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A General Scheme of Solenoid Compensation Using Antisolenoids. An Example: Decoupling of the STAR Detector in RHIC

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A General Scheme of Solenoid Compensation Using Antisolenoids. An Example: Decoupling of the STAR Detector in RHIC

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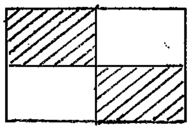
1. Introduction

In this note I would like to describe a general scheme of solenoids compensation[1–3] using, so-called, "projection" approach due to Y. Kobayashi[4] and S. Peggs.[5] The scheme applies to the linear coupling compensation produced by a thin solenoid, thus it applies to the decoupling of detector's solenoids placed at interaction points in large modern colliders. One of the two detectors, called STAR, which will be installed in RHIC provide an opportunity to demonstrate the general decoupling scheme described in the sequel.

2. Description of the Method

Assume that a lattice, without solenoids, is decoupled and that corresponding 4×4 transfer matrix $T_0\left(s'', s'\right)$ is of the block-diagonal form

$$T_0 = \left[\begin{array}{c|c} T_{0x} & \mathbf{0} \\ \hline \mathbf{0} & T_{0y} \end{array}\right] =$$  . (2.1)

In this case the horizontal (x, p_x), and the vertical (y, p_y) variables entering a state-vector

$$z = \begin{bmatrix} x \\ p_x \\ y \\ p_y \end{bmatrix}, \tag{2.2}$$

transform independently from one another when T_0 is applied – they are decoupled.

Imagine now that some insertion containing a number of thin solenoids was installed in a ring, between the points A and B, as shown in Fig. 1. Later on, one of the solenoids will be identified as the STAR detector solenoid, and the other will serve as anti-solenoids which compensate it.

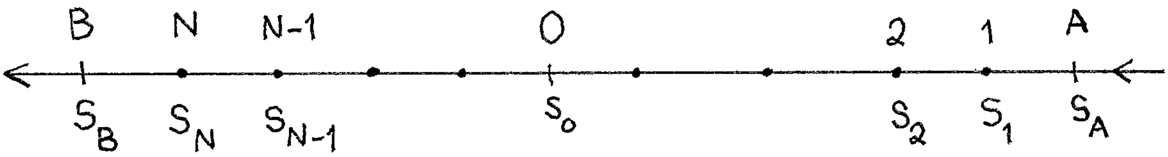
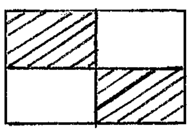


Fig. 1.: Schematic layout of N thin solenoids installed in a ring, around the reference point 0.

In order to ensure that the enriched lattice remains uncoupled we demand that the transfer matrix T_{BA} of the insertion is also of the block-diagonal form

$T_{BA} =$  , (Decoupling Conditions). (2.3)

The product $T_{BA}T_0$ will then also be the block-diagonal. In this way we will find conditions on the solenoid's strengths, at which the insertion is decoupled.

The T_{BA} transfer matrix is given by the expression

$$\begin{aligned} T_{BA} = T_0\left(s_B, s''_N\right) T_N\left(s''_N, s'_N\right) T_0\left(s'_N, s''_N\right) T_{N-1}\left(s''_{N-1}, s'_{N-1}\right) \dots \\ \dots T_2\left(s''_2, s'_1\right) T_0\left(s'_2, s''_1\right) T_1\left(s''_1, s'_1\right) T_0\left(s'_1, s_A\right). \end{aligned} \tag{2.4}$$

where the coordinates s_k are placed in a middle of the k-th solenoid while $s'_k \equiv s_k - 1/2\ \ell_k$ and $s''_k \equiv s_k + 1/2\ \ell_k$ are attached to its ends, as shown in Fig. 2.

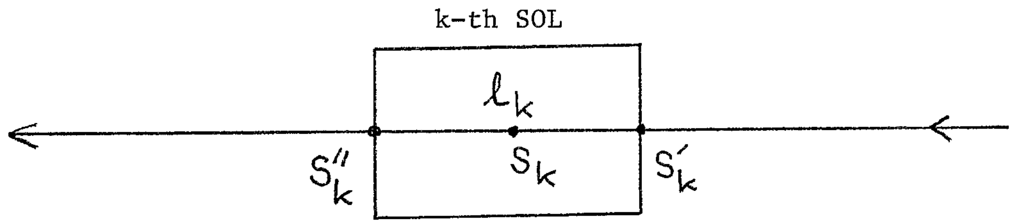


Fig. 2.: Coordinates of the k-th solenoid.

