

## Interpretation of Coupling Impedance Bench Measurements

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# Interpretation of coupling impedance bench measurements

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Coupling impedance values of accelerator components can be obtained from standard bench measurements based on the coaxial wire method. The longitudinal impedance is obtained with one wire and the transverse impedance with a twin wire inserted into the “device under test”. The coupling impedance follows from the interpretation of the scattering coefficients from a network analyzer. In this report, models and formulae applicable to the interpretation of the data are collected and reviewed, with emphasize on lumped and distributed kicker magnets.

## I. INTRODUCTION

The driving terms of instabilities in accelerators/storage rings always depend on the beam surroundings which are conveniently described by impedances.<sup>1,2</sup> Establishing and maintaining a coupling impedance budget becomes an important part of designing a high current accelerator. Theoretical estimates for typical accelerator components have been developed and are available in the standard literature.<sup>3,4,5</sup> For critical devices, the estimates need to be confirmed by bench impedance measurements. The basic concept of bench measurements relies on simulating the beam by a wire for longitudinal or a twin wire *Lecher* line for transverse measurements.<sup>6</sup> The measurements typically involve a measurement of the *Device Under Test* (DUT) and of a *Reference* structure with the difference or ratio of the data used to interpret the coupling impedance. The question to what degree the bench impedance is a valid representation of the beam impedance requires a separate analysis and is beyond the scope of this paper. Theoretical<sup>7,8</sup> and experimental<sup>9</sup> work indicates that a remarkable agreement between actual and measured impedance is achieved with sufficiently thin wires.

Coupling impedance bench measurements discussed here are performed with a network analyzer which provides the scattering coefficients,  $S_{21}$  and  $S_{11}$ , of the DUT and the reference. The standard formulae used to interpret the measured data were all derived in the framework of transmission line theory. The field configuration on an ideal transmission line is a TEM wave with purely transverse components. The finite wall conductivity or a geometrical wall disturbance changes the field into a mode with a local axial component of the electric field responsible for the interaction with the beam. The assumption in the transmission line theory is, however, that the analysis can be performed with ideal walls and the real situation is handled by appropriately modifying the characteristic impedance and propagation constant. At sufficient distance away from the device, the pure TEM mode is reestablished but with modified amplitude and phase of the scattering coefficients. For the purpose of coupling impedance measurements, it is necessary to employ devices with beam tubes attached as part of the unit. Terminal effects, i.e. the local appearance of evanescent modes, at the junction of the device and the transmission line is part of the impedance, but extraneous steps in the transmission line must be avoided. End effects can to some degree be represented by added capacitive elements.<sup>10</sup> It is also plausible that the relative contribution of end effects is smaller for

long distributed impedances than for lumped impedances. Obviously, the bench measurements are limited to the low frequency range where higher order modes do not propagate. Notwithstanding its limitations, transmission line analysis represents the proper framework for the interpretation of coupling impedance bench measurements. The general aspects of impedance bench measurements are discussed in Caspers *Accelerator Handbook* article<sup>11</sup> and need not be repeated here.

The R&D and design work for the construction of the Spallation Neutron Source (SNS) required detailed impedance studies of various components.<sup>12,13</sup> The transverse impedance of the extraction kickers was judged to be critical to the performance of the accumulator ring and received special attention.<sup>14</sup> The measured data is obtained as obtained from the network analyzer as a normalized ratio  $S_{21}^{DUT} / S_{21}^{REF} = S_{21N}$  which is translated via a model to the  $Z^{DUT}$ . In the longitudinal measurement with a single wire, this represents already the coupling impedance. In the measurement with twin wires, spaced apart by  $\Delta$ , the transverse coupling impedance follows from

$$Z_{\perp} = \frac{c}{\omega \Delta^2} Z^{DUT} \quad (1)$$

The interpretation of the measurements was hindered by some inconsistencies of the available publications and pointed to the need for a uniform treatment. In this paper, the relevant models and the applicable interpretation are presented in general terms. The discussion starts with the lumped model for a longitudinal impedance and is extended to the transverse impedance model of a lumped kicker magnet. Then the model for a distributed wall impedance is discussed in view of several *improved* formulae. Finally, the model of a transmission line kicker magnet is analyzed.

