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In this note the nominal parameters for gold ions in Booster, AGS, and RHIC are given for the FY2014 running period.

The “standard setup” parameters are summarized in Sections 17, 18, 19.

“Medium-Energy” parameters are summarized in Sections 20 and 21.

“Low-Energy” parameters are summarized in Sections 22, 23, 24, 25.

1 Mass

A gold ion with charge eQ has $N = 118$ neutrons, $Z = 79$ protons, and $(Z - Q)$ electrons. Here Q is an integer and e is the positive elementary charge. The mass number is

$$A = N + Z = 197. \tag{1}$$

This is also called the number of nucleons. The mass energy equivalent of the ion is

$$mc^2 = am_uc^2 - Qm_ec^2 + E_Q \tag{2}$$

where [1, 2]

$$a = 196.9665687(6) \tag{3}$$

is the relative atomic mass of the neutral gold atom,

$$m_uc^2 = 931.494061(21)\ \text{MeV} \tag{4}$$

is the mass energy equivalent of the atomic mass constant, and

$$m_ec^2 = 0.510998928(11)\ \text{MeV} \tag{5}$$

is the electron mass energy equivalent. The binding energy E_Q is the energy required to remove Q electrons from the neutral gold atom. This amounts to [3, 4] 0.3324 MeV for the helium-like gold ion ($Q = 77$) and 0.5170 MeV for the fully stripped ion. For $Q = 32$ we have $E_Q = 14.5$ KeV.

Thus the mass energy equivalents for the Au32+, Au77+, and Au79+ ions are

$$mc^2(\text{Au32+}) = 183.456851494 \text{ GeV} \tag{6}$$

$$mc^2(\text{Au77+}) = 183.434174442 \text{ GeV} \tag{7}$$

and

$$mc^2(\text{Au79+}) = 183.433337044 \text{ GeV}. \tag{8}$$

2 Kinetic Parameters

In a circular accelerator the ion moves along an orbit of circumference C with revolution frequency f. The radius of the orbit is defined to be $R = C/(2\pi)$. The velocity of the ion is then

$$v = 2\pi R f. \tag{9}$$

This gives momentum, energy, and kinetic energy

$$p = mc\beta\gamma, \quad E = mc^2\gamma, \quad W = mc^2(\gamma - 1) \tag{10}$$

where

$$\beta = v/c, \quad \gamma = 1/\sqrt{1-\beta^2}. \tag{11}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = kcp/Q \tag{12}$$

where $k = 10^9/299792458$ and cp is given in units of GeV. The angular frequency is

$$\omega = 2\pi f. \tag{13}$$

We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{14}$$

where γ_t is the transition gamma. Note that as defined here, η is negative below transition and positive above transition.

3 RF Parameters

1. The stationary bucket area is

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2} \tag{15}$$

   where h is the RF harmonic number, V_g is the total RF gap voltage per turn, and the subscript "s" denotes parameter values for the synchronous particle.

2. The half-height of a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S\,|(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s|^{1/2} \tag{16}$$

   where ϕ_s is the synchronous phase.

3. The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{17}$$

   where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

4. The width of a bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{18}$$

   where the phase ϕ_e satisfies

$$\cos(\pi - \phi_s) - \cos\phi_e = -(\pi - \phi_s - \phi_e)\sin\phi_s. \tag{19}$$

5. The area of a bucket is

$$A_{\rm bk} = \alpha(\phi_s) A_S \tag{20}$$

   where

$$\alpha(\phi_s) = \frac{\sqrt{2}}{8}\int_{\phi_L}^{\phi_R} |(\pi - \phi_s - \phi)\sin\phi_s - \cos\phi_s - \cos\phi|^{1/2}\, d\phi. \tag{21}$$

Below transition we have $\phi_e < \pi - \phi_s$ and the limits of integration are $\phi_L = \phi_e$ and $\phi_R = \pi - \phi_s$. Above transition we have $\pi - \phi_s < \phi_e$ and the limits of integration are $\phi_L = \pi - \phi_s$ and $\phi_R = \phi_e$. The integral $\alpha(\phi_s)$ must be evaluated numerically. An approximate expression is [5]

$$\alpha(\phi_s) \approx \frac{1 - \sin\phi_s}{1 + \sin\phi_s}. \tag{22}$$

6. The synchrotron frequency for small-amplitude oscillations about ϕ_s is

$$F_s = \frac{c}{2\pi R_s}\left\{\frac{-h\eta_s eQV_g\cos\phi_s}{2\pi E_s}\right\}^{1/2} \tag{23}$$

and the corresponding synchrotron tune is $Q_s = 2\pi F_s/\omega_s$. Note that measurement of F_s gives a value for $V_g\cos\phi_s$, while measurement of dB/dt gives a value for $V_g\sin\phi_s$. These two can be used to obtain V_g and ϕ_s.