Using, so called, the "projection" on the k-th solenoid

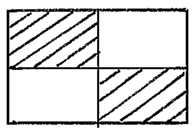
$$P_k\left(s_k, s_0\right) \equiv T_0\left(s_0, s''_k\right) T_k\left(s''_k, s'_k\right) T_0\left(s'_k, s_0\right), k = 1, \ldots, N, \tag{2.5}$$

the transfer matrix T_{BA} of the insertion may be rewritten as follows

$$T_{BA} = T_0\left(s_B, s_0\right) P_N\left(s_N, s_0\right) \cdots P_1\left(s_1, s_0\right) T_0\left(s_0, s_A\right). \tag{2.6}$$

This basic formula can be verified by induction, for example. The decoupling conditions (2.3) can be now expressed as follows

$$P_N\left(s_N, s_0\right) \cdots P_1\left(s_1, s_0\right) =$$



, (Decoupling Conditions), (2.7)

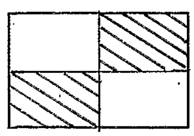
since the matrices T_0 on both ends of the formula (2.6) are block-diagonal, as well.

We shall see shortly, that for a thin solenoid the following asymptotic expansion, in the solenoid's strength θ, holds

$$P_{SOL}\left(s, s_0\right) = \mathbf{1}_4 + \theta S\left(s, s_0\right) + 0\left(\theta^2\right), \tag{2.8}$$

where the matrix $S\left(s, s_0\right)$ is a block-anti-diagonal one

$$S\left(s, s_0\right) =$$



. (2.9)

Therefore, to the first-order in the parameters θ_k, we have

$$\begin{aligned} P_N\left(s_N, s_0\right) \cdots P_1\left(s_1, s_0\right) &= \left[\mathbf{1}_4 + \theta_N S\left(s_n, s_0\right) + \cdots\right] \cdots \left[\mathbf{1}_4 + \theta_1 S\left(s_1, s_0\right) + \cdots\right] = \\ &= \mathbf{1}_4 + \sum_{k=1}^{N} \theta_k S\left(s_k, s_0\right) + \cdots. \end{aligned} \tag{2.10}$$

The product of the projections can be of the block-diagonal form as demanded in (2.7) only if contributions to the anti-diagonal blocks cancel between themselves. Hence, we obtain the decoupling conditions, to the first-order in θ_k's

$$\sum_{k=1}^{N} \theta_k S(s_k, s_0) = 0 \quad \begin{matrix}\text{(Decoupling Conditions)} \\ \text{to the First} - \text{Order}\end{matrix} \quad . \tag{2.11}$$

These matrix conditions may be analyzed, and solved for the parameters θ_k while the S-matrix is given.

Notice, that the above decoupling conditions make the product of the projections equal to the unit matrix, to the second-order, and, as the result, the transfer matrix T_{BA} becomes

$$T_{BA} = T_0(s_B, s_0)(\mathbf{1}_4 + \cdots) T_0(s_0, s_A) = T_0(s_B, s_A) + \cdots \quad . \tag{2.12}$$

This means that the insertion becomes transparent, when decoupled.

3. The S-Matrix for a Solenoid

A transfer matrix for a solenoid of length ℓ and longitudinal field B_s, with the cylindrically symmetric thin fringe fields is known,[6]

$$T_{SOL}(\ell) = \begin{bmatrix} c^2 & K^{-1}sc & sc & K^{-1}s^2 \\ -Ksc & c^2 & -Ks^2 & sc \\ -sc & -K^{-1}s^2 & c^2 & K^{-1}sc \\ Ks^2 & -sc & -Ksc & c^2 \end{bmatrix} =$$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -K & 0 \\ 0 & 0 & 1 & 0 \\ K & 0 & 0 & 1 \end{bmatrix}}_{F} \begin{bmatrix} 1 & K^{-1}sc & 0 & K^{-1}s^2 \\ 0 & c^2-s^2 & 0 & 2cs \\ 0 & -K^{-1}s^2 & 1 & K^{-1}cs \\ 0 & -2cs & 0 & c^2-s^2 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & K & 0 \\ 0 & 0 & 1 & 0 \\ -K & 0 & 0 & 1 \end{bmatrix}}_{F^{-1}} =$$

