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In this note the nominal parameters for helions and gold ions in Booster, AGS, and RHIC are given for the FY2014 running period. (The helion is the bound state of two protons and one neutron. It is the nucleus of a helium-3 atom.)

1 Mass

A gold ion with charge eQ has $N = 118$ neutrons, $Z = 79$ protons, and $(Z - Q)$ electrons. Here Q is an integer and e is the positive elementary charge. The mass number is

$$A = N + Z = 197. \tag{1}$$

This is also called the number of nucleons. The mass energy equivalent of the ion is

$$mc^2 = am_uc^2 - Qm_ec^2 + E_Q \tag{2}$$

where [1, 2]

$$a = 196.9665687(6) \tag{3}$$

is the relative atomic mass of the neutral gold atom,

$$m_uc^2 = 931.494061(21)\text{ MeV} \tag{4}$$

is the mass energy equivalent of the atomic mass constant, and

$$m_ec^2 = 0.510998928(11)\text{ MeV} \tag{5}$$

is the electron mass energy equivalent. The binding energy E_Q is the energy required to remove Q electrons from the neutral gold atom. This amounts to [3, 4] 0.3324 MeV for the helium-like gold ion ($Q = 77$) and 0.5170 MeV for the fully stripped ion. For $Q = 32$ we have $E_Q = 14.5$ KeV. Thus the mass energy equivalents for the Au32+, Au77+, and Au79+ ions are

$$mc^2(\text{Au32+}) = 183.456851494 \text{ GeV} \tag{6}$$

$$mc^2(\text{Au77+}) = 183.434174442 \text{ GeV} \tag{7}$$

and

$$mc^2(\text{Au79+}) = 183.433337044 \text{ GeV}. \tag{8}$$

The mass energy equivalent of the helion is [2]

$$m_h c^2 = 2.808391482(62) \text{ GeV}. \tag{9}$$

2 Helion anomalous g-factor

The helion mass to proton mass ratio is [2]

$$\mathcal{R} = 2.9931526707(25). \tag{10}$$

The helion magnetic moment to nuclear magneton ratio is [2]

$$\mu_h/\mu_N = -2.127625306(25). \tag{11}$$

The helion charge to proton charge ratio is

$$\mathcal{Q} = 2. \tag{12}$$

The helion g-factor is then [5]

$$g_h = 2\,(\mathcal{R}/\mathcal{Q})(\mu_h/\mu_N) = -6.36830736690 \tag{13}$$

and the anomalous g-factor is

$$G_h = (g_h - 2)/2 = -4.18415368345. \tag{14}$$

3 Kinetic Parameters

In a circular accelerator the ion moves along an orbit of circumference C with revolution frequency f. The radius of the orbit is defined to be $R = C/(2\pi)$. The velocity of the ion is then

$$v = 2\pi R f. \tag{15}$$

This gives momentum, energy, and kinetic energy

$$p = mc\beta\gamma, \quad E = mc^2\gamma, \quad W = mc^2(\gamma - 1) \tag{16}$$

where

$$\beta = v/c, \quad \gamma = 1/\sqrt{1-\beta^2}. \tag{17}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = kcp/Q \tag{18}$$

where $k = 10^9/299792458$ and cp is given in units of GeV. The angular frequency is

$$\omega = 2\pi f. \tag{19}$$

We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{20}$$

where γ_t is the transition gamma. Note that as defined here, η is negative below transition and positive above transition.

4 RF Parameters

1. The stationary bucket area is

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2} \tag{21}$$

where h is the RF harmonic number, V_g is the total RF gap voltage per turn, and the subscript "s" denotes parameter values for the synchronous particle.

2. The half-height of a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \left|(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s\right|^{1/2} \tag{22}$$

where ϕ_s is the synchronous phase.

3. The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{23}$$

where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

4. The width of a bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{24}$$

where the phase ϕ_e satisfies

$$\cos(\pi - \phi_s) - \cos\phi_e = -(\pi - \phi_s - \phi_e)\sin\phi_s. \tag{25}$$

5. The area of a bucket is

$$A_{\rm bk} = \alpha(\phi_s) A_S \tag{26}$$

where

$$\alpha(\phi_s) = \frac{\sqrt{2}}{8}\int_{\phi_L}^{\phi_R} \left|(\pi - \phi_s - \phi)\sin\phi_s - \cos\phi_s - \cos\phi\right|^{1/2} d\phi. \tag{27}$$

Below transition we have $\phi_e < \pi - \phi_s$ and the limits of integration are $\phi_L = \phi_e$ and $\phi_R = \pi - \phi_s$. Above transition we have $\pi - \phi_s < \phi_e$ and the limits of integration are $\phi_L = \pi - \phi_s$ and $\phi_R = \phi_e$. The integral $\alpha(\phi_s)$ must be evaluated numerically. An approximate expression is [6]

$$\alpha(\phi_s) \approx \frac{1 - \sin\phi_s}{1 + \sin\phi_s}. \tag{28}$$

6. The synchrotron frequency for small-amplitude oscillations about ϕ_s is

$$F_s = \frac{c}{2\pi R_s} \left\{ \frac{-h\eta_s eQV_g \cos\phi_s}{2\pi E_s} \right\}^{1/2} \tag{29}$$

and the corresponding synchrotron tune is $Q_s = 2\pi F_s/\omega_s$. Note that measurement of F_s gives a value for $V_g \cos\phi_s$, while measurement of dB/dt gives a value for $V_g \sin\phi_s$. These two can be used to obtain V_g and ϕ_s.