7. Let ϕ_l and ϕ_r be the phases at the left and right boundries of a bunch matched to a bucket. We have

$$\phi_l < \phi_s < \phi_r \tag{24}$$

and the width of the bunch is

$$\Delta t = \frac{\Delta\phi}{h\omega_s}, \quad \Delta\phi = \phi_r - \phi_l. \tag{25}$$

In terms of $\Delta\phi$ and ϕ_s we have

$$\phi_r = \frac{\Delta\phi}{2} + \arcsin\left\{\frac{\Delta\phi\sin\phi_s}{2\sin(\Delta\phi/2)}\right\} \tag{26}$$

and

$$\phi_l = -\frac{\Delta\phi}{2} + \arcsin\left\{\frac{\Delta\phi\sin\phi_s}{2\sin(\Delta\phi/2)}\right\}. \tag{27}$$

If $\Delta\phi$ is small we have

$$\sin(\Delta\phi/2) \approx \frac{\Delta\phi}{2}, \quad \frac{\Delta\phi\sin\phi_s}{2\sin(\Delta\phi/2)} \approx \sin\phi_s \tag{28}$$

and

$$\phi_l \approx \phi_s - \frac{\Delta\phi}{2}, \quad \phi_r \approx \phi_s + \frac{\Delta\phi}{2}. \tag{29}$$

8. The half-height of a bunch matched to a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \left|\cos\phi_r - \cos\phi_s + (\phi_r - \phi_s)\sin\phi_s\right|^{1/2}. \tag{30}$$

9. The area of a bunch matched to a bucket is

$$A_{\rm b} = F(\phi_s, \Delta\phi)\, A_S \tag{31}$$

where

$$F(\phi_s, \Delta\phi) = \frac{\sqrt{2}}{8}\int_{\phi_l}^{\phi_r} \left|\cos\phi_l - \cos\phi + (\phi_l - \phi)\sin\phi_s\right|^{1/2} d\phi. \tag{32}$$

The integral $F(\phi_s, \Delta\phi)$ must be evaluated numerically. If $\Delta\phi$ is small we have

$$F(\phi_s, \Delta\phi) \approx \frac{\pi}{64}(\Delta\phi)^2 \left|\cos\phi_s\right|^{1/2}. \tag{33}$$

4 Ring Parameters

Parameter	Booster	AGS	RHIC	Unit
C_I	C_b	C_a	$C_r + \delta C$	m
C_E	$C_a/4$	$4(C_r + \delta C)/19$	$C_r + \delta C$	m
ρ	13.8656	85.378351	242.7806	m
$\gamma_{\rm tr}$	4.832	8.5	22.89	

Here C_I and C_E are the circumferences of the closed orbits in the machines at injection and extraction (or store) respectively. C_b, C_a, and C_r are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$C_b = 201.780, \quad C_a = 2\pi(128.4526), \quad C_r = 3833.845181 \tag{34}$$

meters. δC is the shift (if any) of the RHIC orbit circumference from the design value C_r. Note that $4(C_r/19) = 2\pi(128.4580)$ m which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius (128.4526 m) reported by Bleser [6, 7]. The radius of curvature ρ in the Booster and AGS main dipoles is given in Refs. [6, 7, 8]. The RHIC ring parameters are taken from Ref. [9] and from MAD runs by Steve Tepikian.

5 Bunch Merging

The desired number of ions per bunch in RHIC is achieved by merging bunches in both Booster and AGS as described in Sections 11 and 12 of Reference [10]. Careful measurements by Zeno have shown that the merges conserve the gross longitudinal emittance.

6 Initial Conditions and Assumptions

1. The revolution frequency of the Au32+ ion (from EBIS) at Booster injection is 96.640 kHz. The radius is taken to be the nominal radius $C_b/(2\pi)$.