FRINGE FIELD CONTRIBUTION   FRINGE FIELD CONTRIBUTION

$$= \underbrace{\begin{bmatrix} c & 0 & s & 0 \\ 0 & c & 0 & s \\ -s & 0 & c & 0 \\ 0 & -s & 0 & c \end{bmatrix}}_{R(\theta)} \underbrace{\begin{bmatrix} c & K^{-1}s & 0 & 0 \\ -Ks & c & 0 & 0 \\ 0 & 0 & c & K^{-1}s \\ 0 & 0 & -Ks & c \end{bmatrix}}_{S(\ell, K^2)} \tag{3.1}$$

The matrices, $R(\theta)$ and $S\left(\ell, K^2\right)$, commute and the notations are

$$\theta = K\ell, \tag{3.2}$$

$$K = \frac{B_s}{2(B\rho)}, \tag{3.3}$$

$$(B\rho) - \text{magnetic rigidity}, \tag{3.4}$$

$$c \equiv \cos\theta, \tag{3.5}$$

$$s \equiv \sin\theta. \tag{3.6}$$

Assuming Ksmall, which is the case for STAR solenoid, and taking into account that

$$\begin{aligned} Ks &\simeq 0, \\ K^{-1}s &\simeq \ell, \\ c &\simeq 1, \end{aligned} \tag{3.7}$$

we get for the $S\left(\ell, K^2\right)$ matrix a drift matrix

$$S\left(\ell, K^2\right) \simeq \left[\begin{array}{c|c|c|c} 1 & \ell & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & \ell \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \equiv \left[\begin{array}{c|c} d\left(\ell\right) & \mathbf{0} \\ \hline \mathbf{0} & d\left(\ell\right) \end{array}\right] \equiv D\left(\ell\right), \tag{3.8}$$

where the 2×2 matrix $d\left(\ell\right)$ is, of course

$$d\left(\ell\right) = \left[\begin{array}{c|c} 1 & \ell \\ \hline 0 & 1 \end{array}\right]. \tag{3.9}$$

For the $R\left(\theta\right)$ matrix we get in the thin lens approximation

$$R\left(\theta\right) \simeq \left[\begin{array}{c|c|c|c} 1 & 0 & \theta & 0 \\ \hline 0 & 1 & 0 & \theta \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & -\theta & 0 & 1 \end{array}\right] = \mathbf{1}_4 + \theta \left[\begin{array}{c|c} \mathbf{0} & \mathbf{1}_2 \\ \hline -\mathbf{1}_2 & \mathbf{0} \end{array}\right] + \cdots. \tag{3.10}$$

Hence, for the (thin) solenoid transfer matrix we get according to the formula (3.1), the result

$$T_{SOL}\left(\ell\right) \simeq D\left(\ell\right) R\left(\theta\right) = R\left(\theta\right) D\left(\ell\right). \tag{3.11}$$

We are now in a position to calculate the projection matrix $P_{SOL}\left(s, s_0\right)$ using the definition (2.8)

$$\begin{aligned} P_{SOL}\left(s, s_0\right) &= T_0\left(s_0, s''\right) T_{SOL}\left(s'', s'\right) T_0\left(s', s_0\right) = \mathbf{1}_4 + \theta S\left(s, s_0\right) + 0\left(\theta^2\right), \\ s' &= s - 1/2\ \ell \quad , \quad s'' = s + 1/2\ \ell, \end{aligned} \tag{3.12}$$

