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The Beam Lifetime and Emittance Growth in RHIC under Normal Operating Conditions and with a Hydrogen Gas Jet

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1 Introduction

The inelastic scattering of the beam and the residual gas molecules in RHIC could represent one of the limitations on the beam life time and emittance growth. This report covers the dominant central nuclear collisions' influence on the beam lifetime and transverse emittance growth. The cross sections for the beam-gas electron radiative captures are an order of magnitude smaller. The capture cross sections include the radiative and non-radiative capture, and the capture from the electron-positron pair creation from the "vacuum capture". The beam lifetime is presented as an exponential decay [1] where the loss rate of the beam particles is given by:

$$\frac{1}{\tau} = \frac{1}{N}\frac{dN}{dt} = n\, l\, f\,(\sigma_N + \sigma_C), \tag{1}$$

where f is the revolution frequency, l is the path of the projectiles (the circumference of RHIC is equal to $C = 3833.845$ m). The cross sections [2] for the radiative and non-radiative electron captures, $\sigma_{RC} = a\, Z_P^5 Z_T/\gamma$ and $\sigma_{NRC} = b\, Z_P^5 Z_T^5/\gamma$, respectively, as well as the capture cross section from the beam gas electron-positron creation $\sigma = d\, Z_P^5 Z_T^2\, ln(\gamma/\gamma_o)$, are neglected in this report. The parameters in the cross sections are the charge of the projectiles-P and target-T, Z_P and Z_T, respectively, the relativistic factor γ and parameters $\gamma_o, a, b,$ and d with values provided in ref. [2]. The cross section of the nuclear scattering σ_N in collisions between the projectiles-P ions and the residual gas target-T nuclei, treated as "billiard balls" is presented as:

$$\sigma_N \simeq \pi R_N^2, \quad \text{with } R_N \simeq r_o A^{\frac{1}{3}}, \quad \text{where } r_o = 1.2 \text{ fm}, \tag{2}$$

where the radii of the particle's nuclei is presented as [3]:

$$R_N = 1.2\,(A_P^{\frac{1}{3}} + A_T^{\frac{1}{3}}) \text{ fm}. \tag{3}$$

The luminosity of the head-on collisions between two beams can be calculated from the transverse and longitudinal particle distributions of each beam integrated over the whole collision volume:

$$\mathcal{L} = \int 2cdt\, \rho_1(x,y,s,ct)\rho_2(x,y,s,ct)\, dxdyds, \tag{4}$$

where $\rho_{1,2}(x,y,s,ct)$ are the particle's distributions in three dimensional space. The $\mathcal{L}$ luminosity of a collider ring is directly proportional to the numbers N_1 and N_2 of colliding particles in the two beams. A loss of luminosity due to the lost particles $N_{L1} \cdot N_{L2}$, of the two beams due to collisions with the residual gas molecules is obtained from equation (1) as:

$$N_{L1,L2} = N(0)_{1,2} \, e^{\frac{t}{\tau}}, \tag{5}$$

where the $N(0)_{1,2}$ is the initial number of particles in corresponding beam. Unfortunately at the same time the beam size-emittance changes in all three dimensions due to other effects (the dominant effect is the intra-beam scattering which continuously lowers the luminosity). The emittance growth due to all effects, even from the multiple Coulomb beam-gas scattering, could be ignored in making a rough estimate on the luminosity loss due to the beam-residual gas lifetime. A loss of luminosity under different residual gas conditions and beams will be estimated after 10 hours of the store.

2 The Beam Lifetime in RHIC due to Beam Gas Inelastic Collisions

2.1 RHIC Beam Lifetime in the Gold-Gold Collisions

The cross section depends on the residual gas composition in the bore tube; it is assumed that in the warm bore tube (20% of the circumference) the hydrogen atoms make 90% , while the rest are molecules of methane CH_4 (5%) and carbon-dioxide CO (5%). In the cold ~5K beam tube the assumption is that helium molecules (100%) are the dominant contribution to the vacuum composition. For the RHIC warm vacuum sections the cross sections between the gold ions and the gas molecules is presented as a weighted sum of the three cross sections:

$$\sigma_{N_{tot}} = 0.9\,\sigma_{H_2} + 0.05\,\sigma_{CH_4} + 0.05\,\sigma_{CO} \tag{6}$$