## II. LUMPED IMPEDANCE

The scattering coefficients due to a single, lumped wall impedance,  $Z_w$ , are well known and for convenience repeated here,

$$S_{11} = \frac{Z_w}{2R_0 + Z_w} \text{ and } S_{21} = \frac{2R_0}{2R_0 + Z_w}. \quad (2)$$

Although in principle either coefficient gives correct results, the forward scattering coefficient is applicable to more general configurations and is generally preferred. The effect of the attached beam tubes is eliminated by normalizing the data with impedance,  $S_{21}^{DUT}$ , to that of a reference tube of equal length,  $S_{21}^{REF}$ . The wall impedance is given by the HP-formula<sup>15</sup>

$$Z_w = 2R_C \left( \frac{S_{21}^{REF}}{S_{21}^{DUT}} - 1 \right), \quad (3)$$

It is here assumed that the characteristic impedance,  $R_C$ , of the wire/beam tube is fully matched to the network analyzer impedance,  $R_0$ . Matching can be achieved by an ideal transformer or, with some loss in signal strength, by resistive matching.

### Resistive Matching

Resistive matching is achieved on the input side with a parallel resistor,  $R_p$ , and a series resistor,  $R_m$ , with

$$R_p = R_0 \sqrt{R_C / (R_C - R_0)}, \quad R_m = R_C - R_0 R_p / (R_0 + R_p), \quad (4)$$

and a series resistor on the output side,  $R_{out} = R_C - R_0$ .

### Effect of Mismatch

Matching is typically imperfect and the finite length of the beam tubes leads to errors at higher frequencies. Formal expressions for the forward scattering coefficient associated with a series impedance,  $Z$ , between unmatched coaxial beam tubes are derived here from elementary circuit theory without the explicit use of hybrid matrices. The notation used is exhibited in Fig. 1.

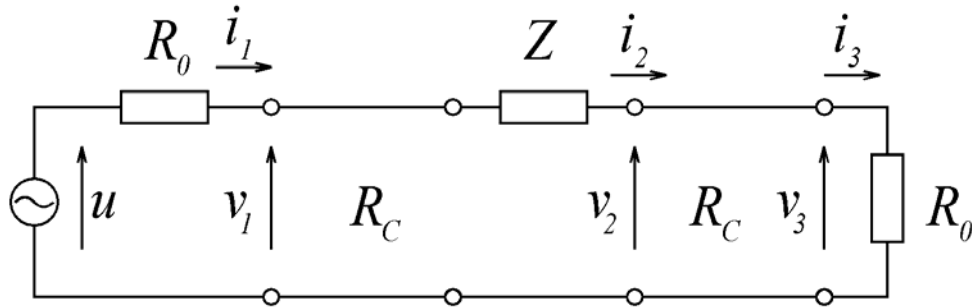


Fig.1. Circuit model for wire impedance measurements

The forward scattering coefficient is defined as

$$S_{21} = 2v_3/u \quad (5)$$

and is obtained by sequential elimination of the currents and voltages as follows,<sup>16</sup>

$$S_{21} = -\frac{ZZ_1}{(Z+Z_2)(R_0+Z_1)} + \frac{Z_1(Z+Z_2)}{(Z+Z_2)(R_0+Z_1)} \cos 2k\ell - j \frac{Z_C(Z+2Z_2)}{(Z+Z_2)(R_0+Z_1)} \sin 2k\ell \quad (6)$$

$$\text{with } Z_1 = \frac{Z+Z_2+jR_C \tan k\ell}{1+(Z+Z_2)R_C^{-1} \tan k\ell}; \quad Z_2 = \frac{R_0+jR_C \tan k\ell}{1+R_0R_C^{-1} \tan k\ell}. \quad (7)$$

The complete expression for the scattering coefficient is obtained via the MAXIMA program but is too large for presentation here. The Taylor expansion for low frequencies,  $k = \omega/c$ , follows as

$$S_{21} \approx \frac{2R_0}{2R_0+Z} - j4k\ell \frac{R_0(R_0^2+R_C^2+R_0Z)}{R_C(4R_0^2+4R_0Z+Z^2)} \quad (8)$$

The coefficient ratio for the interpretation of the measurements yields the formula

$$\frac{S_{21}^{DUT}}{S_{21}^{REF}} \approx \frac{2R_0}{2R_0+Z} + j2k\ell \frac{Z}{Z_C} \frac{Z_C^2 - R_0^2}{4R_0^2 + 4R_0Z + Z^2} \quad (9)$$

which can be used to interpret the measurements. In practice, it gives only an estimate of the error due to a mismatch. Here  $R_0$  represents the nominal instrument impedance (after matching) and  $R_C$  the actual line impedance. Note that toward zero frequency the error

vanishes and consequently the nominal  $R_0$ , rather than the actual  $R_C$  which is less accurate, should be used in the HP formula.