Using the relations

$$T_0\left(s', s_0\right) = T_0\left(s', s\right) T_0\left(s, s_0\right) = D^{-1}\left(\ell/2\right) T_0\left(s, s_0\right), \tag{3.13}$$

and

$$T_0(s_0, s'') = T_0(s_0, s)\, T_0(s, s'') = T_0^{-1}(s, s_0)\, D^{-1}(\ell/2), \tag{3.14}$$

we get using the results (3.8) and (3.10)

$$P_{SOL}(s, s_0) = T_0^{-1}(s, s_0)\, R(\theta)\, T_0(s, s_0) + \cdots =$$

$$= \mathbf{1}_4 + \theta \left[\begin{array}{c|c} \mathbf{0} & T_{0x}^{-1}(s, s_0)\, T_{0y}(s, s_0) \\ \hline -T_{0y}^{-1}(s, s_0)\, T_{0x}(s, s_0) & \mathbf{0} \end{array} \right] + \cdots . \tag{3.15}$$

Comparing with (3.12) one finds that

$$S(s, s_0) = \left[\begin{array}{c|c} \mathbf{0} & \sigma(s, s_0) \\ \hline -\overline{\sigma}(s, s_0) & \mathbf{0} \end{array} \right], \tag{3.16}$$

where the $\sigma(s, s_0)$ 2×2 matrix is of the form

$$\sigma(s, s_0) \equiv T_{0x}^{-1}(s, s_0)\, T_{0y}(s, s_0), \tag{3.17}$$

and $\overline{\sigma}$ is its symplectic conjugate*

$$\overline{\sigma}(s, s_0) \equiv T_{0y}^{-1}(s, s_0)\, T_{0x}(s, s_0) = \sigma^{-1}(s, s_0). \tag{3.18}$$

The matrix $\sigma(s, s_0)$ is simplest when $s = s_0$, i.e., when the solenoid is placed at the reference point 0. We have in this case

$$\sigma(s_0, s_0) = \mathbf{1}_2, \tag{3.19}$$

and we get for the S-matrix the result

$$S(s_0, s_0) = \left[\begin{array}{c|c} \mathbf{0} & \mathbf{1}_2 \\ \hline -\mathbf{1}_2 & \mathbf{0} \end{array} \right]. \tag{3.20}$$

It is interesting that the same result is valid for a solenoid placed a drift away from the reference point since the relations hold

$$\begin{aligned} T_{0x}(s, s_0) &= d(s - s_0), \\ T_{0y}(s, s_0) &= d(s - s_0), \end{aligned} \tag{3.21}$$

* A symplectic conjugate of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\overline{A} \equiv \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. For A symplectic, $\overline{A} = A^{-1}$ holds.

and, as the result we get the remarkable property of the S-matrix

$$S\left(s_0 + d', s_0\right) = S\left(s_0 - d'', s_0\right) = S\left(s_0, s_0\right). \tag{3.22}$$

The decoupling conditions (2.11) simplify considerably because of that

$$\sum_{k=1}^{N} \theta_k S\left(s_k, s_0\right) = \left(\sum_{k=1}^{N} \theta_k\right) \left[\begin{array}{c|c} \mathbf{0} & \mathbf{1}_2 \\ \hline -\mathbf{1}_2 & \mathbf{0} \end{array}\right] = 0, \tag{3.23}$$

and reduce to the single requirement

$$\sum_{k=1}^{N} \theta_k = 0. \tag{3.24}$$

We will now demonstrate the above general formalism on the example provided by the STAR solenoid in RHIC.

4. Application of the General Scheme to Decoupling of the STAR Detector in RHIC

RHIC will contain initially two large detectors, STAR and PHENIX, located at 6 and at 8 o'clock in the rings. The STAR detector will contain a solenoid of 4m long producing longitudinal magnetic field $B_s = 0.5$ Tesla. The PHENIX detector, currently under a construction, will not be considered here.

In order to apply the decoupling procedure, one needs to assume that the RHIC lattice (1991 lattice) is globally decoupled. This means that some correction scheme has been devised, using a part, C_0, of the available a_1-correctors

$$C_0 = C_0^{(Y)} U \, C_0^{(B)}, \tag{4.1}$$

Y-YELLOW, and B-BLUE rings, and, correspondingly–correctors.