$$\begin{aligned}
R_{Au,H_2} &\simeq 1.2\ (197^{\frac{1}{3}} + 2^{\frac{1}{3}})\ (\text{fm}), & \sigma_{Au,H_2} &= 2.267 \cdot 10^{-24}\ (\text{cm}^2),\\
R_{Au,CH_4} &\simeq 1.2\ (197^{\frac{1}{3}} + 8^{\frac{1}{3}} + 12^{\frac{1}{3}})\ (\text{fm}), & \sigma_{Au,CH_4} &= 4.622 \cdot 10^{-24}\ (\text{cm}^2),\\
R_{Au,CO} &\simeq 1.2\ (197^{\frac{1}{3}} + 16^{\frac{1}{3}} + 12^{\frac{1}{3}})\ (\text{fm}), & \sigma_{Au,CO} &= 5.110 \cdot 10^{-24}\ (\text{cm}^2),
\end{aligned}$$

the cross section for the central nuclear collisions between the gold beam and the residual gas in the warm part of RHIC is estimated to be:

$$\sigma_N \simeq 2.53 \cdot 10^{-24}\ \text{cm}^2,$$

while for the cold section the cross section is estimated to be:

$$\sigma_N \simeq 2.48 \cdot 10^{-24}\ \text{cm}^2.$$

The density of molecules within the beam pipe n is equal to:

$$n = \frac{p}{kT} = 9.656 \cdot 10^{18}\, \frac{p(\text{Torr})}{T}, \;\; or \;\; pV = \nu_{mol} R T, \tag{7}$$

where p is the residual gas pressure in the beam pipe, ν_{mol} is the number of moles, R is the molar gas constant $R \equiv kN_A = 8.314510 \cdot 10^7$ (erg/K mol), where N_A is the Avogadro's number $N_A = 6.0221367 \cdot 10^{23}$(molecules/mol), the Boltzmann constant k=$1.380662 \cdot 10^{-16}$ (erg/(molecule K), the gas density n (molecules/cm^3), and the T(K) is the gas temperature. The internationally accepted unit for the gas pressure is 1 Pa=Nt/m^2. The relationships between different pressure units are:

$$1\,\text{Torr} \;=\; 1.33322\,\text{mbar} \;=\; 1.33322 \cdot 10^3\ \text{Bar} \;=\; 1.33322 \cdot 10^2\ \text{Pa}. \tag{8}$$

The revolution frequency is equal to $f = 78.13 \cdot 10^3\,(Hz) = v/C \simeq c/C$, where c is the speed of light.

2.1.1 Gold Beam Lifetime in the "Warm" Part of RHIC

The lifetime of the gold ions in the warm part of RHIC $l = 0.2\ C$, with a pressure $p = 5 \cdot 10^{-10}$ (Torr) is:

$$\tau \;=\; \frac{1}{p(\text{Torr})}\,\frac{1}{0.2} \;\cdot\; 1.137\ 10^{-7}\,\text{hours} \simeq 1140\ \text{(hours)}.$$

Luminosity loss in 10 hours: The luminosity loss in gold-gold collisions, due to the inelastic scattering of gold ions with the residual gas molecules in the warm part of the RHIC circumference is equal to the square of $N_{Au}/N(o)_{Au}$=1.0088 which makes the effect as:

$$\Delta\mathcal{L}(10\,\text{hours}) \;=\; 1.8\%.$$

2.1.2 Gold Beam Lifetime in the "Cold" Part of RHIC

The lifetime of the gold ions in the "cold part" of RHIC where $l = 0.8\ C$, with the temperature of the walls estimated to be 5K, with a pressure of $p = 1 \cdot 10^{-11}$ Torr, is:

$$\tau \;=\; \frac{1}{p(\text{Torr})}\,\frac{1}{0.8} \;\cdot 1.935\,10^{-9} \;\simeq 240\ \text{(hours)}.$$

Luminosity loss in 10 hours: The luminosity loss in gold-gold collisions, due to the inelastic scattering of gold ions with the residual gas molecules in the cold part of the RHIC circumference is equal to the square of $N_{Au}/N(o)_{Au}$=1.0425 which makes the effect as:

$$\Delta\mathcal{L}(10\,\text{hours}) \;=\; 8.7\%.$$

2.1.3 The Beam Lifetime of the Gold Ions due to the Hydrogen Jet

A proposal for the proton elastic scattering experiment with the polarized proton jet requires 10^{14} hydrogen (atoms/cm^2) to cross perpendicular to the beam direction. The "ultra-cold" spin-polarized hydrogen jet will be located in the warm part common to the two RHIC accelerators about six meters away from the interaction region. The "target thickness" in the beam direction is estimated to be $\phi = 10^{13}$ (atoms/cm^2). If the operation mode of RHIC are collisions between gold ions with protons then luminosity and the lifetime of the gold beam will be affected by the jet according to the equation (1):

$$\frac{1}{\tau} \;=\; \frac{1}{N}\frac{dN}{dt} \;=\; \frac{dN_{H_2}}{dV}\,l\,\frac{c}{C}\,\sigma_N \;=\; \frac{\phi}{l}\cdot l\frac{c}{C}\,\sigma_N \;=\; \phi\,\frac{c}{C}\,\sigma_N,$$

where the length of the "target" is $l = 2$ cm, while the density of the gas molecules is:

$$n \;=\; \frac{dN_{H_2}}{V_{vol}} \;=\; \frac{\phi \cdot S}{S\,l} \;=\; \frac{\phi}{l}.$$