2. The revolution frequency of the Au32+ ion at Booster extraction is $f = 658.91$ KHz [11]. The radius is taken to be one fourth the nominal AGS radius $C_a/(2\pi)$. The corresponding magnetic rigidity is $B\rho = 9.4620277$ Tm. The rigidity that can be extracted from Booster into the BTA line is limited by the F3 extraction kicker. The advertised limit is $B\rho = 9.5$ Tm [12].

3. The set revolution frequency of the Au77+ ion at AGS injection is $f = 163.125$ KHz. This gives an energy loss of 2.453 MeV per nucleon in the BTA stripper.

4. The magnetic rigidity of the Au79+ ion at RHIC injection is taken to be $B\rho = 81.1137824$ Tm.

5. The circumference at RHIC injection is C_r.

6. The circumference at Store is taken to be C_r.

7. The energy of the Au79+ ion at RHIC Store is 100 GeV per nucleon.

The parameter values given in the following sections are calculated with these initial conditions and assumptions. For many of the parameters more digits are given than would be warranted by the precision with which the parameter could be measured; this is done for computational convenience.

7 Longitudinal Emittance of Unbunched Beam in Booster at Injection

The longitudinal emittance per nucleon of unbunched beam in Booster at injection is

$$\mathcal{E} = \frac{2}{A}\,\Delta E \Delta T \tag{35}$$

where ΔE is the energy half-width of the beam,

$$\Delta T = \frac{1}{f} = \frac{2\pi R}{c\beta} \tag{36}$$

is the revolution period, and A is the number of nucleons. Using the differential relation

$$\Delta E = \beta^2\,\frac{\Delta p}{p}\,mc^2\gamma \tag{37}$$

we have

$$\mathcal{E} = \frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A}\,\frac{\Delta p}{p} \tag{38}$$

where Δp is the momentum half-width of the unbunched beam. Taking

$$f = 96.640 \text{ kHz} \tag{39}$$

gives

$$\Delta T = 10.3476821192\ \mu\text{s} \tag{40}$$

$$\beta = 0.0650450626079, \quad \gamma = 1.00212216641 \tag{41}$$

and

$$\frac{2\beta^2\gamma}{f} = 87.7450074295 \text{ ns.} \tag{42}$$

For Au32+ ions we have

$$\frac{mc^2}{A} = 0.931253053269 \text{ GeV} \tag{43}$$

which gives

$$\frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A} = 81.7128060778 \text{ eV s} \tag{44}$$

Taking the fractional momentum half-width to be

$$\frac{\Delta p}{p} = 0.001 \tag{45}$$

then gives longitudinal emittance (per nucleon)

$$\mathcal{E} = 0.0817128060778 \text{ eV s}. \tag{46}$$

Measurements by Zeno [13] show that the longitudinal emittance of unbunched Au32+ beam in Booster at injection can be as small as 0.032 eV s per nucleon. This gives a fractional momentum half-width of

$$\frac{\Delta p}{p} = 0.0003916. \tag{47}$$

8 Minimum RF Voltage Required to Capture the Unbunched Beam

In order to capture the unbunched beam into h buckets we must have RF voltage V_g (i.e. total gap voltage per turn) such that

$$\mathcal{E} \leq \frac{hA_S}{A} \tag{48}$$

where A_S is given by (15). Thus we must have

$$\frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A}\,\frac{\Delta p}{p} \leq \frac{8R}{cA}\left\{\frac{2eQV_gE}{\pi h|\eta|}\right\}^{1/2} \tag{49}$$

which gives

$$2\beta^2\gamma\left(\frac{2\pi R}{c\beta}\right)\frac{mc^2}{A}\,\frac{\Delta p}{p} \leq \frac{8R}{c}\left(\frac{2\gamma}{\pi h|\eta|}\right)^{1/2}\frac{mc^2}{A}\left(\frac{eQV_g}{mc^2}\right)^{1/2} \tag{50}$$

$$\beta^2\gamma\left(\frac{\pi}{\beta}\right)\frac{\Delta p}{p} \leq 2\left(\frac{2\gamma}{\pi h|\eta|}\right)^{1/2}\left(\frac{eQV_g}{mc^2}\right)^{1/2} \tag{51}$$

$$\beta^2\gamma^2\pi^2\left(\frac{\Delta p}{p}\right)^2 \leq \left(\frac{8\gamma}{\pi h|\eta|}\right)\left(\frac{Q}{mc^2}\right)eV_g \tag{52}$$

and

$$\frac{1}{8}\,h\pi^3\beta^2\gamma\,|\eta|\left(\frac{mc^2}{Q}\right)\left(\frac{\Delta p}{p}\right)^2 \leq eV_g. \tag{53}$$

