 - 7.75	250	250	250	250	250	1.000	[20.00 - 20.25]	ő	ø	ĕ	ĕ	õ		0.000
7.75 - 8.00	250	250	250	250	250	1.000	[20.25 - 20.50] [20.50 - 20.75]	ä	ŏ	õ	Õ	0		0.000
8.00 - 8.25	250	250	250	250	250	1.000	20.75 - 21.00	ě	ŏ	0	0	0		0.000
8.25 - 8.50	250	250	250	250	250	1.000	21.00 - 21.25	ŏ	ø	ø	0	0		0.000
[8.50 - 8.75]	250	250	250	250	250	1.000 1.000	21.25 - 21.50	ĕ	0	0	0	0		0.000
[8.75 - 9.00]	250	250	250	250	250	1.000	21.50 - 21.75	ē	0	0	0	0		0.000
[9.00 9.25]	250	250	250	250	250	1.000	21.75 - 22.00	ø	0	0	Ø	0		0.000
[9.25 - 9.50]	250	250	250	250	250 250	1.000	[22.00 - 22.25]	Ø	0	0	0	ø		0.000
[9.50 - 9.75]	250	250	250	250 250	250	1.000	22.25 - 22.50]	3	0	0	0	0		0.000
[9.75 - 10.00]	250	250	250	250 250	250	1.000	22.50 - 22.75	1	0	0	0	0		0.000
[10.00 - 10.25]	250	250	250 250	250	250	1.000	22.75 - 23.00]	1	0	0	0	0		0.000
[10.25 - 10.50]	250	250	250	250	250	1.000	23.00 - 23.25]	0	0	0	0	0		0.000 0.000
[10.50 - 10.75]	250	250 250	250	250	250	1.000	23.25 - 23.50	0	0	0	0	9		0.000
[10.75 - 11.00]	250 250	250	250	250	250	1.000	[23.50 - 23.75]	0	0	0	0	9		0.000
[11.00 - 11.25]	250 250	250	250	250	250	1.000	[23.75 - 24.00]	0	0	0	0 0	Ø		0.000
[11.25 - 11.50] [11.50 - 11.75]	250	250	250	250	250	1.000	[24.00 - 24.25]	0	0	0 0	9	ø		0.000
11.75 - 12.00	250	250	250	250	250	1.000	[24.25 - 24.50]	Ø	0	9	ø	ő		0.000
[12.00 - 12.25]	250	250	250	250	250	1.000	[24.50 - 24.75]	0 0	9	ő	ĕ	ø		0.000
12.25 - 12.50	250	250	250	250	250	1.000	[24.75 - 25.00]	U	v	U	· ·	•		
[12.25 - 12.50]	250	250	250	250	250	1.000	(24.70 20.00)							

Table 2. Emittance intervals and number of particles which survive more than a specified number of turns. Case #2