The cross section σ_N for the collisions between gold ions with the hydrogen jet atoms could be estimated as:

$$\sigma_N = \pi R_N^2 = \pi [1.2(197^{\frac{1}{3}} + 2^{\frac{1}{3}})]^2 = 2.27 \cdot 10^{-24} (\text{cm}^2).$$

The lifetime of the gold beam under the influence of the hydrogen jet is calculated to be:

$$\tau_{Au-Jet} = 156 \text{ (hours)}$$

Luminosity loss in 10 hours: The luminosity loss in gold-hydrogen jet collisions, due to the inelastic scattering of only one gold ion beam with hydrogen jet molecules (it is assumed that the hydrogen jet could be active only in the proton-gold collisions) is due to a loss of the gold ions of $N_{Au}/N(o)_{Au}$=1.066 which makes the effect as:

$$\Delta\mathcal{L}(10\,\text{hours}) = 6.6\%.$$

2.2 The Beam Lifetime in Proton-Proton Collisions Store

2.2.1 Proton Beam Lifetime in the "Warm" Part of RHIC

The cross section for the central nuclear collisions between protons and the residual gas molecules, within the warm section of RHIC, is again presented by equation (6) as a weighted sum of the three cross sections. The partial cross sections are calculated as:

$$\sigma_{H_2} = 2.311 \cdot 10^{-25}\,\text{cm}^2,\ \sigma_{CH_4} = 1.265 \cdot 10^{-24}\,\text{cm}^2, \sigma_{CO} = 1.526\, 10^{-24}\,\text{cm}^2,$$

$$\sigma_{N_{tot}} \simeq 3.48 \cdot 10^{-25}\ \text{cm}^2,$$

The lifetime of the proton beams in RHIC within the warm part of RHIC $l = 0.2\,C$, with a pressure $p = 5 \cdot 10^{-10}$ Torr is:

$$\tau = \frac{1}{p(\text{Torr})}\,\frac{1}{0.2} \cdot 1.056 \cdot 10^{-6}\,\text{hours} \simeq 10500 \text{ (hours)},$$

Luminosity loss in 10 hours: The luminosity loss in proton-proton collisions, due to the inelastic scattering of protons on the residual gas molecules in the warm part of the RHIC circumference is equal to the square of $N_{proton}/N(o)_{proton}$=1.00095 which makes the effect as:

$$\Delta\mathcal{L}(10\,\text{hours}) = 0.2\%.$$

2.2.2 Proton Beam Lifetime in the "Cold" Part of RHIC

For the RHIC cold vacuum section the cross section of the proton-helium residual gas collisions is estimated to be:

$$\sigma_N \simeq 3.029 \cdot 10^{-25}\ \text{cm}^2,$$

while the lifetime of the proton beam in the "cold part" of RHIC where $l = 0.8\,C$, with the temperature of the walls estimated to be 5K, with a pressure of $p = 1 \cdot 10^{-11}$ (Torr), is:

$$\tau = \frac{1}{p(\text{Torr})}\,\frac{1}{0.8} \cdot 1.584 \cdot 10^{-8}\,\text{hours} \simeq 1980 \text{ (hours)}$$

Luminosity loss in 10 hours: The luminosity loss in proton-proton collisions, due to the inelastic scattering of protons on the residual gas molecules in the warm part of the RHIC circumference is equal to the square of $N_{proton}/N(o)_{proton}$=1.0051 which makes the effect as:

$$\Delta\mathcal{L}(10\,\text{hours}) \;=\; 1.0\%.$$

2.2.3 The Lifetime of Proton Beams due to the Hydrogen Jet

The beam lifetime of the stored proton beam in RHIC is calculated by the equation (11) where the only difference is the smaller cross section σ_N. The cross section between the stored proton beams and the hydrogen "target" atoms at a temperature of 300 K is calculated to be:

$$\sigma_N \;=\; \pi \cdot [1.2(1 \;+\; 2^{\frac{1}{3}})]^2 \;=\; 2.31 \cdot 10^{-25}\,\text{cm}^2,$$

while the beam lifetime, from equation (11) is:

$$\tau_{p-Jet} \;=\; 1537 \text{ hours}$$

Luminosity loss in 10 hours: The luminosity loss in gold-proton collisions, due to the inelastic scattering of gold ions and proton on the residual gas molecules is equal to the product of $N_{Au}/N(o)_{Au}$=1.0662 % and $N_{proton}/N(o)_{proton}$=1.0074 % which makes for the loss of the total luminosity:

$$\Delta\mathcal{L}(10\,\text{hours}) \;=\; 7.4\%.$$

SUMMARY OF THE RESULTS FOR THE BEAM LIFE TIME DUE TO THE BEAM-GAS AND BEAM JET ELASTIC SCATTERING:

Table 1

Store	p(Torr)	τ(hours)	$\Delta\mathcal{L}$ (%)
Gold-Gold	Warm $5{\cdot}10^{-10}$	1140	1.8
Gold-Gold	Cold $1{\cdot}10^{-11}$	240	8.7
Gold-Jet	$1.55{\cdot}10^{-4}$	156	6.6
proton-proton	Warm $5{\cdot}10^{-10}$	10500	0.2
proton-proton	Cold $1{\cdot}10^{-11}$	1980	1.0
Au-proton-Jet	$1.55\,{\cdot}10^{-4}$	1537	7.4

3 Transverse Emittance Growth Due to Multiple Coulomb Scattering

The residual gas molecules in the beam tube introduce multiple Coulomb scattering. The Rutherford Coulomb scattering cross section for an incident particle with a speed v and charge e approaching a target nucleus of charge Ze with an impact parameter b is [4]:

$$\frac{d\sigma}{d\Omega} = \frac{b}{\theta}\frac{db}{d\theta} = 4\left(\frac{Ze^2}{4\pi\epsilon_o p\; v}\right)^2 \frac{1}{\theta^4}. \tag{9}$$

The multiple elastic Coulomb scattering will cause the transverse emittance of the beam to grow [2]. A particle transverse coordinate will change due to the scattering of the many atoms along its way.

Normalized Emittance Growth: The growth rate of the normalized emittance, $\epsilon_N \equiv \epsilon \cdot \gamma\beta$ (where $\gamma\beta$ are relativistic factors), has been derived [2] from the Fokker-Planck diffusion equation. The transverse beam distribution was assumed to be Gaussian with the beam size defined as:

$$\sigma = \sqrt{\frac{\epsilon_N \, \beta_{\text{TWISS}}}{\gamma\beta}} \tag{10}$$

It is important to note that the beam size in RHIC is defined differently:

$$\sigma_{((RHIC)} = \sqrt{\frac{\epsilon_N \, \beta_{\text{TWISS}}}{6 \, \pi \, \gamma\beta}}, \tag{11}$$

where the relativistic factor $\gamma\beta = \sqrt{\gamma^2 - 1} \simeq \gamma$. The rate of the normalized emittance growth was presented [2] as:

$$\frac{d\epsilon_N}{dt} \simeq \frac{\gamma \, \beta_{\text{TWISS}}}{2} \, \dot{\Theta}^2_{rms}, \tag{12}$$

where β is the average value of the betatron function for the whole length of RHIC, estimated as $\beta_{avr} \simeq 30$ m, γ is the relativistic factor, and where the emittance is defined by the equation (10).

Transverse Scattering Angle: The average rate of change of the transverse scattering angle [4], $\dot{\Theta}^2_{rms}$, is obtained from the variance of the scattering angle in one transverse degree of freedom:

$$< \theta^2 > = < \theta_x^2 > + < \theta_y^2 > = \; 2 < \theta_x^2 >$$

$$< \theta_x^2 > = \frac{1}{\sqrt{2}} \, 4 \, \frac{\pi}{\alpha} \left(\frac{m_e \, c^2}{pv} \right)^2 \frac{l}{L_{RAD}}. \tag{13}$$

So the average rate of change of the transverse scattering angle, Θ^2_{rms}, is:

$$\dot{\Theta}^2_{rms} = \left(\frac{15MeV}{m_p \, c^2 \gamma} \right)^2 \frac{Z_P^2}{A_P^2} \, \frac{c}{L_{RAD}}, \tag{14}$$

where m_p is the proton mass, the charge and atomic number of the projectile ions are Z_P and A_P, respectively.