Here

$$h = 4 \tag{54}$$

and taking revolution frequency

$$f = 96.640 \text{ kHz} \tag{55}$$

we have

$$\eta = -0.952939329734 \tag{56}$$

and

$$\frac{1}{8} h\pi^3 \beta^2 \gamma \, |\eta| = 0.0626374709945. \tag{57}$$

For the Au32+ ion we have mass energy equivalent per unit charge

$$\frac{mc^2}{Q} = 5.73302660918 \text{ GeV}. \tag{58}$$

Taking fractional momentum half-width

$$\frac{\Delta p}{p} = 0.001 \tag{59}$$

then gives

$$359.102287944 \text{ volts} \leq V_g. \tag{60}$$

9 Inflector Voltage

At Booster injection, the voltage V_I required for particles with mass m, velocity $c\beta$, and charge eQ to follow the nominal trajectory through the inflector is given by

$$eV_I = \frac{G}{R_I} \left(\frac{mc^2}{Q} \right) \beta^2 \gamma. \tag{61}$$

Here $G = 0.021$ m is the gap between the cathode and septum of the inflector and $R_I = 8.74123$ m is the radius of curvature along the nominal trajectory. Using the values of β, γ, and mc^2/Q given by (41) and (58), we obtain

$$V_I = 58.396 \text{ kV} \tag{62}$$

for Au32+ ions from EBIS. Because of an unresolved calibration problem, the actual setpoint for the inflector voltage needs to be

$$V_I(\text{setpoint}) = 59.740 \text{ kV}. \tag{63}$$

10 Booster Injection Field

The nominal magnetic field in the Booster dipoles at injection is

$$B = (B\rho)/\rho \tag{64}$$

where $B\rho$ is given by (12) and ρ is the nominal radius of curvature. Writing

$$B\rho = \frac{10^9}{c}\left(\frac{mc^2}{Q}\right)\beta\gamma \tag{65}$$

and using the values of β, γ, and mc^2/Q given by (41) and (58), we obtain

$$B\rho = 1.24651715338 \text{ Tm}. \tag{66}$$

Here we have used the mass energy equivalent mc^2 in units of GeV and the velocity of light in units of m/s. Using

$$\rho = 13.8656 \text{ m} \tag{67}$$

we then obtain

$$B = 898.999793284 \text{ Gauss} \tag{68}$$

for Au32+ ions from EBIS.

The magnetic field is measured with a Hall probe and the Booster Gauss Clock. The Hall probe sits in the reference dipole and gives the value of the field at BT0. The Gauss Clock gives the change in field between BT0 and the time of measurement. The measured field is defined to be the field at BT0 plus the field change given by the Gauss Clock. For Au32+ ions from EBIS the measured field at injection is

$$B(\text{measured}) = 894.0 \text{ Gauss}. \tag{69}$$

11 AGS Injection Field

Similarly, the nominal magnetic field in the AGS dipoles at injection is $B = 454.96$ Gauss for the Au77+ ions. The measured magnetic field is 482.0 gauss [14].

12 BTA Stripper

The stripper used to strip gold ions in the BTA (Booster-To-AGS) transfer line consists of a 6.45 mg/cm^2 aluminum foil followed by a 8.39 mg/cm^2 carbon foil. In Section 26 we use these surface densities to calculate the energy loss of Au77+ ions in the foils.

13 AGS Injection Septum Magnet Current

The field required in the L20 septum magnet is

$$B = (B\rho)/\rho \tag{70}$$

where $B\rho$ is the magnetic rigidity of the beam and $\rho = 18.625$ m [16] is the radius of curvature of the nominal trajectory through the magnet. The required current is given by

$$NI = gB/\mu_0 \tag{71}$$

where $N = 1$ is the number of conductor turns; $g = 0.0467$ m [16] is the magnet gap; and $\mu_0 = 4\pi \times 10^{-7}$ Tm/A.

For Au77+ ions at injection, the magnetic rigidity is $B\rho = 3.88434088$ Tm. This gives $B = 0.208555$ T and $I = 7750$ A.

For comparison, the magnetic rigidity of polarized protons at AGS injection is $B\rho = 7.205178$ Tm. This gives $B = 0.3869$ T and $I = 14380$ A.