EMITTANCE INTERVAL (Pi.mm.mrad)		OF PARTIC 400 600	ES SUF 800	VIVING OVER 1000 turns	SURV.	EMITTANCE INTERVAL (Pi.mm.mrad)	NUMBER 200	OF P	ARTICL 600	ES SUR 800	VIVING OVER 1000 turns	SURV. DENS.
0.00 - 0.25 0.50 - 0.50 0.55 - 0.50 0.575 0.75 - 1.00 1.00 - 1.25 1.25 - 1.50 1.75 0.75 - 2.00 2.00 - 2.25 2.25 - 2.50 2.50 - 2.75 2.75 - 3.00 3.25 - 3.50 3.25 - 3.50 3.25 - 3.50 3.25 - 3.50 3.50 - 3.25 4.50 - 4.50 4.50 - 4.50 4.50 - 4.50 4.50 - 4.50 4.50 - 5.25 5.25 - 5.50 5.25 - 5.50 5.25 - 5.50 5.25 - 6.25 - 6.50 6.50 - 6.25 6.50 - 6.50 - 6.25 6.50 - 6.50 - 6.50 6.50 - 6.50 6.50 - 6.50 6.50 - 7.50 7.50 - 7.50 7.50 - 7.50 7.50 - 7.50 7.50 - 7.50 7.50 - 9.50 9.25 - 9.50 9.25 - 9.50 9.25 - 9.50 9.25 - 9.50 9.25 - 9.50 9.25 - 9.75 9.75 - 10.00 10.00 - 10.25	250 250 250 250 250 250 250 250 250 250	400 600 250 250 250 250 250 250 250 250 250 2	250 250 250 250 250 250 250 250 250 250	250 250 250 250 250 250 250 250 250 250	DENS. 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 0.2448 0.1468 0.1468 0.1468 0.1484 0.1484 0.1044 0.10488 0.1484 0.10488 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000	(Pi.mm.mrad) [12.50 - 12.75] [12.75 - 13.00] [13.00 - 13.25] [13.25 - 13.50] [13.50 - 13.75] [13.75 - 14.00] [14.00 - 14.25] [14.25 - 14.50] [14.50 - 14.75] [14.75 - 15.00] [15.00 - 15.25] [15.50 - 15.75] [15.50 - 15.75] [15.75 - 16.00] [16.00 - 16.25] [16.50 - 16.75] [16.75 - 17.00] [17.00 - 17.25] [17.25 - 17.50] [17.50 - 17.75] [17.75 - 18.00] [17.50 - 17.75] [17.75 - 18.00] [18.00 - 18.75] [18.75 - 19.00] [19.50 - 19.25] [19.25 - 19.50] [19.50 - 19.75] [19.25 - 20.00] [20.00 - 20.25] [20.15 - 21.50] [21.00 - 21.75] [21.75 - 22.00] [22.00 - 22.25] [22.25 - 22.50] [22.25 - 22.75]	20 000000000000000000000000000000000000	49		80 00000000000000000000000000000000000	1000 turns 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	DENS
	000000000000000000000000000000000000000	•	•	•		22.25 - 22.36 22.50 - 22.75 22.75 - 23.00 23.00 - 23.25 23.25 - 23.50 23.50 - 23.75 23.75 - 24.00 24.00 - 24.25 24.25 - 24.50 24.50 - 24.75 24.75 - 25.00	-		0000000000000	-	-	

Table 3. Emittance intervals and number of particles which survive more than a specified number of turns. Case #3

Table 4. Emittance intervals and number of particles which survive more than a specified number of turns. Case #4

							l	<u> </u>					
EMITTANCE INTERVAL (Pi.mm.mrad)	NUMBER 200	OF PAI 400			VIVING OVER 1000 turns	SURV.	EMITTANCE INTERVAL (Pi.mm.mrad)	NUMBER 200	OF P	ARTICLI 600	ES SUR 800	VIVING OVER 1000 turns	SURV. DENS.
(Pi.mm.mrad) 0.00 - 0.25 0.25 - 0.50 0.50 - 0.75 0.75 - 1.00 1.00 - 1.25 1.25 - 1.50 1.50 - 1.75 1.75 - 2.00 2.00 - 2.25 2.25 - 2.50 2.50 - 2.75 2.75 - 3.00 3.00 - 3.25 3.25 - 3.50 3.50 - 3.75 3.75 - 4.00 4.00 - 4.25 4.25 - 4.50 4.50 - 4.50 4.50 - 4.75 4.75 - 5.00 5.00 - 5.25 5.25 - 5.75 5.75 - 6.00 6.00 - 6.25 6.25 - 6.50 6.50 - 6.75 6.75 - 7.00 7.00 - 7.25 7.75 - 8.00 8.00 - 8.25 8.25 - 8.50 8.50 - 8.75	250 250 250 250 250 250 250 250 250 250	250 250 250 250 250 250 250 250 250 250	250 250 250 250 250 250 250 250 250 250	800 2500 255000 25500 25500 25500 25500 25500 25500 25500 25500 25500 255000 25500 25500 25500 25500 25500 25500 25500 25500 25500 25500 255000 25500 25500 25500 25500 25500 25500 25500 25500 25500 255000 25500	250 250 250 250 250 250 250 250 250 250	DENS. 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 0.636 0.588 0.6576 0.536 0.716 0.704 0.640 0.624 0.728 0.720 0.720 0.728 0.720 0.728 0.728 0.268 0.268 0.076 0.008 0.004	(Pi.mm.mrad) 12.50 - 12.75 12.75 - 13.00 13.00 - 13.25 13.25 - 13.50 13.50 - 13.75 13.75 - 14.00 14.00 - 14.25 14.25 - 14.50 14.50 - 15.25 15.25 - 15.25 15.50 - 15.75 15.75 - 16.00 16.00 - 16.25 16.25 - 16.50 16.50 - 16.75 16.75 - 17.00 17.00 - 17.25 17.25 - 17.50 17.50 - 17.75 17.75 - 18.00 18.00 - 18.25 18.25 - 18.50 18.50 - 18.75 18.75 - 19.00 18.00 - 18.25 18.75 - 19.00 19.00 - 19.25 19.25 - 19.50 19.50 - 19.75 19.75 - 20.00 20.00 - 20.25 20.75 - 21.00 21.00 - 21.25	200 00000000000000000000000000000000000		000000000000000000000000000000000000000	80 000000000000000000000000000000000000	1000 turns 000000000000000000000000000000000000	DENS. 0.000
		114000110000000000000000000000000000000	1130001000000000000	-	-			-	000000000000000000	-	-		