7. Let ϕ_l and ϕ_r be the phases at the left and right boundries of a bunch matched to a bucket. We have

$$\phi_l < \phi_s < \phi_r \tag{30}$$

and the width of the bunch is

$$\Delta t = \frac{\Delta\phi}{h\omega_s}, \quad \Delta\phi = \phi_r - \phi_l. \tag{31}$$

In terms of $\Delta\phi$ and ϕ_s we have

$$\phi_r = \frac{\Delta\phi}{2} + \arcsin\left\{ \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \right\} \tag{32}$$

and

$$\phi_l = -\frac{\Delta\phi}{2} + \arcsin\left\{ \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \right\}. \tag{33}$$

If $\Delta\phi$ is small we have

$$\sin(\Delta\phi/2) \approx \frac{\Delta\phi}{2}, \quad \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \approx \sin\phi_s \tag{34}$$

and

$$\phi_l \approx \phi_s - \frac{\Delta\phi}{2}, \quad \phi_r \approx \phi_s + \frac{\Delta\phi}{2}. \tag{35}$$

8. The half-height of a bunch matched to a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \, |\cos\phi_r - \cos\phi_s + (\phi_r - \phi_s)\sin\phi_s|^{1/2}. \tag{36}$$

9. The area of a bunch matched to a bucket is

$$A_\mathrm{b} = F(\phi_s, \Delta\phi)\, A_S \tag{37}$$

where

$$F(\phi_s, \Delta\phi) = \frac{\sqrt{2}}{8} \int_{\phi_l}^{\phi_r} |\cos\phi_l - \cos\phi + (\phi_l - \phi)\sin\phi_s|^{1/2}\, d\phi. \tag{38}$$

The integral $F(\phi_s, \Delta\phi)$ must be evaluated numerically. If $\Delta\phi$ is small we have

$$F(\phi_s, \Delta\phi) \approx \frac{\pi}{64}(\Delta\phi)^2\, |\cos\phi_s|^{1/2}\,. \tag{39}$$

5 Ring Parameters

Parameter	Booster	AGS	RHIC	Unit
C_I	C_b	C_a	$C_r + \delta C$	m
C_E	$C_a/4$	$4(C_r + \delta C)/19$	$C_r + \delta C$	m
ρ	13.8656	85.378351	242.7806	m
$\gamma_{\rm tr}$	4.832	8.5	22.89	

Here C_I and C_E are the circumferences of the closed orbits in the machines at injection and extraction (or store) respectively. C_b, C_a, and C_r are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$C_b = 201.780, \quad C_a = 2\pi(128.4526), \quad C_r = 3833.845181 \tag{40}$$

meters. δC is the shift (if any) of the RHIC orbit circumference from the design value C_r. Note that $4(C_r/19) = 2\pi(128.4580)$ m which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius (128.4526 m) reported by Bleser [7, 8]. The radius of curvature ρ in the Booster and AGS main dipoles is given in Refs. [7, 8, 9]. The RHIC ring parameters are taken from Ref. [10] and from MAD runs by Steve Tepikian.

6 Initial Conditions and Assumptions

1. The revolution frequency of helions and Au32+ ions at Booster injection is 96.640 kHz. The radius is taken to be the nominal radius $C_b/(2\pi)$.

2. The revolution frequency at Booster extraction is 658.91 kHz [11] for Au32+ ions and 1255.0 kHz for helions. The radius is taken to be one fourth the nominal AGS radius $C_a/(2\pi)$. The corresponding magnetic rigidities are 9.4620277 Tm for Au32+ ions and 7.39098924731 Tm for helions. The rigidity that can be extracted from Booster into the BTA line is limited by the F3 extraction kicker. The advertised limit is $B\rho = 9.5$ Tm [12].

3. The set revolution frequency at AGS injection is 163.125 kHz for Au77+ ions and 313.750 kHz for helions.

4. This gives $G_h\gamma = -7.81659212041$ for helions. For polarized helions in future runs we will want to have $G_h\gamma = -7.5$ at AGS injection. This requires a revolution frequency of 308.270877814 kHz. The L10 cavity used for merging bunches on the AGS injection porch then needs to operate at frequency $2 \times 308.270877814 = 0.616541755628$ kHz, which is outside the present range of the cavity.

5. The magnetic rigidity of the Au79+ ion at RHIC injection is taken to be $B\rho = 86.00265$ Tm. The revolution frequency of the helion is required to be the same as that of the Au79+ ion.

6. At RHIC injection the circumference of the Au79+ (yellow) ring is $C_r - 3.5718$ mm. The circumference of the Helion (blue) ring is $C_r + 9.1324$ mm [13].

7. The energy of the Au79+ ion at Store is 100 GeV per nucleon. The revolution frequency of the helion is required to be the same as that of the Au79+ ion. The circumference of the Au79+ (yellow) ring is $C_r - 4.2206$ mm. The circumference of the Helion (blue) ring is $C_r + 6.2708$ mm [13].

The parameter values given in the following sections are calculated with these initial conditions and assumptions. For many of the parameters more digits are given than would be warranted by the precision with which the parameter could be measured; this is done for computational convenience.