14 AGS Injection Kicker Current

The current required in the A5 kicker is [15, 16]

$$I = \frac{B\rho}{K}\sin\phi \tag{72}$$

where

$$K = 1.8718 \times 10^{-5}\ \mathrm{Tm/A} \tag{73}$$

and

$$\phi = 3.35\ \text{milliradians} \tag{74}$$

is the desired kick angle. Using the calculated values of $B\rho$ at AGS injection we obtain a current of 695.2 A for Au77+ ions. The maximum available current is 1100 A.

15 AGS Injection Kicker Short Pulse Waveforms

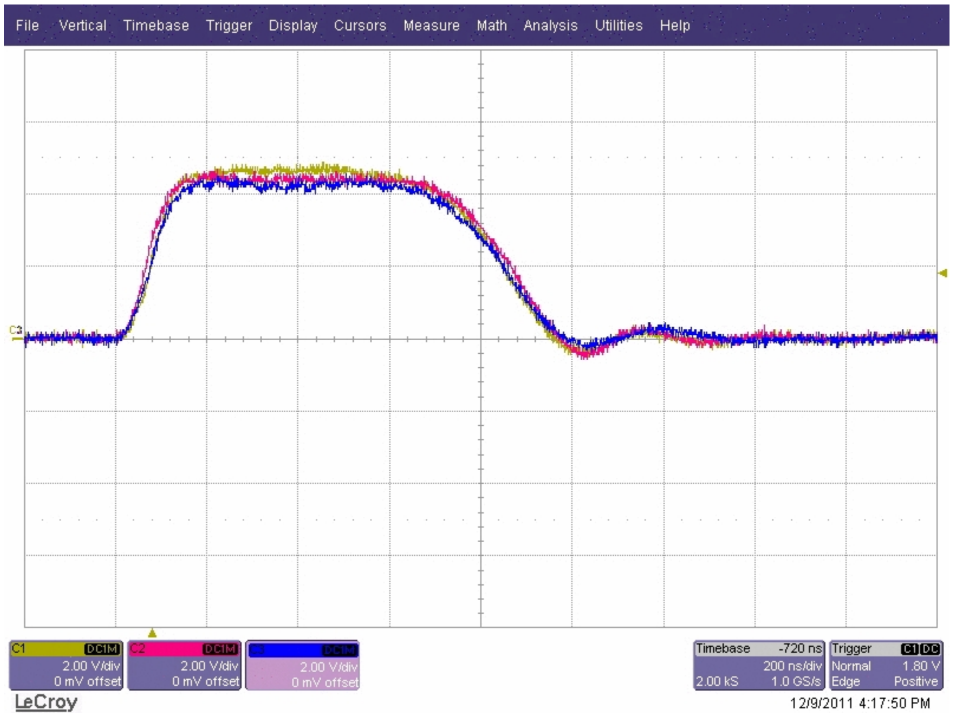

Figure 1: AGS injection kicker waveforms in the short pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan on 9 Dec 2011. The time per division is 200 ns. The RF bucket width on the AGS injection porch is 383 ns for Au77+ ions. In order to put beam into adjacent buckets, the rise time of the kicker must be less than or equal to $T - W$, where T is the bucket width and W the bunch width. The rise time is approximately 100 ns, which implies that the bunch width must be less than or equal to 283 for Au77+ bunches. A single bunch of this width easily fits on the flattop portion of the pulse which is some 600 ns long. The total width of the pulse is approximately 1000 ns. With this kicker pulse one could in principle fill 14 of the 16 RF buckets on the AGS injection porch. The pulse is too wide to fill the remaining buckets without interfering with beam in the adjacent buckets. This is not an issue as only 8 of the buckets need to be filled. One workable filling pattern is four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. Another workable pattern is eight adjacent filled buckets followed eight empty buckets.