The Radiation Length: The L_{RAD} is the radiation length defined [4] as:

$$\frac{1}{L_{RAD}} = 2\alpha \, \frac{N_m}{A_T} \, n_m \, Z_T^2 \, r_e^2 \, ln \left(\frac{R_T}{R_N} \right) \quad (\text{cm}^{-1}), \tag{15}$$

where $\alpha \simeq 1/137$ is the fine structure constant, n_m (g/cm^3) is the gas density, $R_N = 1.2 \cdot A^{\frac{1}{3}}$ is the effective radius of the target nucleus, N_m (molecules/g) is another kind of gas "density", Z_P is equal to 79 for the fully ionized gold ions $^{197}Au^{79+}$, $r_e = 2.82 \cdot 10^{-13}$ cm, the fine-structure constant $\alpha = 7.29735308 \cdot 10^{-3} \simeq 1/137$, R_T is the Thomas-Fermi screening radius of the target atom $R_T = 1.4 \cdot a/A^{\frac{1}{3}}$ where a is the Bohr radius $a = 5.2917706 \cdot 10^{-11}$ m. The radiation length of materials can also be obtained from the unit of radiation length usually denoted by X_o calculated in units of g/cm^2. The unit radiation length X_o(g/cm^2) was previously [5] tabulated for every element in the periodic system which is assumed to be free. The unit radiation lengths of the elements of concern are presented in table 2.

Table 2

Element	Z	A	X_o(g/cm^2)
H	1	1.0080	63.0470
He	2	4.0026	94.3221
C	6	12.0111	42.6983
O	8	15.9994	34.2381

The unit radiation length of molecules as H_2 due to screening effects changed the value for Hydrogen from 63.047 (g/cm^2) to 61.283 (g/cm^2). When the molecular binding is neglected the radiation length of the complex molecules such as CH_4 could be calculated [5] from the following equation:

$$\frac{A(C)+4A(H)}{X_o(CH_4)} = \frac{A(C)}{X_o(C)} + \frac{4A(H)}{X_o(H)}. \tag{16}$$

The unit radiation lengths for the complex molecules CH_4 and CO are calculated as:

$$X_o(CH_4) = 46.446 \text{ (g/cm}^2\text{)}, \quad X_o(CO) = 37.415 \text{ (g/cm}^2\text{)}.$$

The unit radiation length of the gas composition in the RHIC warm vacuum section is calculated following the prescription given [5] as:

$$\frac{1}{X_o(\text{RHIC-WARM})} = \frac{0.900}{X_o(H_2)} + \frac{0.05}{Xo(CH_4)} + \frac{0.05}{Xo(CO)}, \tag{17}$$

which makes the value for the effective radiation length of the warm part of the RHIC vacuum equal to $X_o(\text{RHIC-WARM}) = 58.483$ (g/cm^2). The unit radiation length for the cold part of RHIC vacuum system is estimated as the 100% helium gas as $X_o(\text{RHIC-COLD}) = 94.3221$ (g/cm^2), while the unit radiation length for the hydrogen jet atoms H_2 is given above. The factor l/L_{RAD} in equation (24) can be written as:

$$\frac{l}{L_{RAD}} = \frac{m_{gas}/S}{X_o} = \frac{fMl}{V\,X_o} = \frac{pM\,l}{R\,T\,X_o}, \tag{18}$$

where m_{gas} (g) is the mass of a gas from which the ions are scattered, $V = Sl$ (cm^3) is a volume of the gas, ν_{mol} is the number of moles, M (g/mol) is the molar mass, and X_o(g/cm^2) is the unit radiation length.

$$\frac{l}{L_{RAD}} = \frac{p(\text{Bar})\,M(\text{g/mol})\,l(\text{cm})}{R(\text{erg/K mol})\ \text{T(K)}\ X_o(\text{g/cm}^2)}.$$

3.0.4 The Transverse Emittance Growth in Gold-Gold Collisions

RHIC "Warm" Section (Gold on Gold Store): The dependence on the residual gas pressure p(Torr) for the warm part (T=300K) of the RHIC vacuum, where the unit radiation length is $X_o(\text{RHIC-WARM}) = 58.483$ (g/cm^2), and the molar mass of the gas composition $M \simeq 4$, can now be written as:

$$\frac{l}{L_{RAD}} = 2.06 \cdot 10^{-9}\, p(\text{Torr})\, l(\text{cm}).$$

The variance of the scattering angle in the transverse plane is calculated for gold $^{197}Au^{+79}$ ion beam as:

$$\Theta^2_{rms} = \left(\frac{15MeV}{m_p c^2 \gamma}\right)^2 \frac{Z_P^2}{A_P^2} \frac{l}{L_{RAD}} = \frac{8.467 \cdot 10^{-14}}{\gamma^2} \, l\,(\mathrm{cm}) \cdot p(\mathrm{Torr}).$$

