

BNL-105669-2014-TECH

BNL/SNS Technical Note No. 101;BNL-105669-2014-IR

# Estimates of the Linear Coupling Effects

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October 2001

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### **U.S. Department of Energy**

USDOE Office of Science (SC)

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October 3, 2001

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#### Linear coupling effects

In this note the effects of linear coupling are estimated for the proposed operating point,  $\nu_x = 6.23$ ,  $\nu_y = 6.20$  suggested by A. Fedotov [1, 2].

Two linear coupling effects are measureable and are used to correct linear coupling due to skew quadrupole field errors. In this note, the magnitude of these two effects will be computed for the proposed operating point  $\nu_x = 6.23$ ,  $\nu_y = 6.20$ .

In the absence of skew quadrupole field errors, the linear horizontal motion has the tune  $\nu_x$ , and the linear vertical motion has the tune  $\nu_y$ . In the presence of the skew quadrupole field errors the horizontal and vertical motions each show the presence of two tunes,  $\nu_1$  and  $\nu_2$ . Theory indicates that there are now two modes present, one with the tune  $\nu_1$  and the other with the tune  $\nu_2$ . The difference  $|\nu_1 - \nu_2|$  is called the tune splitting .  $\nu_1$  and  $\nu_2$  and the tune splitting can be measured. Global correction of the linear coupling effects is done by adjusting the skew quadrupole field correctors to minimize the tune splitting. It is estimated below that the increase in tune splitting caused by a rms rotation error of 1 mr in the location of the quadrupoles is about .0012 for the proposed tune of 6.23, 6.20. This is a small increase in the tune splitting compared to the tune splitting of .030 present without the coupling.

A second linear coupling effect has to do with the coupling of the horizontal and vertical motions. One finds that in one of the modes, the horizontal motion is larger than the vertical motion, while in the other mode the vertical motion is larger. Let us assume that the mode with the larger horizontal motion is the mode with the tune  $\nu_1$ , and the mode with the larger vertical motion is the mode with the tune  $\nu_2$ . The local coupling at a particular location in the ring is described by the two parameters  $C_1$  and  $C_2$  which are defined as follows: If one excites the betatron motion so that only the  $\nu_1$  mode is present then

$$C_1 = [y_{max}/x_{max}]_1 \tag{1}$$

where  $x_{max}$  and  $y_{max}$  are defined as the maximum values of the horizontal and vertical motions. Similarly,  $C_2$  is defined as

$$C_2 = [x_{max}/y_{max}]_2 (2)$$

 $C_1$  and  $C_2$  are measureable at any point in the ring where there are beam position monitors. They can be used to locally correct the coupling at that point by adjusting the skew quadrupole field correctors to minimize  $C_1$  and  $C_2[3, 4, 5]$ . It is estimated below that the local coupling caused by a rms rotation error of 1 mr in the location of the quadrupoles is about 14% for the proposed tune of 6.23, 6.20. This is probably small compared to the non linear coupling due to non linear space charge forces and the chromaticity correcting sextupoles.

#### Tune splitting estimate

 $\nu_1$  and  $\nu_2$  can be computed from [6]

$$\nu_{1} - \nu_{x} = -(\nu_{x} - \nu_{y})/2 + [((\nu_{x} - \nu_{y})/2)^{2} + |\Delta\nu|^{2}]^{1/2}$$
(3)  

$$\nu_{2} - \nu_{y} = -(\nu_{1} - \nu_{x})$$
  

$$\Delta\nu = \frac{1}{4\pi} \int ds (\beta_{x}\beta_{y})^{1/2} \frac{B_{1}}{B\rho} exp[i(\psi_{x} - \psi_{y})]$$
  

$$B_{x} = B_{1}x$$

which holds when  $\nu_x \geq \nu_y$  and  $\nu_x, \nu_y$  are near the  $\nu_x = \nu_y$  resonance line.  $B_x = B_1 x$  is the skew quadrupole field on the median plane. In the quadrupoles,  $B_1 = 2G\Delta\theta$  where  $\Delta\theta$  is the rotation error in the quadrupoles and G the quadrupole field gradient. The tune splitting depends on  $\Delta\nu$ , the coupling stopband. An estimate of  $\Delta\nu$  can be found for the 6.23, 6.20 working point using the following result for the rms  $|\Delta\nu|$  due an rms rotation error,  $\Delta\theta_{rms}$ , in the quadrupoles.

$$\Delta \nu_{rms} = \frac{1}{4\pi} \frac{2\Delta \theta_{rms}}{B\rho} \left[\sum L^2 G^2 \beta_x \beta_y\right]^{1/2} \tag{4}$$

where the sum is over the quadrupoles and L is the quadrupole length. For the 6.23, 6.20 lattice this gives

$$\Delta \nu_{rms} = .0042 \tag{5}$$

To estimate the tune splitting, one can use  $\Delta \nu = \Delta \nu_{rms} = .0042$ . Using this  $|\Delta \nu|, \nu_1 - \nu_x = -(\nu_2 - \nu_x)$  can be found, giving  $\nu_1 - \nu_x = -(\nu_2 - \nu_u) = .00058$ . The tune splitting estimate is then

$$\nu_1 - \nu_2 = .0012 + \nu_x - \nu_y \tag{6}$$

The additional tune splitting due to random skew quadrupole field errors is about .0012 .

#### Local coupling estimate

 $C_1$  and  $C_2$  at any point around can be estimated from the result

$$C_{1} = |(\nu_{1} - \nu_{x})/\Delta\nu|(\beta_{y}/\beta_{x})^{1/2}$$

$$C_{2} = |(\nu_{2} - \nu_{y})/\Delta\nu|(\beta_{x}/\beta_{y})^{1/2}$$
(7)

which holds when  $\nu_x, \nu_y$  are near the  $\nu_x = \nu_y$  resonance line and no coupling correction has been done. This gives for the 6.23, 6.20 working point with an rms rotation error of 1 mr in the quadrupoles

$$C_{1} = .14(\beta_{y}/\beta_{x})^{1/2}$$

$$C_{2} = .14(\beta_{x}/\beta_{y})^{1/2}$$
(8)

This result may be described as showing 14% coupling between the horizontal and vertical motions. It seems likely that this amount of linear coupling will be small compared to the non linear coupling due to non linear space charge forces and the chromaticity correcting sextupoles.

#### Vertical dispersion estimate

An estimate of the size of the vertical dispersion generated by the skew quadrupole error fields can be obtained by computing the rms vertical dispersion. Assuming that the vertical dispersion is largely generated by the rotation error in the quadrupoles , then the rms vertical dispersion,  $Y_{p,rms}$ , can be computed from the following result for the vertical dispersion  $Y_p$ 

$$Y_{p}(s) = \frac{1}{2sin(\pi\nu_{y})} \int ds' (\beta_{y}(s)\beta_{y}(s'))^{.5} \frac{2G\Delta\theta X_{p}(s')}{B\rho} cos(\pi\nu_{y} - |\psi_{y}(s) - \psi_{y}(s')|)$$
(9)

For a rotation error of 1 mr rms in the quadrupoles, one finds

$$Y_{p,rms} = .040m \tag{10}$$

at the vetical quadrupole in the straight section for the 6.23, 6.20 lattice. At  $\Delta p/p = .007$ , this gives a vertical displacement of .28 mm rms, which is small compared to the available aperture or the full beam size.

#### Acknowledgements

I wish to express my thanks for the information I have received from N. Catalan-Lasheras, A. Fedotov, C. Gardener, Y. Papaphilippou, J. Wei.

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