7 Longitudinal Emittance of Unbunched Beam in Booster at Injection

The longitudinal emittance per nucleon of unbunched beam in Booster at injection is

$$\mathcal{E} = \frac{2}{A}\,\Delta E \Delta T \tag{41}$$

where ΔE is the energy half-width of the beam,

$$\Delta T = \frac{1}{f} = \frac{2\pi R}{c\beta} \tag{42}$$

is the revolution period, and A is the number of nucleons. Using the differential relation

$$\Delta E = \beta^2\,\frac{\Delta p}{p}\,mc^2\gamma \tag{43}$$

we have

$$\mathcal{E} = \frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A}\,\frac{\Delta p}{p} \tag{44}$$

where Δp is the momentum half-width of the unbunched beam. Taking

$$f = 96.640\ \text{kHz} \tag{45}$$

gives

$$\Delta T = 10.3476821192\ \mu\text{s} \tag{46}$$

$$\beta = 0.0650450626079, \quad \gamma = 1.00212216641 \tag{47}$$

and

$$\frac{2\beta^2\gamma}{f} = 87.7450074295\ \text{ns}. \tag{48}$$

<u>For Au32+ ions</u> we have

$$\frac{mc^2}{A} = 0.931253053269\ \text{GeV} \tag{49}$$

which gives

$$\frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A} = 81.7128060778\ \text{eV s} \tag{50}$$

Taking the fractional momentum half-width to be

$$\frac{\Delta p}{p} = 0.001 \tag{51}$$

then gives longitudinal emittance (per nucleon)

$$\mathcal{E} = 0.0817128060778 \text{ eV s}. \tag{52}$$

Measurements by Zeno [14] show that the longitudinal emittance of unbunched Au32+ beam in Booster at injection can be as small as 0.032 eV s per nucleon. This gives a fractional momentum half-width of

$$\frac{\Delta p}{p} = 0.0003916. \tag{53}$$

For helions we have

$$\frac{mc^2}{A} = 0.936130494000 \text{ GeV} \tag{54}$$

which gives

$$\frac{2\beta^2\gamma}{f}\frac{mc^2}{A} = 82.1407771510 \text{ eV s} \tag{55}$$

Taking the fractional momentum half-width to be

$$\frac{\Delta p}{p} = 0.001 \tag{56}$$

then gives longitudinal emittance (per nucleon)

$$\mathcal{E} = 0.082140777151 \text{ eV s}. \tag{57}$$

Measurements by Zeno [15] show that the longitudinal emittance (per nucleon) of unbunched helion beam in Booster at injection is at least twice that of Au32+ beam.

8 Minimum RF Voltage Required to Capture the Unbunched Beam

In order to capture the unbunched beam into h buckets we must have RF voltage V_g (i.e. total gap voltage per turn) such that

$$\mathcal{E} \leq \frac{hA_S}{A} \tag{58}$$

where A_S is given by (21). Thus we must have

$$\frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A}\,\frac{\Delta p}{p} \leq \frac{8R}{cA}\left\{\frac{2eQV_gE}{\pi h|\eta|}\right\}^{1/2} \tag{59}$$

which gives

$$2\beta^2\gamma\left(\frac{2\pi R}{c\beta}\right)\frac{mc^2}{A}\,\frac{\Delta p}{p} \leq \frac{8R}{c}\left(\frac{2\gamma}{\pi h|\eta|}\right)^{1/2}\frac{mc^2}{A}\left(\frac{eQV_g}{mc^2}\right)^{1/2} \tag{60}$$

$$\beta^2\gamma\left(\frac{\pi}{\beta}\right)\frac{\Delta p}{p} \leq 2\left(\frac{2\gamma}{\pi h|\eta|}\right)^{1/2}\left(\frac{eQV_g}{mc^2}\right)^{1/2} \tag{61}$$

$$\beta^2\gamma^2\pi^2\left(\frac{\Delta p}{p}\right)^2 \leq \left(\frac{8\gamma}{\pi h|\eta|}\right)\left(\frac{Q}{mc^2}\right)eV_g \tag{62}$$

and

$$\frac{1}{8}\,h\pi^3\beta^2\gamma\,|\eta|\left(\frac{mc^2}{Q}\right)\left(\frac{\Delta p}{p}\right)^2 \leq eV_g. \tag{63}$$

Here

$$h = 4 \tag{64}$$

and taking revolution frequency

$$f = 96.640 \text{ kHz} \tag{65}$$

we have

$$\eta = -0.952939329734 \tag{66}$$

and

$$\frac{1}{8}\,h\pi^3\beta^2\gamma\,|\eta| = 0.0626374709945. \tag{67}$$

For the Au32+ ion we have mass energy equivalent per unit charge

$$\frac{mc^2}{Q} = 5.73302660918 \text{ GeV}. \tag{68}$$

Taking fractional momentum half-width

$$\frac{\Delta p}{p} = 0.001 \tag{69}$$

then gives

$$359.102287944 \text{ volts} \leq V_g. \tag{70}$$

For helions we have mass energy equivalent per unit charge

$$\frac{mc^2}{Q} = 1.40419574100 \text{ GeV}. \tag{71}$$

Taking fractional momentum half-width

$$\frac{\Delta p}{p} = 0.001 \tag{72}$$

then gives

$$87.9552699975 \text{ volts} \leq V_g. \tag{73}$$

9 Bunch Merging

The desired number of ions per bunch in RHIC is achieved by merging bunches in both Booster and AGS. The process is described for gold ions in Sections 11 and 12 of Reference [16]. During the setup of helions for the FY2014 RHIC run it was discovered that the standard scheme for merging bunches on the AGS injection porch required an injection kicker pulse width narrower than what was available. To overcome this difficulty, K. Zeno [17] proposed and developed a clever and unusual alternative which uses RF harmonic numbers 12, 4, 2 (rather than the standard 8, 4, 2) to merge 8 helion bunches into 2. Careful measurements by Zeno have shown that the merges in both machines conserve the gross longitudinal emittance.