The length of the residual gas can be written as $l \simeq c(\mathrm{cm/s}) \cdot t(\mathrm{s})$, where this is only 20% of the circumference. The emittance growth per hour of the gold ion beam in the warm sections of RHIC (T=300K), with the previously estimated value of the average beta function of $\beta_{avr} \simeq 30$ m, is calculated using the above result as:

$$\frac{d\epsilon_N}{dt} \simeq 4.57 \cdot 10^6 \cdot 0.2 \cdot \beta \ : (\mathrm{m}) \, \frac{p(\mathrm{Torr})}{\gamma} \quad (\mathrm{mm\,mrad/hour}).$$

The normalized emittance growth in the warm part of the RHIC (20%) at the estimated pressure of $p = 5 \cdot 10^{-10}$ (Torr) and $\gamma = 100$, is calculated as:

$$\frac{d\epsilon_N}{dt} \simeq 1.37 \cdot 10^{-4} \quad (\mathrm{mm\,mrad/hour}).$$

The normalized transverse emittance (defined by equation (11) of the gold beam during a store with the maximum energy γ=100 is estimated to be $\epsilon_N = 40\,\pi$ mm mrad. The beam σ used in the Fokker-Planck diffusion equation has to have the same value as the one obtained from the above emittance definition. The normalized emittance for the gold ions to be used in the emittance growth calculation is equal to:

$$\epsilon_N = \frac{\epsilon_{(\mathrm{RHIC})N}}{6\,\pi} = 6.67 \quad (\mathrm{mm\,mrad}).$$

The transverse emittance growth due to multiple elastic scattering after 10 hours of the maximum energy gold-gold collisions' store within the "warm" part of RHIC is equal to $1.37 \cdot 10^{-3}$ mm mrad which is to be compared to ϵ=6.67 mm mrad. This makes only 0.02% of the total transverse emittance.

RHIC "Cold" Section (Gold on Gold Store): The radiation length factor for the RHIC $5K$ cold vacuum chamber walls depends mostly on helium gas. The parameters in equation (33) are: M=4 (g/mol), T=5 K, and the unit radiation length $X_o(\mathrm{RHIC{-}COLD}) = 94.3221$ (g/cm^2):

$$\frac{l}{L_{RAD}} = 7.651 \cdot 10^{-8} \, p(\mathrm{Torr}) \, l(\mathrm{cm})$$

The variance of the scattering angle in the transverse plane for the cold section of RHIC is:

$$\Theta^2_{rms} = \frac{3.144 \cdot 10^{-12}}{\gamma^2} \, l\,(\mathrm{cm}) \cdot p(\mathrm{Torr}),$$

The length of the "cold" residual gas $l \simeq c(\mathrm{cm}/s) \cdot t(\mathrm{s})$ makes 80% of the circumference. The emittance growth per hour of the gold ion beam in the cold section of RHIC $5K$ is:

$$\frac{d\epsilon_N}{dt} \simeq 1.696 \cdot 10^8 \cdot 0.8 \cdot \beta\,(\mathrm{m}) \, \frac{p(\mathrm{Torr})}{\gamma} \quad (\mathrm{mm\,mrad/hour}).$$

The normalized emittance growth in the cold part of the RHIC (80%) at the estimated pressure of $p = 1 \cdot 10^{-11}$ (Torr) and $\gamma = 100$, is calculated as:

$$\frac{d\epsilon_N}{dt} = 4.07 \cdot 10^{-4} \quad (\mathrm{mm\,mrad/hour}).$$

The transverse emittance growth due to multiple elastic scattering after 10 hours of the maximum energy gold-gold collisions' store in the "cold" part of RHIC is equal to $4.07 \cdot 10^{-3}$ mm mrad which is to be compared to ϵ=6.67 mm mrad. This makes only 0.06% of the total transverse emittance.

3.0.5 The Transverse Emittance Growth in RHIC in Proton-Proton Collisions

RHIC "Warm" Section (Proton on Proton Store): The factor l/L_{RAD} is the same as in the gold on gold "warm" collision store. The variance of the scattering angle of the proton beam in the transverse plane, with the factor $Z_P^2/A_P^2 = 1$ and temperature of T=300K, is calculated as:

$$\Theta_{rms}^2 = \frac{5.265 \cdot 10^{-13}}{\gamma^2} \; l\,(\mathrm{cm}) \cdot p(\mathrm{Torr}),$$

while the normalized transverse emittance growth rate in the proton-proton store in the warm part (20% of circumference) of the RHIC, is predicted as:

$$\frac{d\epsilon_N}{dt} = 2.841 \cdot 10^7 \cdot 0.2 \cdot \beta\,(\mathrm{m}) \, \frac{p(\mathrm{Torr})}{\gamma} \; (\mathrm{mm\,mrad/hour}).$$

The proton transverse normalized emittance growth rate, in the warm part of the RHIC circumference, with an estimated pressure of $p = 5 \cdot 10^{-10}$ Torr, at the maximum energy of $\gamma = 250$, provides for the normalized emittance growth rate:

$$\frac{d\epsilon_N}{dt} \simeq 3.41 \cdot 10^{-4} \; (\mathrm{mm\,mrad/hour}).$$

The transverse emittance growth due to multiple elastic scattering after 10 hours of the maximum energy proton-proton collisions' store is equal to $3.41 \cdot 10^{-3}$ mm mrad which is to be compared to ϵ=3.33 mm mrad. This makes only 0.1% of the total transverse emittance.

