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NSNS Space Charge Effect

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Abstract

A space charge formulation is presented, then the envelope oscillation, emittance growth are discussed. The relevant experiments performed on several low energy proton synchrotrons are reviewed, which agree with the theoretical prediction. The space charge effect on the NSNS is discussed based on these informations.

1 Introduction

In this note, the space charge effect formulation will be discussed. After a brief review of the prediction of the envelope oscillation and simulation, the observations and experiments of the space charge effect performed on Fermilab Booster, CERN PS, PS Booster, PSR, and AGS Booster will be presented. These are regarded as the useful reference for the NSNS storage ring space charge effect. Finally, some comments will be made for the NSNS storage ring.

2 Space Charge Effect Formulation

The space charge incoherent tune spread can be written as,

$$\Delta\nu_{inc} = \frac{-NRr_0}{2\pi\nu_0\beta^2\gamma^3a^2} \tag{1}$$

where N is the total number of particles, R is the machine average radius, r_0 is the classical radius of proton, 1.535×10^{-18} m, ν_0 is the betatron tune with zero beam current, and a is the average radius of the beam.

This formulation applies to the coasting symmetric beam with uniform distribution, also under the conditions of the non-penetrating fields and circular chamber.

For Gaussian distribution, the beam radius should be,

$$a = \sqrt{2}\sigma \tag{2}$$

where σ is the standard deviation, or the *rms* beam radius.

For bunched beam, simply adding the bunching factor B_f in the denominator, we get,

$$\Delta\nu_{inc} = \frac{-Nr_0R}{4\pi\nu_0B_f\beta^2\gamma^3\sigma^2} \tag{3}$$

Using the relation between the beam size and the normalized emittance including 95% particles,

$$\epsilon_{N,95\%} = \frac{6\beta\gamma\nu_0\sigma^2}{R} \tag{4}$$

we have

$$\Delta\nu_{inc} = \frac{-3Nr_0}{2\pi B_f \beta\gamma^2 \epsilon_{N,95\%}} \tag{5}$$

The space charge tune shift presented here represents, in fact, the maximum tune shift, which applies only to the particles with the smallest betatron tune amplitudes. For bunched beams, it applies further only to the particles at the peak current.

Serving as a convention in discussing the space charge effect, this formulation has been commonly used in literature. In the following review of the theory, simulation, and experiments, we need indeed only indicate out that someone who is not using this formulation.

3 Envelope Oscillation, Emittance growth and simulation

The theory of envelope oscillation, the emittance growth, and the associated simulation will be briefly reviewed, based on the formulation presented in the last section.

3.1 Envelope oscillation

In discussing the space charge effect, a ring with uniform transverse focusing can be assumed. The direct space charge, on the other hand, is a defocusing force. In the form, it is similar to the quadrupole, or half integer, force.

As an internal force, the space charge is essentially different from the external focusing force, and it is unequally distributed among the particles. Also, a strong space charge force produces larger than the original beam size, in turn, a larger beam size implies weaker space charge force.

Therefore, the approach in analyzing the space charge effect has to be self-consistent. The self-consistency can be represented in the relation between the beam *rms* beam size σ and the space charge force k, as $k = \alpha/\sigma^2$, where α is a constant. An envelope equation developed in this way can be shown as,

$$\sigma'' - \frac{\epsilon_0^2}{\sigma^3} + K\sigma - \frac{\alpha}{\sigma} = 0 \tag{5}$$

where ϵ_0 is the equilibrium beam emittance, K is the external focusing force. Sacherer [2] predicted that if the space charge incoherent tune spread is depressed beyond half integer, or integer, by a third of the tune shift, envelope oscillation will happen. For instance, taking the AGS tune at injection as 8.85, then an envelope oscillation may happen if we have $|\Delta\nu_{inc}| \geq 0.47$. Let us define the variable χ to represent the percentage of the tune spread beyond the half integer, or integer. In this case we have $\chi = 33$.

Note that the half integer error does not show up in the equation (5), and therefore, it is not a necessary condition in an envelope oscillation. It is, however, a strong exciting force, because of its resonance to the inherent mode of space charge.