Table 5. Emittance intervals and number of particles which survive more than a specified number of turns. Case #5

									ADTIOL	FC CUD	VIVINO OVER	I SURV.
EMITTANCE INTERVAL (Pi.mm.mrad)			TICLES SU 600 800	RVIVING OVER 1000 turns	SURV.	EMITTANCE INTERVAL (Pi.mm.mrad)	200	400	600		VIVING OVER 1000 turns	
[0.00 - 0.25]	250	250 2	250 250	250	1.000	[12.50 - 12.75]	243	206	183	163	154	0.616
0.25 - 0.50			250 250	250	1.000	[12.75 - 13.00]	194	138	111	94	77	0.308
1 0.50 - 0.751			250 250	250	1.000	[13.00 - 13.25]	190	108	76	60	48 5.6	0.192
0.75 - 1.00	250	250 2	250 250	250	1.000	[13.25 - 13.50]	213	148	101	71	56 14	0.224 0.056
[1.00 - 1.25]			250 250	250	1.000	[13.50 - 13.75]	82	37	22 8	17 6	4	0.016
[1.25 - 1.50]			250 250	250	1.000	[13.75 - 14.00]	33 153	18 47	14	4	2	0.008
[1.50 - 1.75]			250 250	250	1.000	[14.00 - 14.25] [14.25 - 14.50]	153 166	46	22	11	9	0.036
[1.75 - 2.00]			250 250 250 250	250 250	1.000 1.000	14.50 - 14.75	153	82	46	40	38	0.152
[2.00 - 2.25]			250 250 250 250	250	1.000	14.75 - 15.00	126	41	15	7	2	0.008
[2.25 - 2.50] 2.50 - 2.75			250 250 250 250	250	1.000	15.00 - 15.25	75	26	14	7	5	0.020
2.75 - 3.00			250 250 250 250	250	1.000	[15.25 - 15.50]	78	29	9	3	1	0.004
1 3.00 - 3.251			250 250	250	1.000	[15.50 - 15.75]	48	13	. 7	6	6	0.024
3.25 - 3.50			250 250	250	1.000	[15.75 - 16.00]	45	19	15	12	9	0.036
3.50 - 3.75	250		250 250	250	1.000	[16.00 - 16.25]	_2	1	1	1 7	1	0.004 0.016
[3.75 - 4.00]			250 250	250	1.000	[16.25 - 16.50]	35 4	16	7 1	1	4	0.004
[4.00 - 4.25]			250 250	250	1.000	[16.50 - 16.75]	12	2 5	4	4	4	0.016
[4.25 - 4.50]			250 250	250	1.000	[16.75 - 17.00] [17.00 - 17.25]	1	1	õ	õ	o o	0.000
[4.50 - 4.75]			250 250 250 250	250 250	1.000 1.000	17.25 - 17.50	ė	ė	ŏ	ĕ	ő	0.000
[4.75 - 5.00]			250 250 250 250	250 250	1.000	17.50 - 17.75	1	ĕ	ø	0	Ø	0.000
[5.00 - 5.25] 5.25 - 5.50			250 250 250 250	250 250	1.000	17.75 - 18.00	Ó	ø	0	0	0	0.000