### III. LUMPED KICKER MAGNET

#### Nassibian & Sacherer Model<sup>17</sup>

The interpretation of wire measurements on a kicker magnet differs if the unit is designed as transmission or lumped magnet. The lumped magnet is at “low” frequencies characterized by a position-independent bus bar current. In spite of its finite length, the lumped magnet can thus be analyzed with the help of a transformer model as developed in the seminal paper by Nassibian & Sacherer (NS). The illustrative example assumes a perfect magnet with  $\ell, h, w$ , representing length, height, and width respectively. The kicker has an inductance  $L_K = \mu_0 h \ell / w$  and is terminated with the power supply impedance,  $Z_g$ . The expression for the coupling impedance seen by the beam is

$$Z_{\perp}^{NS} = \frac{c}{h^2} \frac{\omega L_K^2}{j\omega L_K + Z_g} \quad (10)$$

$$= \frac{c}{h^2} \left\{ \frac{\omega L_K^2 \operatorname{Re} Z_g}{\operatorname{Re} Z_g^2 + (\omega L_K + \operatorname{Im} Z_g)^2} - j \frac{\omega L_K^2 (\omega L_K + \operatorname{Im} Z_g)}{\operatorname{Re} Z_g^2 + (\omega L_K + \operatorname{Im} Z_g)^2} \right\}$$

with  $h$  the aperture in kick direction. In addition to the impedance coupled to the kicker termination, the beam sees an uncoupled impedance from image currents on the bus bar and the ferrite core. The uncoupled impedance is essentially inductive and the resistive part can be neglected. An estimate for the uncoupled inductance,  $L_I$ , is obtained from the simple model of a dipole current between metal plates spaced apart by the width,  $w$ ,

corresponding to <sup>14</sup>  $Z_{\perp} \approx \frac{j\pi\ell}{6w^2} Z_0$ . (11)

#### Lumped kicker bench measurement

The transverse coupling impedance is obtained from a twin wire bench measurement in which the magnet is coupled to the twin wire transmission line by the mutual inductance,  $M$ . The line has a nominal wire spacing,  $\Delta$ , a characteristic impedance of  $R_C$ , and is assumed fully matched to the network analyzer impedance. The line has in free space the inductance,  $L_C = R_C \ell / c$  and a negligible radiation resistance at the “low” frequencies of interest here, i.e. when the line is short compared with a wavelength. The wire measurements are interpreted with regard to a model represented by the equivalent circuit in Fig. 2. This model incorporates the impedance contributions from the uncoupled and coupled impedances, as well as that attributed to the leakage flux,  $(1 - \kappa^2)L_K$ .

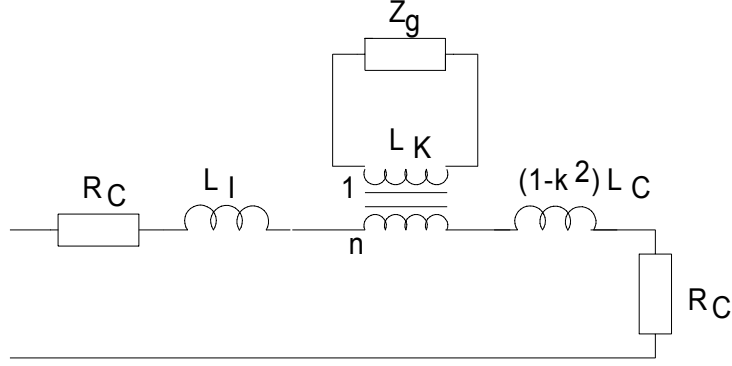


FIG. 2. Lumped kicker magnet circuit model

The coupling coefficient  $\kappa$  and transformer ratio  $n$  are given by

$$\kappa = \sqrt{\frac{M^2}{L_K L_C}} \text{ and } n = \frac{M}{L_K} = \frac{\Delta}{h} \quad (12)$$