10 Inflector Voltage

At Booster injection, the voltage V_I required for particles with mass m, velocity $c\beta$, and charge eQ to follow the nominal trajectory through the inflector is given by

$$eV_I = \frac{G}{R_I}\left(\frac{mc^2}{Q}\right)\beta^2\gamma. \tag{74}$$

Here $G = 0.021$ m is the gap between the cathode and septum of the inflector and $R_I = 8.74123$ m is the radius of curvature along the nominal trajectory. Using the values of β, γ, and mc^2/Q given by (47) and (68), we obtain

$$V_I = 58.396 \text{ kV} \tag{75}$$

for Au32+ ions from EBIS. Because of an unresolved calibration problem, the actual setpoint for the inflector voltage needs to be

$$V_I(\text{setpoint}) = 59.740 \text{ kV}. \tag{76}$$

For helions we have

$$V_I = 14.303 \text{ kV} \tag{77}$$

and

$$V_I(\text{setpoint}) = 14.60 \text{ kV}. \tag{78}$$

11 Booster Injection Field

The nominal magnetic field in the Booster dipoles at injection is

$$B = (B\rho)/\rho \tag{79}$$

where $B\rho$ is given by (18) and ρ is the nominal radius of curvature. Writing

$$B\rho = \frac{10^9}{c}\left(\frac{mc^2}{Q}\right)\beta\gamma \tag{80}$$

and using the values of β, γ, and mc^2/Q given by (47) and (68), we obtain

$$B\rho = 1.24651715338 \text{ Tm}. \tag{81}$$

Here we have used the mass energy equivalent mc^2 in units of GeV and the velocity of light in units of m/s. Using

$$\rho = 13.8656 \text{ m} \tag{82}$$

we then obtain

$$B = 898.999793284 \text{ Gauss} \tag{83}$$

for Au32+ ions from EBIS.

The magnetic field is measured with a Hall probe and the Booster Gauss Clock. The Hall probe sits in the reference dipole and gives the value of the field at BT0. The Gauss Clock gives the change in field between BT0 and the time of measurement. The measured field is defined to be the field at BT0 plus the field change given by the Gauss Clock. For Au32+ ions from EBIS the measured field at injection is

$$B(\text{measured}) = 894.0 \text{ Gauss.} \tag{84}$$

Similarly, for helions we have

$$B\rho = 0.305310649535 \text{ Tm.} \tag{85}$$

$$B = 220.192887099 \text{ Gauss} \tag{86}$$

and

$$B(\text{measured}) = 215.6 \text{ Gauss.} \tag{87}$$

12 AGS Injection Field

Similarly, the nominal magnetic field in the AGS dipoles at injection is $B = 454.96$ Gauss for the Au77+ ions. The measured magnetic field is 482.0 gauss [18].

For helions we have $B = 865.674864967$ Gauss and measured field 888.8 gauss.

13 BTA Stripper

The stripper used to strip gold ions in the BTA (Booster-To-AGS) transfer line consists of a 6.45 mg/cm^2 aluminum foil followed by a 8.39 mg/cm^2 carbon foil. In Section 24 we use these surface densities to calculate the energy loss of Au77+ ions in the foils.

14 AGS Injection Septum Magnet Current

The field required in the L20 septum magnet is

$$B = (B\rho)/\rho \tag{88}$$

where $B\rho$ is the magnetic rigidity of the beam and $\rho = 18.625$ m [20] is the radius of curvature of the nominal trajectory through the magnet. The required current is given by

$$NI = gB/\mu_0 \tag{89}$$

where $N = 1$ is the number of conductor turns; $g = 0.0467$ m [20] is the magnet gap; and $\mu_0 = 4\pi \times 10^{-7}$ Tm/A.

For Au77+ ions at injection, the magnetic rigidity is $B\rho = 3.88434088$ Tm. This gives $B = 0.208555$ T and $I = 7750$ A.

For helions at injection, the magnetic rigidity is $B\rho = 7.39098924731$ Tm. This gives $B = 0.396832$ T and $I = 14747$ A.

For comparison, the magnetic rigidity of polarized protons at AGS injection is $B\rho = 7.205178$ Tm. This gives $B = 0.3869$ T and $I = 14380$ A.

15 AGS Injection Kicker Current

The current required in the A5 kicker is [19, 20]

$$I = \frac{B\rho}{K} \sin\phi \tag{90}$$

where

$$K = 1.8718 \times 10^{-5} \text{ Tm/A} \tag{91}$$

and

$$\phi = 3.35 \text{ milliradians} \tag{92}$$

is the desired kick angle. Using the calculated values of $B\rho$ at AGS injection we obtain a current of 695.2 A for Au77+ ions and 1323 A for helions. The maximum available current is 1100 A.