16 AGS Injection Kicker <u>Long</u> Pulse Waveforms

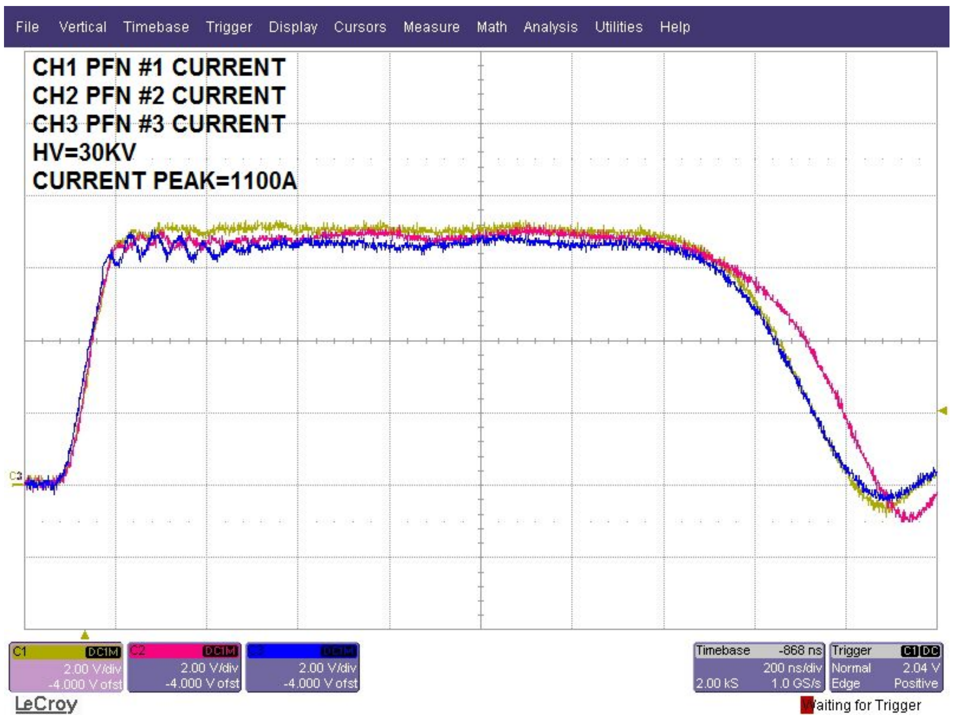

Figure 2: AGS injection kicker waveforms in the long pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan in Oct 2010. The time per division is 200 ns. The RF bucket width on the AGS injection porch is 383 ns for Au77+ ions. Here the flattop portion of the pulse is some 1300 ns long. The total pulse width is approximately 2000 ns. In principle this kicker pulse could be used to fill 8 buckets wtih a filling pattern of four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets.

17 Gold in Booster

Parameter	Injection	Merge porch	Extraction	Unit
Q	32	32	32	
mc^2	183.456851	183.456851	183.456851	GeV
W/A	1.9762739452	49.259795	107.75879	MeV
cp/A	60.701960016	306.87652	460.77475	MeV
E/A	0.9332293272	0.98051285	1.0390118	GeV
$B\rho$	1.24651715338	6.3017214	9.4620277	Tm
β	0.065045062608	0.31297552	0.44347401	
$\gamma - 1$	0.002122166406	0.052896251	0.11571376	
η	-0.953	-0.859	-0.7605	
ϵ_H (95%)	12.1π	12.1π	12.1π	mm mrad
ϵ_V (95%)	5.68π	5.68π	5.68π	mm mrad
h	4	1	1	
hf	386.560	465.000	658.910	KHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parm	Injection	Injection	Ext	Ext	Ext	Unit
V_g	0.403	5.730	25.2	25.2	25.2	kV
A_S	4.263	16.076	318.54	318.54	318.54	eV s
dB/dt	0	0	**70**	**35**	**0**	G/ms
ϕ_s	0	0	50.999	22.866	0	deg
F_s	0.3065	1.1557	0.8139	0.9849	1.0260	kHz
A_{bk}	4.263	16.076	36.294	140.88	318.54	eV s
A_b	1.576	3.8907	17.533	17.533	17.533	eV s
Δt	1196	947.0	299.3	263.8	257.7	ns
ΔE	0.8596	2.654	37.98	42.50	43.45	MeV

Parameter	Injection	Injection	Extraction	Unit
No. Bunches	4	4	1	
Bucket Width	2586.92053	2586.92053	1517.65795	ns
Ions/Bunch	1.12/4	1.12/4	1.007	10^9 [17]
Bunch Area	0.032/4 [13]	0.0790/4 [18]	0.089 [19]	eV s/A