5.50 - 5.75			250 250	250	1.000	18.00 - 18.25	0	0	0	0	0	0.000
5.75 - 6.001			250 250	250	1.000	[18.25 - 18.50]	0	0	0	0	0	0.000
6.00 - 6.25			250 250	250	1.000	[18.50 - 18.75]	0	0	0	0	0	0.000
6.25 - 6.501	250	250 2	250 250	250	1.000	[18.75 - 19.00]	0	0	0	0	0	0.000 0.000
[6.50 - 6.75]			250 250		1.000	[19.00 - 19.25]	0	0	0 0	0 0	0 0	0.000
[6.75 7.00]			250 250		1.000	19.25 - 19.50	0 0	0	ő	ø	0	0.000
[7.00 - 7.25]			250 250		1.000	[19.50 - 19.75] [19.75 - 20.00]	ő	ø	ő	ø	ĕ	0.000
[7.25 - 7.50]			250 250		1.000 1.000	20.00 - 20.25	ő	ø	ő	ĕ	ě	0.000
[7.50 - 7.75]			250 250 250 250		1.000	20.25 - 20.50	ŏ	ĕ	ĕ	ĕ	ø	0.000
[7.75 - 8.00]			250 250 250 250		1.000	20.50 - 20.75	ŏ	ŏ	Õ	Ø	0	0.000
[8.00 - 8.25] [8.25 - 8.50]			250 250 250 250		1.000	20.75 - 21.00	0	0	0	0	Ø	0.000
8.50 - 8.75			250 250		1.000	21.00 - 21.25]	0	0	0	0	0	0.000
8.75 - 9.001			250 250		1.000	[21.25 - 21.50]	0	0	Ø	0	0	0.000
9.00 - 9.25		250	250 250		1.000	[21.50 - 21.75]	0	0	0	0 0	0 0	0.000 0.000
9.25 - 9.50]			250 250		1.000	[21.75 - 22.00]	0 0	0 0	0 0	9	0	0.000
[9.50 - 9.75]			250 250		1.000	[22.00 - 22.25] [22.25 - 22.50]	0	a	ø	ő	ø	0.000
[9.75 - 10.00]			250 250		1.000 1.000	22.50 - 22.75	ő	ĕ	ĕ	ĕ	ĕ	0.000
[10.00 - 10.25]			250 250 250 250		1.000	22.75 - 23.00	ĕ	ĕ	ĕ	ŏ	Ö	0.000
[10.25 - 10.50]			250 250		1.000	23.00 - 23.25	ø	ø	0	0	0	0.000
[10.50 - 10.75] [10.75 - 11.00]			250 250		1.000	[23.25 - 23.50]	0	0	0	0	0	0.000
11.00 - 11.251			250 250		1.000	[23.50 - 23.75]	0	0	0	0	0	0.000
11.25 - 11.501		250	250 250	250	1.000	[23.75 - 24.00]	Ø	. 0	Ø	0	Q Ø	0.000 0.000
11.50 - 11.75]			250 250		1.000	[24.00 - 24.25]	9	0 0	0 0	0 0	0	0.000
[11.75 - 12.00]			250 250		0.996	$ \begin{bmatrix} 24.25 - 24.50 \\ 24.50 - 24.75 \end{bmatrix} $	a	ø	ø	ő	ő	0.000
[12.00 - 12.25]			240 229		0.852 0.772	24.75 - 25.00	ě	ő	ĕ	ĕ	ĕ	0.000
[12.25 - 12.50]	250	244	228 215	193	0.772	[[24.70 - 20.00]	•	•	-	=		