16 AGS Injection Kicker: Au77+ Injection

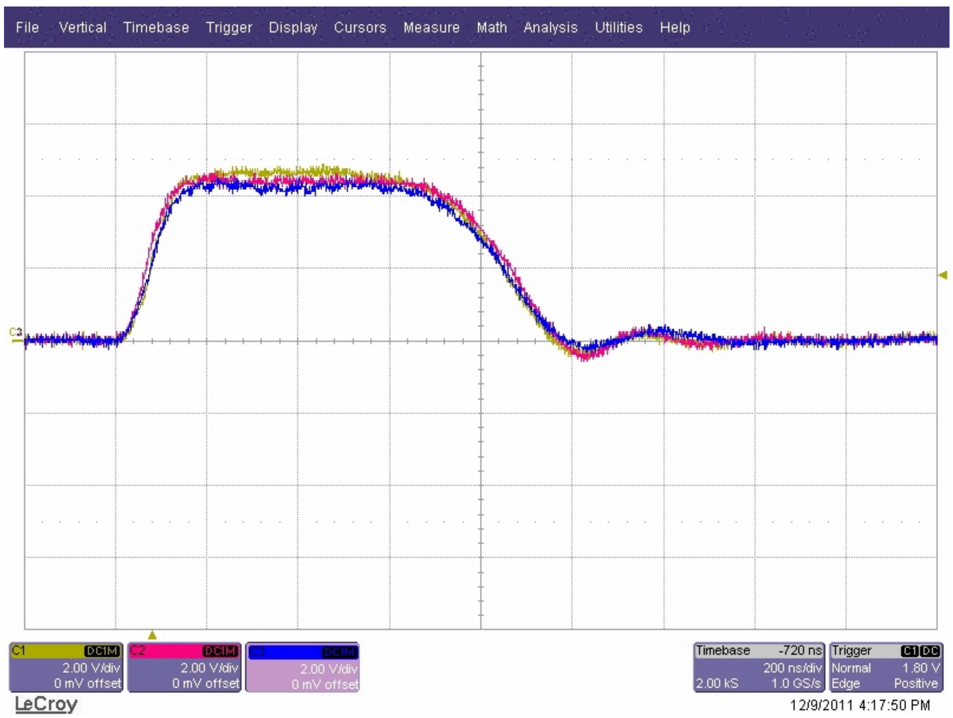

Figure 1: AGS injection kicker waveforms in the short pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan on 9 Dec 2011. The time per division is 200 ns. The RF bucket width on the AGS injection porch is 383 ns for Au77+ ions. In order to put beam into adjacent buckets, the rise time of the kicker must be less than or equal to $T - W$, where T is the bucket width and W the bunch width. The rise time is approximately 100 ns, which implies that the bunch width must be less than or equal to 283 for Au77+ bunches. A single bunch of this width easily fits on the flattop portion of the pulse which is some 600 ns long. The total width of the pulse is approximately 1000 ns. With this kicker pulse one could in principle fill 14 of the 16 RF buckets on the AGS injection porch. The pulse is too wide to fill the remaining buckets without interfering with beam in the adjacent buckets. This is not an issue as only 8 of the buckets need to be filled. One workable filling pattern is four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. Another workable pattern is eight adjacent filled buckets followed eight empty buckets.

17 AGS Injection Kicker: Helion Injection

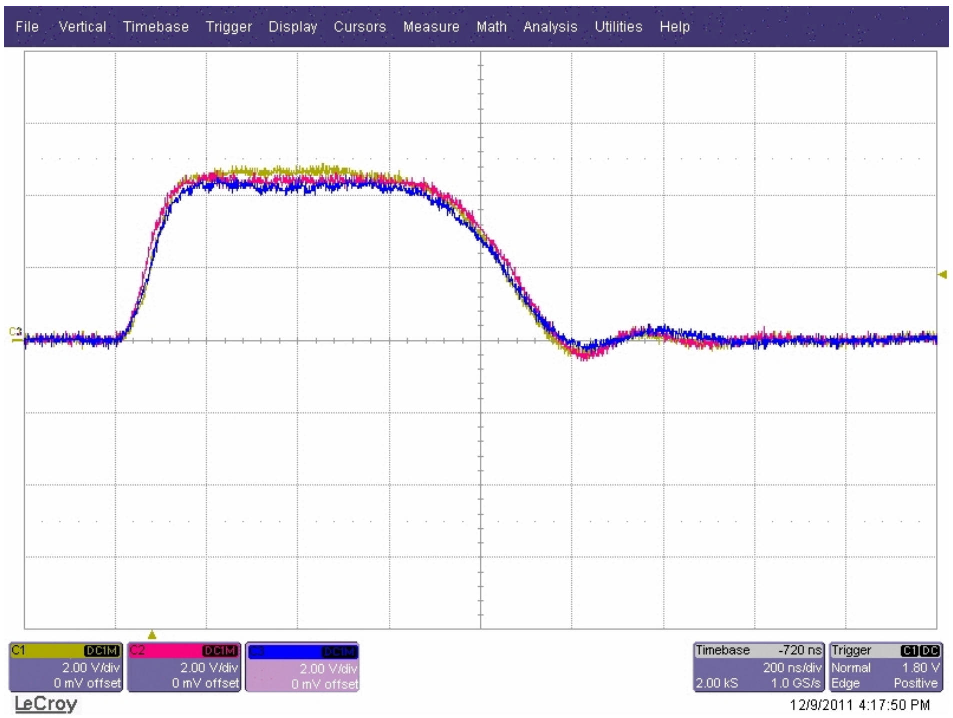

Figure 2: AGS injection kicker waveforms in the short pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan on 9 Dec 2011. The time per division is 200 ns. For helions the RF bucket width on the AGS injection porch is 266 ns. In order to put beam into adjacent buckets, the rise time of the kicker must be less than or equal to $T - W$, where T is the bucket width and W the bunch width. The rise time is approximately 100 ns, which implies that the bunch width must be less than or equal to 166 ns. A single bunch of this width easily fits on the flattop portion of the pulse which is some 600 ns long. The total width of the pulse is approximately 1000 ns. With this kicker pulse one could in principle fill 10 of the 12 RF buckets on the AGS injection porch. The pulse is too wide to fill the remaining buckets without interfering with beam in the adjacent buckets. This is not an issue as only 8 of the buckets need to be filled. The workable filling pattern in this case was four adjacent filled buckets followed by two adjacent empty buckets, followed by another four adjacent filled buckets. This allowed for a gap of width $4T - W$ for the kicker pulse, where T is the bucket width and W the bunch width.