18 Gold in AGS

Parameter	Injection	Transition	Extraction	Unit
Q	77	77	77	
mc^2	183.434174	183.434174	183.434174	GeV
W/A	0.10529199	6.98353456	8.86486832	GeV
cp/A	0.45515837	7.85970883	9.75165221	GeV
E/A	1.0364299	7.91467250	9.79600626	GeV
$B\rho$	3.88434088	67.0750887	83.2210138	Tm
β	0.43915981	0.993055472	0.995472231	
γ	1.1130788	8.5000	10.5204673	
η	-0.793	0.0	0.00481	
ϵ_H (95%)	$\leq 12\pi$	$\leq 12\pi$	$\leq 12\pi$	mm mrad
ϵ_V (95%)	$\leq 12\pi$	$\leq 12\pi$	$\leq 12\pi$	mm mrad
h	16	12	12	
hf	2.610000	4.42642072	4.43700723899	MHz
R	128.4526	128.4526	128.45798	m

Parameter	Injection	Injection	Extraction	Unit
h	16	4	12	
V_g	22.048	7.324	192.0	kV
A_S	28.248	130.25	5070	eV s
dB/dt	0	0	0	G/ms
ϕ_s	0	0	180	degrees
F_s	1.522	0.4387	0.0985	kHz
A_{bk}	28.248	130.25	5070	eV s
A_b	22.261	89.044	137.9	eV s
Δt	287.0	1034 [20]	26.8	ns
ΔE	53.47	58.22	3281	MeV

Parameter	Injection	Injection	Extraction	Unit
h	16	4	12	
Bucket Width	383.142	1532.567	225.377	ns
No. of Bunches	8	2	2	
Ions/Bunch	0.55725	2.229	2.055	10^9 [17]
Bunch Area	0.113 [21]	0.452	0.70 [22]	eV s/A

19 Gold in RHIC

Parameter	Injection	Transition	Store	Unit
Q	79	79	79	
mc^2	183.433337	183.433337	183.433337	GeV
W/A	8.86482785	20.3825165	99.0688663	GeV
cp/A	9.75160770	21.2933012	99.9956649	GeV
E/A	9.79596155	21.3136502	**100.000000**	GeV
$B\rho$	81.1137824	177.117482	831.763013	Tm
β	0.995472231	0.999045259	0.999956649	
γ	10.5204673	22.8900	107.395964	
η	-0.00713	0.0	0.00182	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	360	360	2520	
hf	28.0232036147	28.1237867061	197.046100581	MHz
δC	0	0	0	mm

Parameter	Injection	Store	Unit
h	360	2520	
V_g	393.1	3000	kV
A_S	174.4	164.4	eV s
dB/dt	0	0	G/ms
ϕ_s	0	180	degrees
F_s	0.200	0.232	kHz
A_{bk}	174.4	164.4	eV s
A_b	137.9	137.9	eV s
A_b	0.70	0.70	eV s/A [23]
Δt	26.8	4.00	ns
ΔE	3549	24052	MeV

20 Gold in AGS

Parameter	Injection	Transition	Extraction	Unit
Q	77	77	77	
mc^2	183.434174	183.434174	183.434174	GeV
W/A	0.10529199	6.98353456	8.86486832	GeV
cp/A	0.45515837	7.85970883	9.75165221	GeV
E/A	1.0364299	7.91467250	9.79600626	GeV
$B\rho$	3.88434088	67.0750887	83.2210138	Tm
β	0.43915981	0.993055472	0.995472231	
γ	1.1130788	8.5000	10.5204673	
η	-0.793	0.0	0.00481	
ϵ_H (95%)	$\leq 12\pi$	$\leq 12\pi$	$\leq 12\pi$	mm mrad
ϵ_V (95%)	$\leq 12\pi$	$\leq 12\pi$	$\leq 12\pi$	mm mrad
h	16	12	12	
hf	2.610000	4.42642072	4.437007239	MHz
R	128.4526	128.4526	128.45798	m

21 Medium-Energy Gold in RHIC

	Energy 1	Energy 2	Energy3	Injection	Unit
Q	79	79	79	79	
mc^2	183.433337	183.433337	183.433337	183.433337	GeV
W/A	30.268866	18.568866	12.568866	8.8648279	GeV
cp/A	31.186103	19.477756	13.467850	9.7516077	GeV
E/A	**31.2**	**19.5**	**13.5**	9.7959615	GeV
$B\rho$	259.40571	162.01580	112.02545	81.1137824	Tm
β	0.99955457	0.99885930	0.99761854	0.99547223	
γ	33.5075407	20.942213	14.498455	10.520467	
η	0.001018	-0.0003715	-0.002849	-0.00713	
h	360	360	360	360	
hf	28.1381241	28.118552	28.0836235	28.0232036	MHz
δC	0	0	0	0	mm