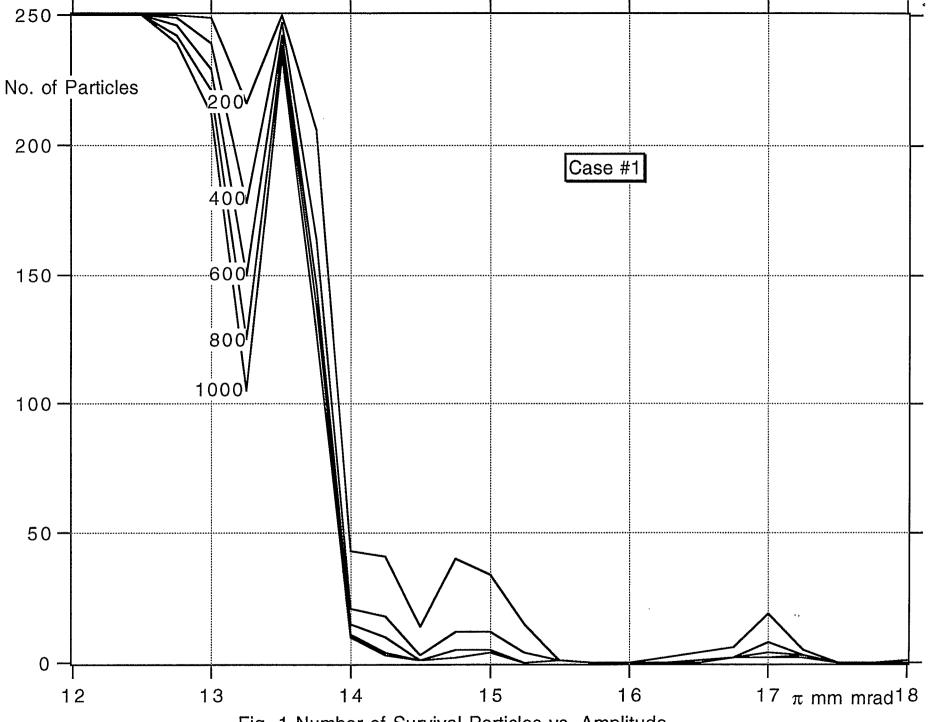


Fig. 1 Number of Survival Particles vs. Amplitude