3.2 Emittance growth and simulation

The envelope oscillation implies a significant equilibrium emittance growth only for very large space charge force, which happens for the low energy end of Linac. For all synchrotrons, therefore, the envelope oscillation will not give rise to essential emittance growth.

In the simulation performed for LEB of SSC [3], Machida found that the beam emittance grows when the bare tune becomes smaller than 11.66, for a fixed tune spread $\Delta\nu_{inc} = -0.33$. With half integer resonance, the particle distribution change has been observed, which is believed to cause the emittance growth.

In reality, other factors might contribute to the emittance growth if the envelope oscillation becomes strong. For instance, the nonlinearity of the quadrupole focusing, and the nonlinearity of the space charge force itself.

It is interesting, therefore, to confirm the relevance between the onset of the envelope oscillation and the emittance growth.

4 Study of Low Energy Proton Synchrotrons

In the past two decades, experiments have been performed on several low energy proton synchrotrons. The emittance growth is indeed observed, when the space charge incoherent tune spread is depressed beyond half integer, or integer, by about a third of the tune shift, i.e. at the onset of the envelope oscillation.

4.1 Fermilab Booster

In an experiment performed at the Fermilab Booster, the beam intensity was increased, while the emittance was observed [4]. The Linac beam emittance was $\epsilon_{N,95\%} = 7\ \pi mmmr$. At $N = 1.2 \times 10^{12}$, emittance growth was observed. The tune spread is calculated as $\Delta\nu_{inc} = -0.38$, which is kept constant for higher intensities, because of the emittance growth. For example, at $N = 1.2 \times 10^{12}$, the emittance is proportionally enlarged to $\epsilon_{N,95\%} = 17$ $\pi mmmr$. The bare tune was $\nu_y = 6.8$, and we have $\chi = 21$.

4.2 CERN PS

At the CERN Proton Synchrotron (PS), for a fixed beam intensity, the RF voltage variation was used for the space charge tune shift adjustment, instead of using the intensity variation. The growth of the emittance was observed [5]. For the vertical bare tune of $\nu_y = 6.22$, at $\Delta\nu_{inc} = -0.28$, emittance growth was observed. For shorter bunches, i.e. stronger space charge effects, the emittance grows such that the real incoherent tune spread is kept constant. We calculate that $\chi = 21$.

Another set-up shows that at the vertical bare tune of $\nu_y = 6.28$, at $\Delta\nu_{inc} = -0.37$, emittance grows. In this case, we have $\chi = 24$.

4.3 PS Booster

No dedicated study for the emittance growth was available. Instead, some information can be learned in the intensity push reported for the PS Booster [6].

The space charge formulation in [6] uses a form factor for the transverse particle distribution, which has been, in fact, accounted in the equation (2). Therefore, this factor is double counted. Moreover, other complications involved in the intensity push, such as that a debuncher in Linac is used to change the particle distribution, the vertical beam mis-steering and x-y coupling are used in the injection, double RF is used, the beam horizontal emittance is 3 times of the vertical one, and the ring is filled up, with heavy beam losses. All these made the use of the original data very difficult.

It is known that the PS Booster vertical limiting physical half aperture is 32 mm, similar to the AGS Booster. The limiting average beam rms size is, therefore, about 10 mm. Using $\sigma = 10$ mm and the typical high intensity $N = 6 \times 10^{12}$ particle per ring for PSB, we get $\Delta\nu_{inc} = -0.66$, meanwhile $\nu_y = 5.45$. That is $\chi = 32$.

4.4 PSR

Various set up for the bare tune and intensity were made to observe the emittance growth at the Proton Storage Ring (PSR) at the Los Alamos [7].

The space charge incoherent tune shift defined in [7] is smaller than (5) by a factor of 2, which for Gaussian distribution happens to be the rms tune spread. For consistency, we still use the tune spread (5). Let the original rms beam size be 4.25 mm, then the emittance growth with respect to the intensity increase and also the bare tune is shown in Table 1. We observe that only in two cases, i.e. A.1 and B.1, there are no emittance growth. In other cases, the emittance grows proportionally to the intensity increase. This happens for each bare tune set up. We may also see that this experiment failed to identify the emittance growth thresholds, for that we can only say $\chi < 34$, which comes from the cases A.2 and C.1.