RHIC "Cold" Section (Proton on Proton Store): The factor l/L_{RAD} has the same value as in the gold on gold "cold" collision store. The variance of the scattering angle of the proton beam in the transverse plane, with the factor $Z_P^2/A_P^2 = 1$ and temperature of $T = 5K$, is calculated as:

$$\Theta_{rms}^2 = \frac{1.955 \cdot 10^{-11}}{\gamma^2} \; l\,(\mathrm{cm}) \cdot p(\mathrm{Torr}).$$

while the normalized transverse emittance growth rate in proton-proton collision store in the cold part (80% of circumference) of the RHIC, is predicted as:

$$\frac{d\epsilon_N}{dt} = 1.055 \cdot 10^9 \cdot 0.8 \cdot \beta\,(\mathrm{m}) \, \frac{p(\mathrm{Torr})}{\gamma} \; (\mathrm{mm\,mrad/hour})$$

The proton transverse normalized emittance growth rate, with an estimated pressure of $p = 1 \cdot 10^{-11}$ Torr, at the maximum energy of $\gamma = 250$ provides for the normalized emittance growth rate:

$$\frac{d\epsilon_N}{dt} \simeq 1.01 \cdot 10^{-3} \; (\mathrm{mm\,mrad/hour}).$$

The transverse emittance growth due to multiple elastic scattering after 10 hours of the maximum energy proton-proton collisions' store is equal to $1.01 \cdot 10^{-2}$ mm mrad which is to be compared to ϵ=3.33 mm mrad. This makes only 0.3% of the total transverse emittance.

3.0.6 The Transverse Emittance Growth in RHIC due to the Hydrogen Jet Target

Gold Beam Emittance Growth due to the Hydrogen Jet: The betatron functions at the jet, located 6 m from an interaction region with $\beta^\star = 10$ m, are equal to:

$$\beta = \beta^\star + \frac{s^2}{\beta^\star} = 13.6 \text{ m}.$$

The emittance growth is estimated by the equations (12),(13), and (18). The residual gas pressure at the jet target is calculated as:

$$p = \frac{0.5 \cdot 10^{13}\ (1/\text{cm}^3) \cdot 300\ (\text{K})}{9.656 \cdot 10^{18}} = 1.55 \cdot 10^{-4}\ (\text{Torr})$$

The factor l/L_{RAD} in equation (24), in the case of the hydrogen jet where the unit of radiation length $X_o = 61.283$ (g/cm^2), can be written as:

$$\frac{l}{L_{RAD}} \simeq 1.52 \cdot 10^{-13}\ l\,(\text{cm})$$

The variance of the scattering angle of the gold beam in the transverse plane with a hydrogen jet is calculated as:

$$\Theta^2_{rms} = \frac{6.252 \cdot 10^{-18}}{\gamma^2}\ l\,(\text{cm}),$$

where the length of the target of only l= 2 cm reduces the multiple multiple Coulomb scattering by a factor:

$$l = c \cdot t', \quad \text{where}\, t' = t \cdot l/C = 5.217 \cdot 10^{-6}.$$

The normalized transverse emittance growth rate in the gold store with a presence of the hydrogen jet is predicted as:

$$\frac{d\epsilon_N}{dt} = 1.76 \cdot 10^{-3}\,\beta\,(\text{m})\,\frac{1}{\gamma}\ \ (\text{mm mrad/hour}) \tag{19}$$

or:

$$\frac{d\epsilon_N}{dt} = 2.394 \cdot 10^{-4}\ (\text{mm mrad/hour}).$$

The transverse emittance growth due to multiple elastic scattering from the jet target, after 10 hours of the maximum energy gold-proton collisions' store, is equal to $2.39{\cdot}10^{-3}$ mm mrad which is to be compared to ϵ=6.67 mm mrad. This makes only 0.035% of the total transverse emittance.