From the standard electrical engineering description of a transformer follows the forward scattering coefficient directly as

$$\begin{aligned} S_{21}^{DUT} &= 2R_C \left\{ 2R_C + n^2 \frac{j\omega L_K Z_g}{j\omega L_K + Z_g} + j(1-\kappa^2)\omega L_C + j\omega L_I \right\}^{-1} \\ &= 2R_C \left\{ 2R_C + \frac{\omega^2 M^2}{j\omega L_K + Z_g} + j\omega L_C + j\omega L_I \right\}^{-1} \end{aligned} \quad (13)$$

The scattering coefficient of the reference line in the beam tube is in the low frequency approximation

$$S^{REF} = 2R_C (2R_C + j\omega L_w)^{-1} \quad (14)$$

Interpretation of the wire measurement via the lumped HP formula yields

$$Z^{DUT} = n^2 \frac{\omega^2 L_K^2}{j\omega L_K + Z_g} + j\omega L_I \quad (15)$$

and the transverse coupling impedance (in kick direction) becomes

$$Z_{\perp} = \frac{c}{\omega} \frac{Z^{DUT}}{\Delta^2} = \frac{c}{h^2} \frac{\omega L_K^2}{j\omega L_K + Z_g} + j \frac{c}{\Delta^2} L_I \quad (16)$$

As expected, the wire measurement yields the theoretical Nassibian & Sacherer impedance estimate plus the uncoupled image impedance.

### Reference Calibration

The above interpretation of the measurements implies the calibration of the twin line to obtain  $S_{21}^{REF}$  in a beam tube with a diameter equal to the aperture of the magnet. The calibration can also be done in free space, which is simpler in the case of rigid lines.



However, the line inductance (and correspondingly the characteristic impedance) is reduced by

$$\Delta L \approx \frac{2\mu_0\ell}{\pi} \left( \frac{\Delta}{w} \right)^2 \quad (17)$$

The results from measurements based on the tube as reference will differ from that on air by the reactive transverse impedance per unit length,<sup>18</sup>

$$Z_{\perp}^{PT} = -j \frac{2}{\pi w^2} Z_0 \quad (18)$$

as long as radiation from the line in air and wall losses of the tube are negligible. It is to be noted that the instability driving resistive part does not change with the reference taken as tube or air.

Kicker magnets with access to the bus bar offer the possibility to short it,  $Z_g = 0$ , and use the shorted magnet as reference, leading to the expression for the coupled impedance<sup>14</sup>

$$Z_{\perp}^{DH} = \frac{c}{\omega h^2} \frac{j\omega L_K Z_g}{j\omega L_K + Z_g} = \frac{c}{\omega h^2} \left\{ \frac{(\omega L_K)^2 \operatorname{Re} Z_g}{\operatorname{Re} Z_g^2 + (\omega L_K + \operatorname{Im} Z_g)^2} + j \dots \right\} \quad (19)$$

with the resistive part identical to the NS value. Using the shorted magnet as reference simplifies and shortens the time between measurements and effectively eliminates instrument drift.

### Frequency effect

The current induced voltage in the magnet is  $u_K \approx j\omega M i_C$  if the magnet length is short compared to the wavelength on the transmission line, but at higher frequencies one finds<sup>17</sup>

$$u_K = j\omega \{MG\} i_C \quad \text{with} \quad G = \frac{\sin \frac{1}{2}k\ell}{\frac{1}{2}k\ell} \quad (20)$$

leading to the measured impedance of the kicker magnet,

$$Z^{DUT} \approx \frac{(\omega MG)^2}{(Z_g + j\omega L_K)} + \dots \quad (21)$$

and a corresponding correction of the measured transverse impedance.

## IV. DISTRIBUTED IMPEDANCE

The transmission line analysis of a distributed impedance of length,  $\ell$ , can be based on the Faltens et al.<sup>6</sup> model in which the total impedance of the device,  $Z$ , is represented by a uniformly distributed wall impedance,  $Z/\ell$ . The bench wire measurements are interpreted by comparing the wave propagation through the device with that in a “perfect” reference tube. In this model, propagation in the device is described by the changed characteristic impedance and propagation constants,

$$Z_w = \eta Z_C = Z_C \sqrt{1 - j \frac{Z}{\Theta Z_C}}, \quad \text{and} \quad k_w = \eta k = k \sqrt{1 - j \frac{Z}{\Theta Z_C}} \quad (22)$$

where  $\Theta = k\ell = \omega\ell / c$  and  $Z_C$  are the electric length and the characteristic impedance of the reference tube. In this model, the coupling impedance is fully described by the changed propagation constant  $k_W$  or through the single complex value  $\eta$ . The expression for the coupling impedance can now be formally written as is<sup>19</sup>