Since both, YELLOW and BLUE rings are identical, apart from their opposite directions, it is enough to consider the insertion in one of them, say the BLUE ring, only. The relevant parameters of the STAR's solenoid are:

$$\begin{aligned}
\ell_* &= 4m, \\
B_* &= 0.5\text{T}, \quad \text{longitudinal field inside the detector}, \\
K_* &= (4B\rho)^{-1} = \begin{cases} 0.0026, & \text{at injection}, \\ 0.0003, & \text{at top energy}, \end{cases} \\
\theta_* &= K_* \ell_* = \begin{cases} 0.0104, & \text{at injection}, \\ 0.0012, & \text{at top energy}. \end{cases}
\end{aligned} \tag{4.2}$$

The K_* and θ_* parameters are rather small, indeed, due to large values of the rigidity. We shall consider two decoupling schemes:

A. Single anti-solenoid placed before or after the detector, see Fig. 3.

B. Two anti-solenoids, one on either side of the detector as shown in Fig. 4.

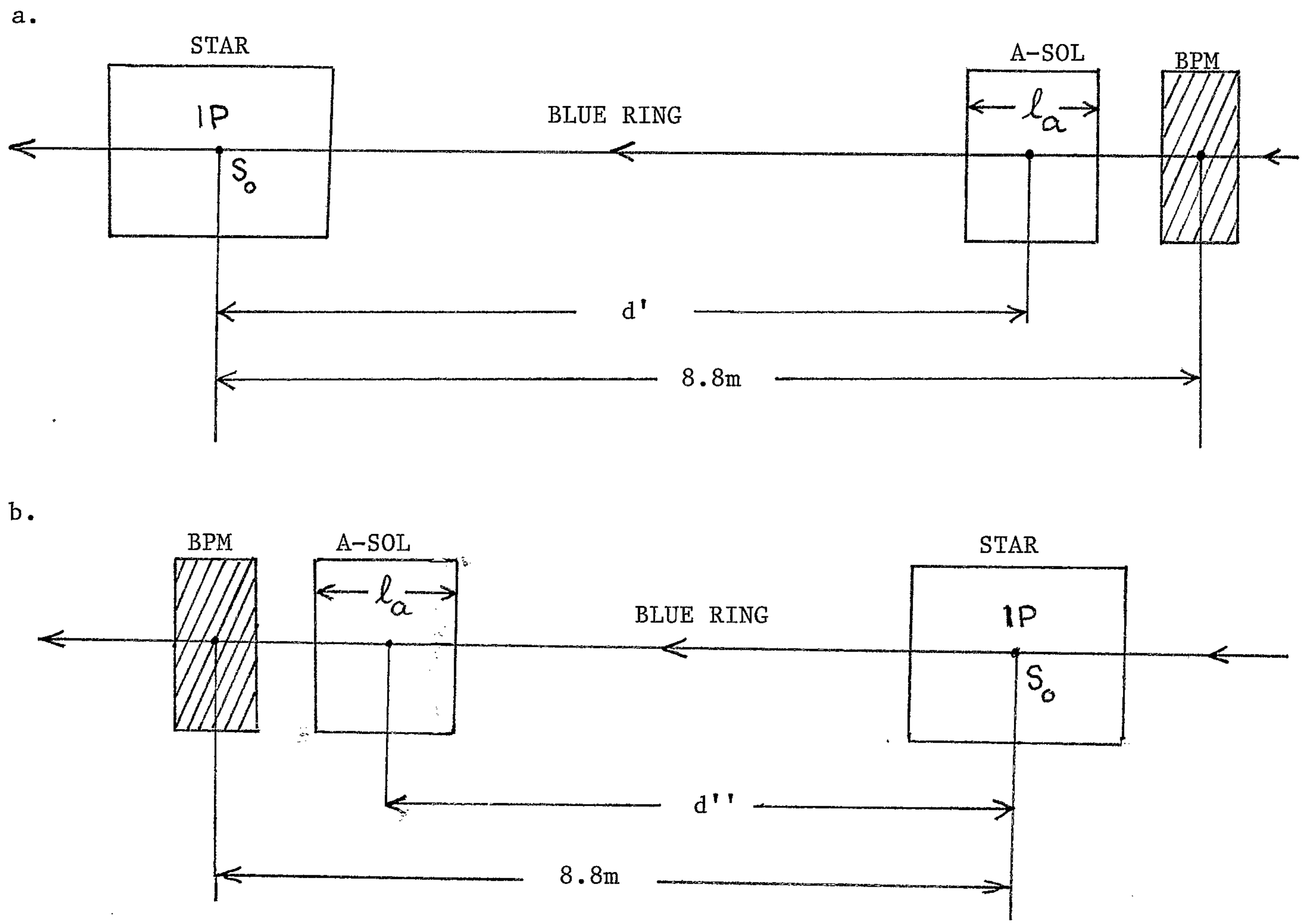


Fig. 3.: Decoupling scheme of the STAR detector solenoid using a single anti-solenoid before the detector (a), or a single anti-solenoid after the detector (b).