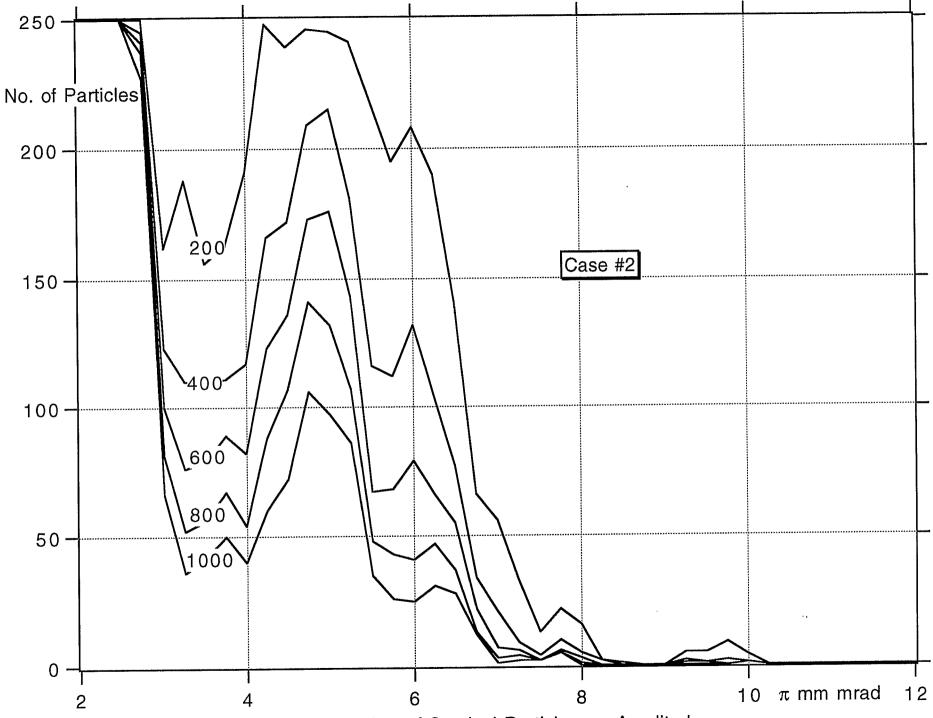
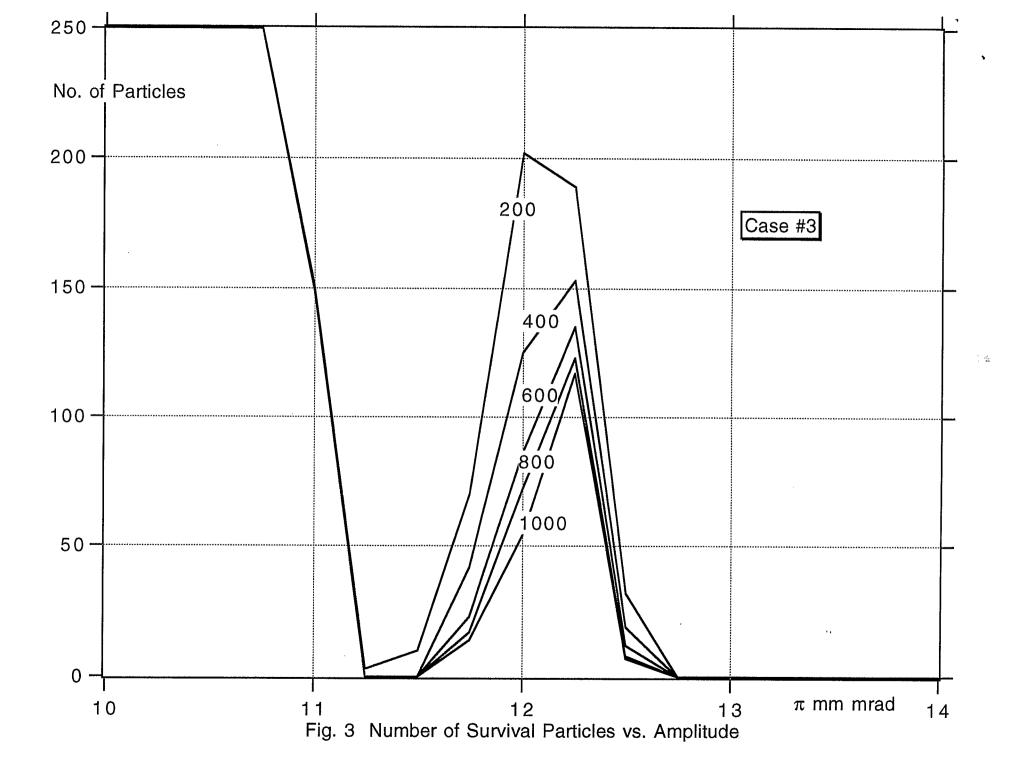
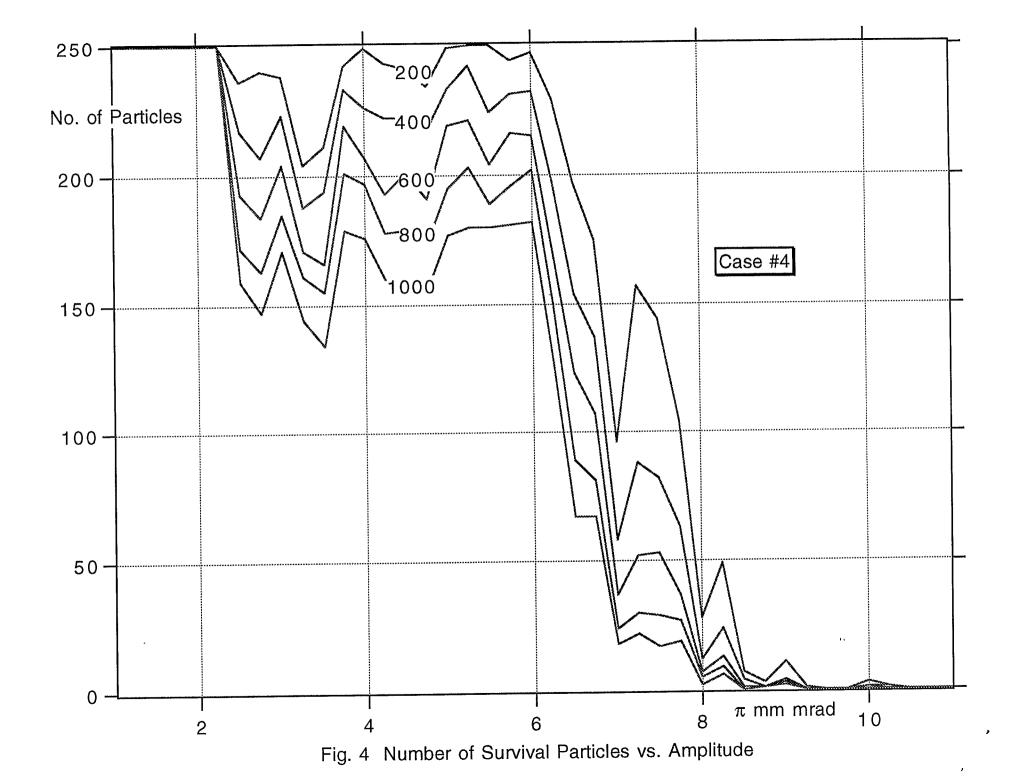