18 Gold in Booster

Parameter	Injection	Merge porch	Extraction	Unit
Q	32	32	32	
mc^2	183.456851	183.456851	183.456851	GeV
W/A	1.9762739452	49.259795	107.75879	MeV
cp/A	60.701960016	306.87652	460.77475	MeV
E/A	0.9332293272	0.98051285	1.0390118	GeV
$B\rho$	1.24651715338	6.3017214	9.4620277	Tm
β	0.065045062608	0.31297552	0.44347401	
$\gamma - 1$	0.002122166406	0.052896251	0.11571376	
η	-0.953	-0.859	-0.7605	
ϵ_H (95%)	12.1π	12.1π	12.1π	mm mrad
ϵ_V (95%)	5.68π	5.68π	5.68π	mm mrad
h	4	1	1	
hf	386.560	465.000	658.910	kHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parm	Injection	Injection	Ext	Ext	Ext	Unit
V_g	0.403	5.730	25.2	25.2	25.2	kV
A_S	4.263	16.076	318.54	318.54	318.54	eV s
dB/dt	0	0	**70**	**35**	**0**	G/ms
ϕ_s	0	0	50.999	22.866	0	deg
F_s	0.3065	1.1557	0.8139	0.9849	1.0260	kHz
A_{bk}	4.263	16.076	36.294	140.88	318.54	eV s
A_b	1.576	3.8907	17.533	17.533	17.533	eV s
Δt	1196	947.0	299.3	263.8	257.7	ns
ΔE	0.8596	2.654	37.98	42.50	43.45	MeV

Parameter	Injection	Injection	Extraction	Unit
No. Bunches	4	4	1	
Bucket Width	2586.92053	2586.92053	1517.65795	ns
Ions/Bunch	1.12/4	1.12/4	1.007	10^9 [21]
Bunch Area	0.032/4 [14]	0.0790/4 [22]	0.089 [23]	eV s/A

19 Gold in AGS

Parameter	Injection	Transition	Extraction	Unit
Q	77	77	77	
mc^2	183.434174	183.434174	183.434174	GeV
W/A	0.10529199	6.98353456	9.45010636221	GeV
cp/A	0.45515837	7.85970883	10.3394011153	GeV
E/A	1.0364299	7.91467250	10.3812443035	GeV
$B\rho$	3.88434088	67.0750887	88.2368878770	Tm
β	0.43915981	0.993055472	0.995969347511	
γ	1.1130788	8.5000	11.1489864635	
η	-0.793	0.0	0.005796	
ϵ_H (95%)	$\leq 12\pi$	$\leq 12\pi$	$\leq 12\pi$	mm mrad
ϵ_V (95%)	$\leq 12\pi$	$\leq 12\pi$	$\leq 12\pi$	mm mrad
h	16	12	12	
hf	2.610000	4.42642072	4.43922711616	MHz
R	128.4526	128.4526	128.457861713	m

Parameter	Injection	Injection	Extraction	Unit
h	16	4	12	
V_g	22.048	7.324	192.0	kV
A_S	28.248	130.25	4752	eV s
dB/dt	0	0	0	G/ms
ϕ_s	0	0	180	degrees
F_s	1.522	0.4387	0.1051	kHz
A_{bk}	28.248	130.25	4752	eV s
A_b	22.261	89.044	137.9	eV s
Δt	287.0	1034 [24]	27.7	ns
ΔE	53.47	58.22	3177	MeV

Parameter	Injection	Injection	Extraction	Unit
h	16	4	12	
Bucket Width	383.142	1532.567	225.264	ns
No. of Bunches	8	2	2	
Ions/Bunch	0.55725	2.229	2.055	10^9 [21]
Bunch Area	0.113 [25]	0.452	0.70 [26]	eV s/A

20 Gold in RHIC

Parameter	Injection	Transition	Store	Unit
Q	79	79	79	
mc^2	183.433337	183.433337	183.433337	GeV
W/A	9.45006322133	20.3825165	99.0688663	GeV
cp/A	10.3393539147	21.2933012	99.9956649	GeV
E/A	10.3811969119	21.3136502	**100.000000**	GeV
$B\rho$	86.00265	177.117482	831.763013	Tm
β	0.995969347511	0.999045259	0.999956649	
γ	11.1489864634	22.8900	107.395964	
η	-0.00614	0.0	0.00182	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	360	360	2520	
hf	28.0372238915	28.1238129076	197.046317505	MHz
δC	-3.5718	-3.5718	-4.2206	mm

Parameter	Injection	Store	Unit
h	360	2520	
V_g	294.1	3000	kV
A_S	167.4	164.4	eV s
dB/dt	0	0	G/ms
ϕ_s	0	180	degrees
F_s	0.156	0.232	kHz
A_{bk}	167.4	164.4	eV s
A_b	137.9	137.9	eV s
A_b	0.70	0.70	eV s/A [27]
Δt	27.7	4.00	ns
ΔE	3461	24052	MeV