The decoupling condition (3.24), at $N = 2$ reads, in both cases (a) and (b),

$$\theta_* + \theta_a = 0, \tag{4.3}$$

or, equivalently, using (3.2)

$$B_* \ell_* + B_a \ell_a = 0, \tag{4.4}$$

where B_a is a longitudinal field inside the anti-solenoid. Hence, the decoupling strength is

$$B_a = -\ell_* / \ell_a \; B_*. \tag{4.5}$$

The distances d', d'' can be arbitrary, within the insertion limits. Assuming, for example, $\ell_a = 1$m, we get for the decoupling strength the value

$$B_a = -4 \times 0.5 \text{ T} = -2 \text{ T}. \tag{4.6}$$

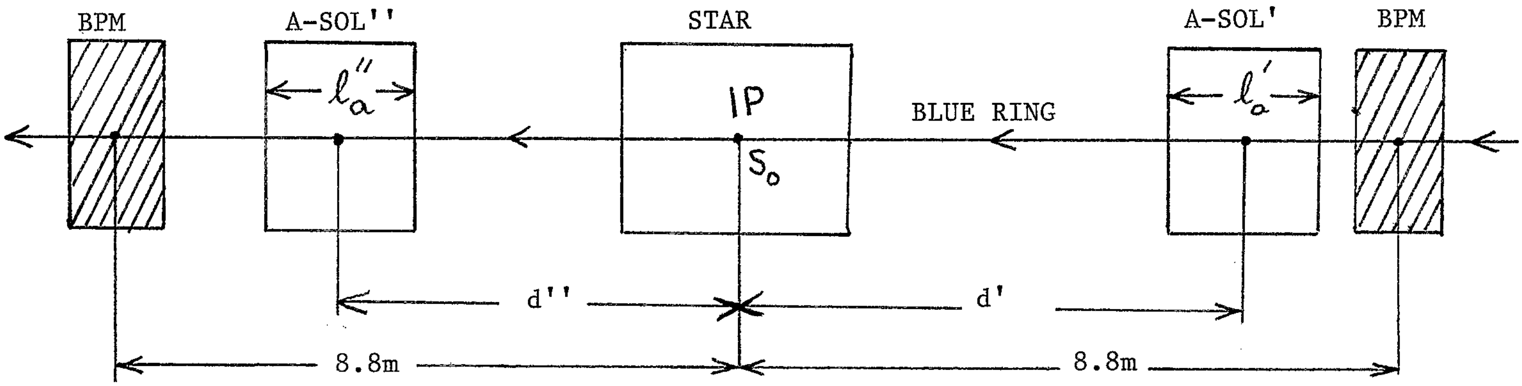


Fig. 4.: Decoupling scheme of the STAR detector solenoid using two anti-solenoids, one on either side.

For the decoupling scheme using two anti-solenoids one gets from the condition (3.24), at $N = 3$

$$\theta_* + \theta'_a + \theta''_a = 0, \tag{4.7}$$

or, in terms of strengths and lengths

$$B_* \ell_* + B'_a \ell'_a + B''_a \ell''_a = 0. \tag{4.8}$$

Assuming, for simplicity, the same lengths, and strengths

$$\begin{aligned} \ell'_a &= \ell''_a = \ell_a, \\ B'_a &= B''_a = B_a, \end{aligned} \tag{4.9}$$

we get for the decoupling strength

$$B_a = -\ell_*/2\ell_a \; B_*. \tag{4.10}$$

The distances d', d'' are arbitrary, within the limits. Taking, for example, shorter solenoids

$$\ell_a = 0.5 \text{ m}, \tag{4.11}$$

we get for the decoupling field strength

$$B_a = -2 \text{ T}, \tag{4.12}$$

which is quite acceptable.