$$Z = j\Theta Z_C (k_W^2 - k^2) / k^2, \quad (23)$$

or alternatively by

$$Z = j\Theta Z_C (\eta + 1)(\eta - 1) \quad (24)$$

Representing the amplitude of the forward and reflected wave by  $A$  and  $B$  respectively, one can apply field matching (i.e. voltage and current matching in the transmission line) which leads to the conditions,<sup>20</sup>

at the input port of the wall impedance

$$A_{in} + B_{in} = A_W + B_W, \quad (A_{in} - B_{in}) = \frac{1}{\eta} (A_W - B_W) \quad (25)$$

at the output port ( $A_{out} = 0$ )

$$A_W e^{-j\eta\Theta} + B_W e^{j\eta\Theta} = B_{out}, \quad \frac{1}{\eta} (A_W e^{-j\eta\Theta} - B_W e^{j\eta\Theta}) = B_{out} \quad (26)$$

With the scattering coefficients defined as

$$S_{21} = \frac{B_{out}}{A_{in}} \quad \text{and} \quad S_{11} = \frac{B_{in}}{A_{in}} \quad (27)$$

one finds after simple manipulations

$$S_{21} = \frac{4\eta e^{-j\eta\Theta}}{(\eta + 1)^2 - (\eta - 1)^2 e^{-j2\eta\Theta}} = \frac{2\eta}{2\eta \cos \eta\Theta + j(\eta^2 + 1) \sin \eta\Theta}, \quad (28)$$

$$S_{11} = \frac{(\eta^2 - 1)(1 - e^{-j2\eta\Theta})}{(\eta + 1)^2 - (\eta - 1)^2 e^{-j2\eta\Theta}} = \frac{j(\eta^2 - 1) \sin \eta\Theta}{2\eta \cos \eta\Theta + j(\eta^2 + 1) \sin \eta\Theta}, \quad (29)$$

These equations are exact within the limitations of the model and either of them could be used to extract numerically the value of  $\eta$  for use in Eq. (24). This task is simplified by combining the values of forward and reflected scattering coefficients to form the relation

$$\eta = \frac{1}{\Theta} \arccos \frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}} \quad (30)$$

Together with Eq. (24), this expression provides an exact value for the impedance but, due to its complexity, is of limited value for the routine interpretation of measurements.

### Wang & Zhang Formula<sup>21</sup>

An alternate formula for the interpretation of the wire measurements was derived by Wang & Zhang by introducing a corrected S-parameter,  $S_C \equiv e^{-jk_W\ell}$ , obtained from

$$S_C^2 + \frac{S_{11}^2 - S_{21}^2 - 1}{S_{21}} S_C + 1 = 0 \quad (31)$$

Inserting the above expressions for the scattering coefficients in Eqs. (28) and (29) confirms Eq. (30). The propagation constants  $k_w \ell = j \log S_C$  and  $kl = j \log S_{21}^{REF}$  are combined with Eq. (23) to yield the expression

$$Z^{WZ} = -Z_C \ln \left( \frac{S_C}{S_{21}^{REF}} \right) \left( 1 + \frac{\ln S_C}{\ln S_{21}^{REF}} \right) \quad (32)$$

This expression is exact but also of limited practical use.

### **Walling et al. Log-formula<sup>22</sup>**

Taking the ratio of scattering coefficients provided by the network analyzer, and treating the wall impedance as a perturbation of the reference tube, i.e.  $Z \ll Z_C$  leads to

$$\frac{S_{21}^{DUT}}{S_{21}^{REF}} = \frac{4\eta e^{-j(\eta-1)\Theta}}{(\eta+1)^2 - (\eta-1)^2 e^{-j2\eta\Theta}} \approx \exp(-j(\eta-1)\Theta) \quad (33)$$

Approximating Eq. (24) by

$$\eta - 1 \approx -j \frac{Z}{2\Theta Z_C}$$

leads to the well known log-formula for distributed impedances by Walling et al.,

$$Z = -j2Z_C \ln \frac{S_{21}^{DUT}}{S_{21}^{REF}} \quad (34)$$

### **Improved log-formulae by Vaccaro<sup>19</sup> and Jensen<sup>23</sup>**

Under the assumption that the reflection coefficient is small,  $S_{11} \approx 0$ , Vaccaro makes the approximation that  $S_{21}^{DUT} \approx \exp(-jk_w \ell)$ . Using Eq. (23), rather than (24), an improved expression for the measured impedance can be obtained,<sup>19</sup>

$$Z = -Z_C \ln \left( \frac{S_{21}^{DUT}}{S_{21}^{REF}} \right) \left( 1 + \frac{\ln S_{21}^{DUT}}{\ln S_{21}^{REF}} \right). \quad (35)$$