21 Helions in Booster

Parameter	Injection	Extraction	Extraction	Unit
$G_h\gamma$	-4.19303315384	-7.5	-7.81659212041	
W/A	1.98662468605	741.862055071	812.693950956	MeV
cp/A	61.0198867185	1392.59423126	1477.17522233	MeV
E/A	0.938117118686	1.67799254907	1.74882444496	GeV
$B\rho$	0.305310649535	6.96779152093	7.39098924731	Tm
β	0.0650450626079	0.829916814608	0.844667528862	
γ	1.00212216641	1.79247718115	1.86814173469	
η	-0.9529	-0.2684	-0.2437	
h	4	1	1	
hf	386.560	1233.08351126	1255.0	kHz
R	$201.780/(2\pi)$	128.4526/4	128.4526/4	m

Parm	Injection	Injection	Ext	Ext	Ext	Unit
V_g	0.244	2.7	25.2	25.2	25.2	kV
A_S	0.1026	0.3413	22.52	22.52	22.52	eV s
dB/dt	0	0	**70**	**35**	**0**	G/ms
ϕ_s	0	0	50.999	22.866	0	deg
F_s	0.4819	1.6030	0.7195	0.8706	0.9070	kHz
A_{bk}	0.1026	0.3413	2.566	9.961	22.52	eV s
A_b	0.09820	0.1342	0.6	0.6	0.6	eV s
Δt	2338	1239	106.7	95.82	93.76	ns
ΔE	0.03080	0.0708	3.605	3.994	4.080	MeV

Parameter	Injection	Injection	Extraction	Unit
No. Bunches	4	4	1	
Bucket Width	2586.92053	2586.92053	796.81275	ns
Ions/Bunch	3.148/4	3.184/4	2.020	10^{10} [28]
Bunch Area	0.1309/4 [15]	0.1789/4 [29]	0.2 [30]	eV s/A

The Booster merging porch is at $G_h\gamma = -4.5$:

Parameter	$G_h\gamma = -4.5$	$G_h\gamma = -5.5$	Unit
$G_h\gamma$	-4.5	-5.5	
W/A	70.6650354427	294.397375319	MeV
cp/A	0.370536012162	0.798660463134	GeV
E/A	1.00679552944	1.23052786932	GeV
$B\rho$	1.85396264453	3.99606682134	Tm
β	0.368035019352	0.649038906836	
γ	1.07548630869	1.31448326617	
η	-0.8217	-0.5359	
h	4	4	
hf	2.18721623712	3.85721021346	MHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	m

Parameters for integer values of $G_h\gamma$:

Parameter	$G_h\gamma = -5.0$	$G_h\gamma = -6.0$	Unit
$G_h\gamma$	-5.0	-6.0	
W/A	182.531205381	406.263545257	MeV
cp/A	0.612424440943	0.962123409359	GeV
E/A	1.11866169938	1.34239403926	GeV
$B\rho$	3.06424206781	4.81394736767	Tm
β	0.547461704716	0.716722051218	
γ	1.19498478743	1.43398174492	
η	-0.6575	-0.4435	
h	4	1	
hf	3.25354128492	1.06486205490	MHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	m

22 Helions in AGS

Parameter	Injection	Injection	Transition	Unit
$G_h\gamma$	-7.5	-7.81659212041	-35.5653063093	
W/A	0.741862055071	0.812693950956	7.02097870500	GeV
cp/A	1.39259423126	1.47717522233	7.90185082769	GeV
E/A	1.67799254907	1.74882444496	7.95710919900	GeV
$B\rho$	6.96779152093	7.39098924731	39.5366058259	Tm
β	0.829916814608	0.844667528862	0.993055471537	
γ	1.79247718115	1.86814173469	8.5000	
η	-0.2974	-0.2727	0.0	
h	12	12	6	
hf	3.69925053377	3.765	2.21321035945	MHz
$4f$	1.23308351126	1.255		MHz
$2f$	0.616541755628	0.6275		MHz
f	0.308270877814	0.31375		MHz
R	128.4526	128.4526	128.4526	m

Helions and Au77+ ions at AGS extraction:

Parameter	Extraction	Extraction	Unit
$G_h\gamma$	-46.6681443155	Au77+	
W/A	9.50504263138	9.45010636221	GeV
cp/A	10.3991228444	10.3394011153	GeV
E/A	10.4411731254	10.3812443035	GeV
$B\rho$	52.0316100366	88.2368878770	Tm
β	0.995972647853	0.995969347511	
γ	11.1535445029	11.1489864635	
η	0.005802	0.005796	
h	6	12	
hf	4.43922711615/2	4.43922711616	MHz
R	128.458287384	128.457861713	m

RF parameters and ion intensities for helions in AGS:

Parameter	Injection	Injection	Extraction	Unit
h	12	2	10	
V_g	47.0	22.0	188.0	kV
A_S	2.798	28.13	115.5	eV s
dB/dt	0	0	0	G/ms
ϕ_s	0	0	180	degrees
F_s	1.135	0.3169	0.1319	kHz
A_{bk}	2.798	28.13	115.5	eV s
A_b	0.6047	2.42	3.677	eV s
Δt	91.5	339.7	34.7	ns
ΔE	4.262	4.557	67.57	MeV