Case	ν_y	N	σ_y	$\epsilon_{N,95\%}$	χ
		10^{12}	rms size	emittance	%
A.1	2.193	6	4.25	11.75	-30
A.2	2.193	11.8	4.95	15.94	34
A.3	2.193	23	6.65	28.76	66
B.1	2.142	6	4.2	11.47	7
B.2	2.142	11.8	5.75	21.51	53
B.3	2.142	23	7.75	39.07	76
C.1	2.100	6	4.75	14.68	35
C.2	2.100	11.8	7.25	34.19	67
C.3	2.100	23	10.75	75.17	83
D.1	2.059	6	6.3	25.82	63
D.2	2.059	11.8	10.3	69.01	81
Loss Inhibits	2.059	23			

Table 1

4.5 AGS Booster

At the AGS Booster multiturn injection, the vertical mis-steering is used to define the vertical emittance, which we assume to be equivalent to $\epsilon_{N,95\%} = 30$ $\pi mmmr$. For '95 and '96 HEP run, the beam vertical emittance observed at the BTA multiwire 006 clearly indicates the emittance growth along with the intensity increase, which is shown in Fig.1. It is known that the Booster extraction septum has a vertical aperture at about $\epsilon_{N,95\%} = 65$ $\pi mmmr$. This limitation is shown in Fig.1. Since there is emittance growth, the Booster vertical acceptance is not determined at 200 MeV, but at about 10 ms after the injection, at 340 MeV, where the bare tune has been reduced from $\nu_y = 4.95$ at the injection to $\nu_y = 4.80$. If we take $\epsilon_{N,95\%} = 40\ \pi mmmr$ for $N = 40 \times 10^{12}$, and $\epsilon_{N,95\%} = 60\ \pi mmmr$ for $N = 60 \times 10^{12}$, shown by a solid line in Fig.1, then we can calculate that $\Delta\nu_{inc} = -0.36$. Therefore, we get $\chi = 17$.

4.6 Summary

The studies performed on the low energy proton synchrotrons have shown the emittance growth at the threshold predicted as the onset of the envelope oscillation. Therefore, it is convinced that the envelope oscillation causes the beam emittance growth. The results are summarized in Table 2.

Parameter		FermiB	CPS	CPS	PSB	PSR	AGSB	Unit
Vertical Tune	ν_y	6.80	6.22	6.28	5.45	2.19	4.80	
Tune Spread	$-\Delta\nu_y$	0.38	0.28	0.37	0.66	0.29	0.36	
Threshold	χ	21	21	24	32	<34	17	%

Table 2

5 Space Charge Effect in NSNS

Once the maximum space charge tune spread is depressed beyond the half integer by a certain amount, the envelope oscillation can be excited, and the emittance will grow. Below that threshold, the emittance growth should not be a big concern. However, the conservation of the emittance does not necessarily imply no beam loss. In fact, the beam losses have been observed at many synchrotrons at the tune spread below that threshold, the reason is the resonance stopband crossing. At the threshold of the onset of the envelope oscillation, a large number of particles have crossed the half integer, or integer resonance. Therefore, resonance modes have been established, which give rise to the envelope oscillation. Below the threshold, a smaller number of particles have crossed the half integer, or integer resonance, which may, however, still cause beam loss, because of the resonance crossing.

With the beam loss allowed at 10^{-3} to 10^{-4} for the NSNS storage ring, the maximum space charge tune spread $\Delta\nu_{inc}$ cannot be depressed beyond the half integer and integer. Probably even more, the higher order resonances should also be avoided, or carefully corrected. An acceptable criterion for the space charge incoherent tune spread can be $\Delta\nu_{inc} < 0.2$.

The ratio of the aperture and the beam size of the emittance defined according to this criterion probably should be at least large than 2, better at 3, for the 2 MW machine.

For NSNS, only the multiturn injection and extraction are concerned, therefore, the space charge effect must be studied by using the simulation. However, the simulation code should be able to apply to some real machine. Say, to get similar results like the emittance growth, beam loss, that have been actually observed. For this purpose, the results presented in this note can be used as a reference.

Just like that the emittance needs to be large enough to hold itself, there might be a similar constraint for the particle density in phase space for vertical mis-steering, or smoke ring, injection. If this is true, then the vertical painting will be helpful in the injection with vertical mis-steering. Also, x-y linear coupling could be considered in the injection.