An identical equation, although written in a more convenient form by means of  $S_{21}^{REF} = \exp(-j\Theta)$ , was recently presented by Jensen,<sup>23</sup>

$$Z = -2Z_C \ln \left( \frac{S_{21}^{DUT}}{S_{21}^{REF}} \right) \left\{ 1 + \frac{j}{2\Theta} \ln \left( \frac{S_{21}^{DUT}}{S_{21}^{REF}} \right) \right\} \quad (36)$$

The improved impedance expressions require the knowledge of the electrical length of the device under test and its accuracy decreases for shorter devices. In contrast, the simple log-formula is generally applicable including lumped components, provided that no strong resonance is present and the perturbation treatment is justified. The lumped impedance formula is applicable to single resonances and has the advantage that the scattering coefficient ratio is directly converted into an impedance by the network analyzer.

## **V. TRAVELING WAVE KICKER**

One of the more important distributed impedance is represented by traveling wave kicker magnets and the interpretation of bench measurements is typically done with the log-

formula. The above concept of treating the lumped kicker magnet as a transformer can be generalized and applied to traveling wave kickers.<sup>24</sup> The kicker properties are characterized by its characteristic impedance,  $Z_K$ , propagation constant,  $k_K$ , and electrical length,  $\Theta = k_K \ell$ . The kicker and the Lecher line are treated as transmission lines, coupled via the mutual inductance,  $M$ , for which the differential equations are well known. The general solution becomes unwieldy and several simplifications can be adopted without reducing the value of the results. The major part of the impedance is due to the coupled flux between the beam and the external terminations at either end of the bus-bar, so that the contribution of the uncoupled flux can be neglected. In order to let the Lecher line represent the “stiff” ultra-relativistic beam, its current is considered externally imposed and thus unchanged by the current in the bus bar. The impedance seen by the beam is then obtained by the voltage generated by the bus-bar current via the mutual inductance. One finds, with the time dependence  $e^{j\omega t}$  suppressed, the following set of differential equations in the position dependent variables,  $i_K$ ,  $u_K$ ,  $i_B$ ,  $u_B$  representing the kicker current and voltage, and the beam current and voltage respectively

$$\frac{\partial u_K}{\partial s} = -jk_K Z_K i_K + j \frac{\Delta}{h} k_K Z_k i_B \quad (37)$$

$$\frac{\partial i_K}{\partial s} = -j \frac{k_K}{Z_k} u_K \quad (38)$$

$$\frac{\partial u_B}{\partial s} = j \frac{\Delta}{h} k_K Z_K i_K \quad (39)$$

where  $k_K = k \sqrt{L'C'}$ ,  $Z_K = \sqrt{L'/C'}$ , and  $k = \omega/c$ .  $L'$  and  $C'$  are the kicker inductance and capacity per unit length and  $\Delta/h = M'/L'$ . Assuming an ultra-relativistic beam current,  $i_B = I e^{-jks}$ , associated with the dipole strength  $I\Delta$ , one finds the impedance measured in the bench measurement

$$Z^{DUT} = -2Z_L \ln \frac{S_{21}^{DUT}}{S_{21}^{REF}} = -\frac{1}{I} \int_0^\ell \frac{\partial u_B}{\partial s} e^{jks} \quad (40)$$

This value yields the transverse coupling impedance according to

$$Z_\perp = \frac{c}{\omega \Delta^2} Z^{DUT} \quad (41)$$

The solution of the above differential equations are found by imposing the boundary conditions established by the kicker input and output terminations,  $R_i$  and  $R_o$ ,

$$u_K(0) = R_i i_K(0), \quad u_K(\ell) = -R_o i_K(\ell) \quad (42)$$

The general expression for the coupling impedance is somewhat lengthy, but reduces in typical kickers where  $k \ll k_K$  to a manageable size. Furthermore, in the low frequency range of interest, one can take  $i_B \approx I$ . The case of input and output terminated with the characteristic impedance follows in this approximation as

$$Z_\perp = \frac{c}{\omega h^2} Z_K [(1 - \cos \Theta) + j(\Theta - \sin \Theta)] \quad (43)$$

which differs from Nassibian's expression<sup>24</sup>

$$Z_{\perp}^{NS} = \frac{c}{\omega W^2} Z_K [(1 - \cos \Theta) + j(\Theta - \sin \Theta)] \quad (44)$$

only in its dependence on geometry. (Note that the corresponding handbook formula contains typographical errors).<sup>25</sup>