Parameter	Injection	Injection	Extraction	Unit
h	12	2	10	
Bucket Width	265.604	1593.6255	269.627	ns
No. of Bunches	8	2	2	
Ions/Bunch	1.895	7.58	7.282	10^{10} [28]
Bunch Area	0.2016 [30]	0.8067	1.23 [31]	eV s/A

For polarized helions in future runs we will want to have $G_h\gamma = -48.5$ or -49.5 at AGS extraction:

Parameter	Extraction	Extraction	Unit
$G_h\gamma$	-48.5	-49.5	
W/A	9.91488798999	10.1386203299	GeV
cp/A	10.8105625126	11.0351151108	GeV
E/A	10.8510184840	11.0747508239	GeV
$B\rho$	54.0902325463	55.2137728102	Tm
β	0.996271689014	0.996421074056	
γ	11.5913524381	11.8303493956	
η	0.006398	0.006696	
h	6	6	
hf	4.44057057338/2	4.44123641065/2	MHz
R	128.457981391	128.457981391	m

23 Helions in RHIC

Helions and Au79+ ions at injection:

Parameter	Injection	Injection	Unit
$G_h\gamma$	-46.6681443155	Au79+	
W/A	9.50504263138	9.45006322133	GeV
cp/A	10.3991228444	10.3393539147	GeV
E/A	10.4411731254	10.3811969119	GeV
$B\rho$	52.0316100366	86.00265	Tm
β	0.995972647853	0.995969347511	
γ	11.1535445029	11.1489864634	
η	-0.006130	-0.006136	
h	360	360	
hf	28.0372238915	28.0372238915	MHz
δC	9.1324	-3.5718	mm

For polarized helions in future runs we will want to have $G_h\gamma = -48.5$ or -49.5 at RHIC injection:

Parameter	Injection	Injection	Unit
$G_h\gamma$	-48.5	-49.5	
W/A	9.91488798999	10.1386203299	GeV
cp/A	10.8105625126	11.0351151108	GeV
E/A	10.8510184840	11.0747508239	GeV
$B\rho$	54.0902325463	55.2137728102	Tm
β	0.996271689014	0.996421074056	
γ	11.5913524381	11.8303493956	
η	-0.005534	-0.005236	
h	360	360	
hf	28.0457088845	28.0499141725	MHz
δC	0	0	mm

Helions and Au79+ ions at store:

Parameter	Store	Store	Unit
$G_h\gamma$	-464.251851687	Au79+	
W/A	102.932022576	99.0688663094	GeV
cp/A	103.863934461	99.9956648563	GeV
E/A	103.868153070	100	GeV
$B\rho$	519.679189835	831.763013151	Tm
β	0.999959384969	0.999956648563	
γ	110.954780060	107.395963664	
η	0.001827	0.001822	
h	2520	2520	
hf	197.046317505	197.046317505	MHz
δC	6.2708	-4.2206	mm

Helion RF parameters at injection and store:

Parameter	Injection	Store	Unit
h	360	2520	
V_g	369.1	3000	kV
A_S	3.694	3.285	eV s
dB/dt	0	0	G/ms
ϕ_s	0	180	degrees
F_s	0.225	0.294	kHz
$A_{\rm bk}$	3.694	3.285	eV s
A_b	3.677	3.0	eV s
A_b	1.23	1.0	eV s/A [27]
Δt	34.7	4.337	ns
ΔE	81.27	495.2	MeV

24 Au77+ Energy Loss in the BTA Stripper Foils

The stripper used to strip gold ions consists of a 6.45 mg/cm^2 aluminum foil followed by a 8.39 mg/cm^2 "glassy" carbon foil [32, 33]. We can estimate the energy loss in the foils as follows:

The kinetic energy of a proton that has the same velocity as the Au77+ ion just upstream of the aluminum foil is

$$W_p = 108.6 \text{ MeV}. \tag{93}$$

The rate of energy loss of a proton passing through the foil with kinetic energy W_p is [34]

$$-\frac{dE_p}{dx} = 5.348 \text{ MeV cm}^2/\text{g}. \tag{94}$$

The rate of energy loss of the Au77+ ion is obtained by scaling the Bethe-Bloch result for protons [35]. Thus

$$-\frac{dE}{dx} = -Z^2 \frac{dE_p}{dx} \text{ cm}^2/\text{g} \tag{95}$$

where $Z = 77$. Multiplying this by the surface density of the aluminum foil (6.45 mg/cm^2) gives

$$\Delta E_a = 1.038 \text{ MeV per nucleon}. \tag{96}$$

This is the energy lost by the Au77+ ion upon passing through the aluminium foil. The kinetic energy of a proton that has the same velocity as the Au77+ ion just downstream of the aluminum foil is then

$$W_p = 107.5 \text{ MeV}. \tag{97}$$

The rate of energy loss of a proton passing through the carbon foil with this kinetic energy is [34]

$$-\frac{dE_p}{dx} = 6.180 \text{ MeV cm}^2/\text{g}. \tag{98}$$

Using this result in (95) with $Z = 77$, and multiplying by the surface density of the carbon foil (8.39 mg/cm^2) gives

$$\Delta E_c = 1.561 \text{ MeV per nucleon}. \tag{99}$$

The total energy lost upon passing through both foils is then

$$\Delta E = \Delta E_a + \Delta E_c = 2.599 \text{ MeV per nucleon}. \tag{100}$$

This agrees reasonably well with the value 2.453 MeV per nucleon obtained in Section 6.

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