Fig. 2 Number of Survival Particles vs. Amplitude





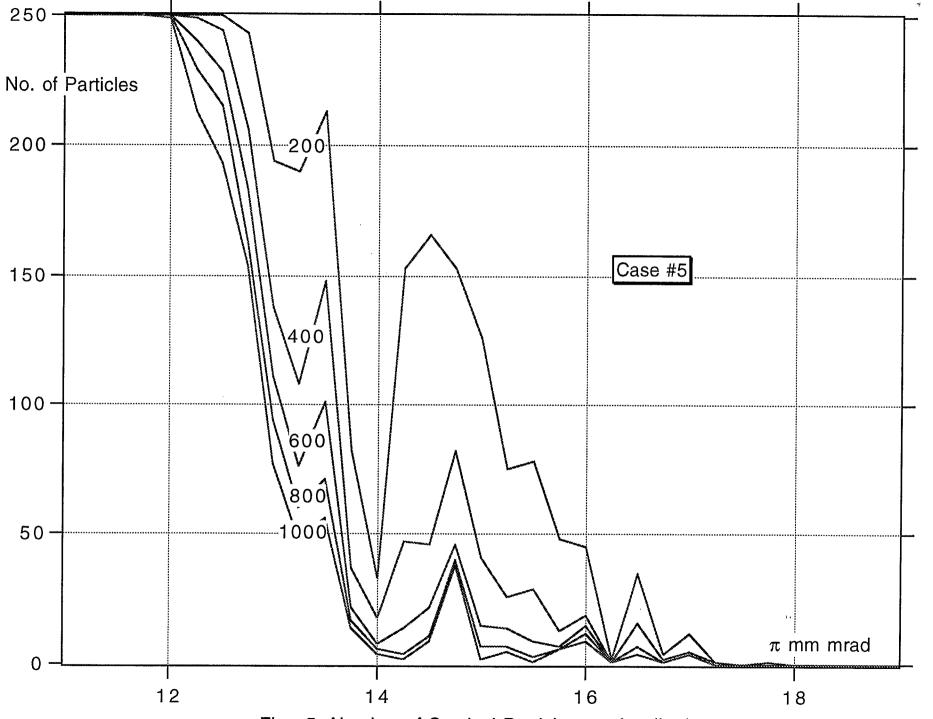


Fig. 5 Number of Survival Particles vs. Amplitude