## References

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- <sup>1</sup> V.G. Vaccaro, CERN Report No. ISR-RF 66-35 (1966)
- <sup>2</sup> A. Sessler, CERN Yellow Report 67-2 (1967)
- <sup>3</sup> G. Lambertson, in *Physics of Particle Accelerators, AIP Conference Proc. 153*, edited by M. Month and M. Dienes (AIP, New York, NY, 1987), Vol. **2**, p.1413.
- <sup>4</sup> K. Y. Ng, in *Handbook of Accelerator Physics and Engineering*, edited by A. W. Chao and M. Tigner (World Scientific, Singapore, 1998), p. 203.
- <sup>5</sup> K. Y. Ng, in *Physics of Particle Accelerators, AIP Conference Proceedings 184*, edited by M. Month and M. Dienes (AIP, New York, NY, 1989), Vol. **1**, p.472.
- <sup>6</sup> A. Faltens, E. C. Hartwig, D. Mohl, and A. M. Sessler, Proceedings of the 8th Intern. Conf. High-Energy Accelerators, Geneva, p. 338, (CERN, Geneva, 1971).
- <sup>7</sup> R. L. Gluckstern and R. Li, *Particle Accelerators* **29**, 159 (1990).
- <sup>8</sup> H. Hahn, *Phys. Rev. ST Accel. Beams* **3**, 122001 (2000).
- <sup>9</sup> D. Davino, M. R. Masullo, V. G. Vaccaro, and L. Verolino, *Il Nuovo Cimento* **114B**, 1319 (1999).
- <sup>10</sup> R. W.P. King, in *Encyclopedia of Physics*, edited by S. Fluegge (Springer-Verlag, Berlin, 1958), Vol. **XVI**, Chap. 17, p.210.
- <sup>11</sup> F. Caspers, in *Handbook of Accelerator Physics and Engineering*, edited by A. W. Chao and M. Tigner (World Scientific, Singapore, 570), p.570.
- <sup>12</sup> J. Wei, Proceedings of the 2002 European Particle Conference, Paris, France, 1067 (CERN, Geneva, 2002),
- <sup>13</sup> H. Hahn, Proceedings of the 2003 Particle Accelerator Conference, Portland, OR, RPPB010, (2003).
- <sup>14</sup> D. Davino and H. Hahn, *Phys. Rev. ST Accel. Beams* **6**, 012001 (2003).
- <sup>15</sup> H. Hahn and F. Pedersen, BNL Report , BNL 50870 (1978)
- <sup>16</sup> H. Hahn, BNL/SNS Technical Note No. 120
- <sup>17</sup> G. Nassibian and F. Sacherer, *Nucl. Instr. Methods* **159**, 21 (1979).
- <sup>18</sup> A. Mostacci, F. Caspers, and U. Iriso, Particle Accelerator Conference, Portland, OR, TPPB090, (2003).
- <sup>19</sup> V.G. Vaccaro, Report INFN/TC-94/023 (1994)
- <sup>20</sup> H. Hahn, M. Morvillo, and A. Ratti, Report BNL, AD/RHIC/RD-95 (1995)
- <sup>21</sup> J. G. Wang and S. Y. Zhang, *Nucl.Instr. Methods* **A459**, 389 (2001).
- <sup>22</sup> L. S. Walling, D. E. McMurray, D. V. Neuffer and H.A. Thiessen, *Nucl. Instr. & Methods* **A281**, 433 (1989).
- <sup>23</sup> E. Jensen, CERN PS/RF/Note 2000-001 (2000)
- <sup>24</sup> H. Hahn, in *Workshop on Instabilities of High Intensity Hadron Beams in Rings*, edited by T. Roser and S.Y. Zhang (American Institute of Physics, Melville, NY, 1999), p.266.
- <sup>25</sup> K. N. Ng, in *Handbook of Accelerator Physics and Engineering*, edited by A. W. Chao and M. Tigner (World Scientific, Singapore, 1998), p.206.