Of course, it is possible to install more anti-solenoids but it does not seem practical, in the case at hand.

5. Decoupling of the YELLOW RING Using Anti-Solenoids

The decoupling of the YELLOW RING is completely analogous to the decoupling scheme for the BLUE RING. One should only notice that reversed (YELLOW) beam sees the same magnetic field of the STAR solenoid as the BLUE beam, and the decoupling field is directly opposite to it. Therefore, all the anti-solenoids, in the BLUE and the YELLOW RINGS should be equally powered as shown in Fig. 5.

$$(B_a)_{\text{YELLOW}} = (B_a)_{\text{BLUE}} \,. \tag{5.1}$$

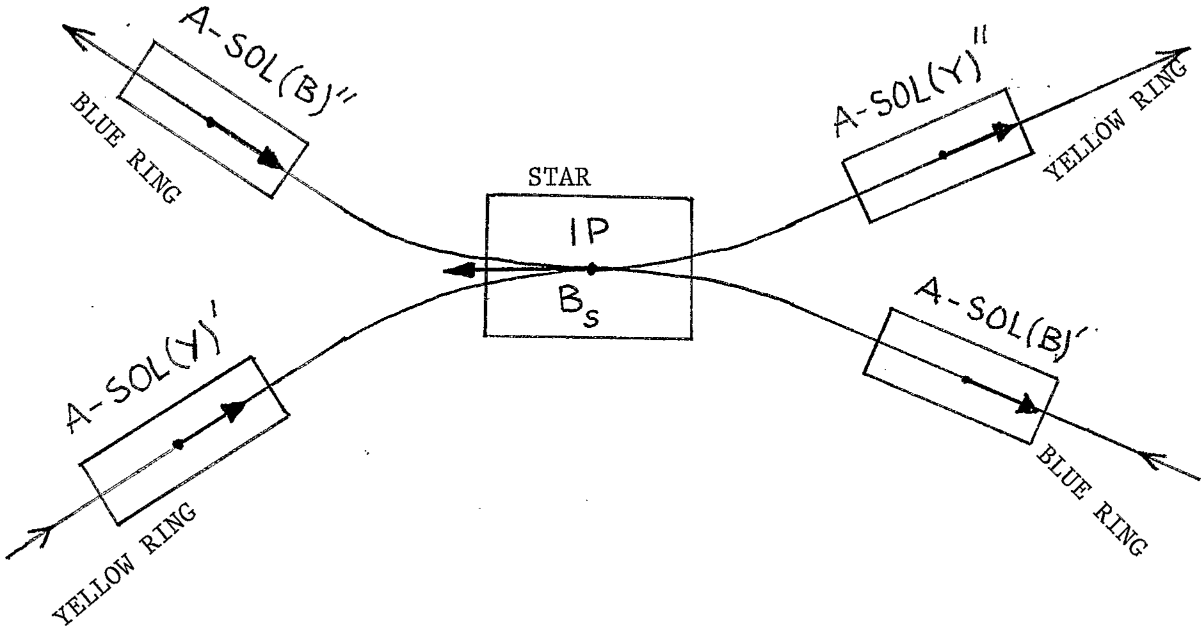


Fig. 5.: Anti-solenoids in the YELLOW RING are powered in the same way as corresponding anti-solenoids in the BLUE RING.

6. Conclusion

In conclusion we would like to collect main pros and cons for using anti-solenoids for the decoupling of the STAR detector.

ADVANTAGES	DISADVANTAGES
1. Anti-solenoids can be placed arbitrarily in the insertion. Phase advances do not enter the decoupling condition.	1. Consume valuable space around the 1P.
2. Particles of all energies decouple in the same way, no dependence on dispersion.	2. Anti-solenoids are costly.
3. Strength of the decoupling field is constant during acceleration period, does not depend on the magnetic rigidity.	3. The scheme with anti-solenoids does not utilize already available a_1- correctors.
	4. Anti-solenoids would have to be moved sideways when different species are accelerated in both RHIC's rings, and geometry of beams changes.

It seems that the displayed above disadvantages suggest that a different decoupling scheme, using skew-quadrupoles placed sufficiently far away from 1P, should also be considered. Such scheme will be presented in the next technical note.

7. Acknowledgments

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