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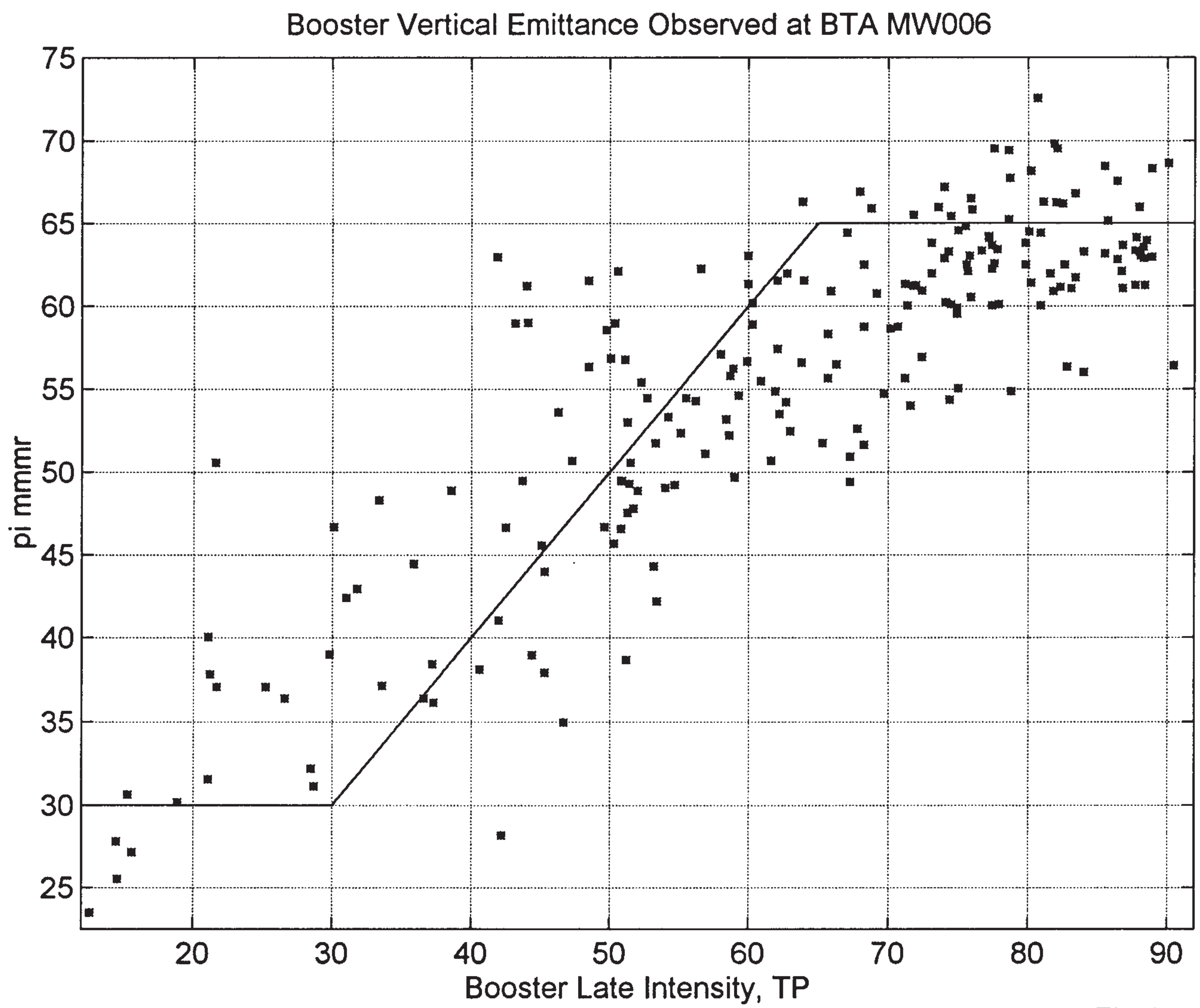
Booster Vertical Emittance Observed at BTA MW006
75
70
65
60
55
50
45
40
35
30
25
pi mmmr
20
30
40
50
60
70
80
90
Booster Late Intensity, TP

Fig.1