The Proton Beam Emittance Growth due to the Hydrogen Jet: A factor $l/L_{RAD} \simeq 1.52 \cdot 10^{-13} l\,(\text{cm})$ in equation (24), has the same values as in the gold ion store, as well as the the unit of radiation length $X_o = 61.283\,(g/\text{cm}^2$. The variance of the scattering angle of the proton beam in the transverse plane with a hydrogen jet is calculated by using the equation (33) as:

$$\Theta^2_{rms} = \frac{3.89 \cdot 10^{-17}}{\gamma^2}\ l\,(\text{cm}),$$

where the effect of the l=2 cm jet target is included by the following correction; $l = c \cdot t'$ with $t' = t \cdot l/C = 5.217 \cdot 10^{-6}$. The normalized proton transverse emittance growth rate, in the proton-proton store, with a presence of the hydrogen jet is predicted as:

$$\frac{d\epsilon_N}{dt} = 2.098 \cdot 10^3 \, \beta \, (\text{m}) \, \frac{1}{\gamma} \; (\text{mm mrad/hour})$$

or with the maximum energy $\gamma = 250$ and $\beta = 13.6 \, m$:

$$\frac{d\epsilon_N}{dt} = 5.95 \cdot 10^{-4} \; (\text{mm mrad/hour}).$$

The transverse emittance growth due to multiple elastic scattering of the jet atoms, after 10 hours of the maximum energy gold-proton collisions' store, is equal to $5.95 \cdot 10^{-3}$ mm mrad which is to be compared to ϵ=3.33 mm mrad. This makes 0.17% of the total proton transverse emittance.

SUMMARY OF THE RESULTS FOR THE TRANSVERSE EMITTANCE GROWTH DUE TO THE BEAM-GAS AND BEAM- JET MULTIPLE COULOMB SCATTERING

Table 3

Store	p(Torr)	ϵ_N(mm mrad)
Gold-Gold	Warm $5 \cdot 10^{-10}$	$1.37 \cdot 10^{-4}$
Gold-Gold	Cold $1 \cdot 10^{-11}$	$4.07 \cdot 10^{-4}$
Gold-Jet	$1.55 \cdot 10^{-4}$	$2.39 \cdot 10^{-4}$
proton-proton	Warm $5 \cdot 10^{-10}$	$3.41 \cdot 10^{-4}$
proton-proton	Cold $1 \cdot 10^{-11}$	$1.01 \cdot 10^{-3}$
Au-proton-Jet	$1.55 \cdot 10^{-4}$	$5.95 \cdot 10^{-4}$

APPENDIX-Emittance Growth Calculation by D. Trines-HERA:

A similar way to calculate the emittance growth due to the multiple Coulomb scattering was reported earlier [1]. The average growth in emittance due to changes in $\delta x'$ from the Coulomb scattering of residual gas molecules, around the circumference, was presented as:

$$\Delta\epsilon = \beta \cdot \delta x', \tag{20}$$

where $\delta x'$ is given by the length of the transversed gas measured in units of radiation length X_o:

$$\delta x' = \left(\frac{14.1}{p}\right)^2 \cdot \frac{X}{X_o}, \tag{21}$$

For high energies:

$$\delta x' = \frac{2.26 \cdot 10^{-4}}{\gamma^2} \cdot \frac{X}{X_o}, \tag{22}$$

expressed by pressure [1]:

$$\frac{X}{X_o} = 3.6 \cdot 10^5 \frac{p(\text{mbar}) \cdot M(\text{g})}{X_o\,(\text{g/cm}^2) \cdot T\,(\text{K})}\, t(\text{s}), \tag{23}$$

or for the average emittance growth $\Delta\epsilon$ (this is not normalized emittance ϵ_N instead $\epsilon = \epsilon_N/\gamma$) is given by:

$$\Delta\epsilon = \frac{81.3}{\gamma^2}\,\beta\,\frac{p(\text{mbar}) \cdot M(\text{g})}{X_o(\text{g/cm}^2) \cdot T(\text{K})}\, t(\text{s}), \tag{24}$$

Applying the equation (24) for the proton-proton store in RHIC, assuming that the whole circumference is "cold", where the pressure is p=$1{\cdot}10^{-11}$ (Torr), β=30 m, $X_o = 94.3221$ (g/cm^2), $\gamma \simeq$250, M=4 (He), and T=5 (K), the emittance growth per hour is:

$$\Delta\epsilon = 8.94 \cdot 10^{-12}\ (\text{m rad},\quad or\ \Delta\epsilon_N = 2.23 \cdot 10^{-3}\ (\text{mm mrad/hour}).$$

The result presented above was calculated for the whole circumference. With a correction for the 80% of the circumference the normalized emittance growth is $\Delta\epsilon_N = 1.78{\cdot}10^{-3}$(mm mrad/hour), which is to be compared to the previously calculated normalized transverse emittance growth of $\Delta\epsilon_N = 1.01 \cdot 10^{-3}$ (mm mrad/hour).

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