22 Low-Energy Gold at AGS Extraction I

	Energy 1	Energy 2	Energy 3	Energy 4	Unit
Q	77	77	77	77	
mc^2	183.434174	183.434174	183.434174	183.434174	GeV
W/A	8.0689031	4.8188883	2.9188796	1.5688735	GeV
cp/A	8.9517441	5.6741329	3.7357218	2.3201378	GeV
E/A	9.0000411	5.7500262	3.8500176	2.5000114	GeV
$B\rho$	76.394564	48.423291	31.880808	19.800154	Tm
β	0.99463369	0.98680121	0.97031292	0.92805087	
γ	9.6656367	6.1752679	4.1347446	2.6848991	
η	0.00314	-0.0124	-0.0447	-0.1249	
h	12	12	12	12	
hf	4.4332697	4.3983589	4.3248675	4.13649751	MHz
R	128.45798	128.45798	128.45798	128.45798	m

23 Low-Energy Gold at RHIC Injection I

	Energy 1	Energy 2	Energy 3	Energy 4	Unit
Q	79	79	79	79	
mc^2	183.433337	183.433337	183.433337	183.433337	GeV
W/A	8.0688663	4.8188663	2.9188663	1.5688663	GeV
cp/A	8.9517032	5.6741070	3.7357048	2.3201272	GeV
E/A	**9.000**	**5.750**	**3.850**	**2.500**	GeV
$B\rho$	74.460184	47.197169	31.073558	19.298796	Tm
β	0.99463369	0.98680121	0.97031292	0.92805087	
γ	9.6656367	6.1752679	4.1347446	2.6848991	
η	-0.00880	-0.0243	-0.0566	-0.1368	
h	360	363	369	387	
hf	27.9995981	28.0106013	27.9978263	28.0846410	MHz
δC	0	0	0	0	mm

24 Low-Energy Gold at AGS Extraction II

	Energy 1	Energy 2	Energy 3	Energy 4	Unit
Q	77	77	77	77	
mc^2	183.434174	183.434174	183.434174	183.434174	GeV
W/A	6.5688963	6.368895384	6.0688940	5.81889287	GeV
cp/A	7.44200885	7.24040528	6.9378260	6.685499094	GeV
E/A	7.5000342	7.300033325	7.0000320	6.75003081	GeV
$B\rho$	63.510419	61.78992595	59.207701	57.05433297	Tm
β	0.99226332	0.99183181253	0.99111347	0.9904397887	
γ	8.0546973	7.83990534719	7.5177175	7.249227547	
η	-0.00157	-0.00243	-0.00385	-0.00519	
h	12	12	12	12	
hf	4.42270450	4.42078120761	4.41757944	4.414576695	MHz
R	128.45798	128.45798	128.45798	128.45798	m

25 Low-Energy Gold at RHIC Injection II

	Energy 1	Energy 2	Energy 3	Energy 4	Unit
Q	79	79	79	79	
mc^2	183.433337	183.433337	183.433337	183.433337	GeV
W/A	6.5688663	6.36886631	6.0688663	5.81886631	GeV
cp/A	7.4419749	7.24037223	6.9377943	6.68546857	GeV
E/A	**7.500**	**7.300**	**7.000**	**6.750**	GeV
$B\rho$	61.902278	60.2253491	57.708509	55.6096656	Tm
β	0.99226332	0.99183181253	0.99111347	0.9904397887	
γ	8.0546973	7.83990534748	7.5177175	7.249227547	
η	-0.0135	-0.0144	-0.0158	-0.0171	
h	360	360	363	363	
hf	27.9328705	27.9207234165	28.1330059	28.1138832	MHz
δC	0	0	0	0	mm

26 Au77+ Energy Loss in the BTA Stripper Foils

The stripper used to strip gold ions consists of a 6.45 mg/cm^2 aluminum foil followed by a 8.39 mg/cm^2 "glassy" carbon foil [24, 25]. We can estimate the energy loss in the foils as follows:

The kinetic energy of a proton that has the same velocity as the Au77+ ion just upstream of the aluminum foil is

$$W_p = 108.6 \text{ MeV}. \tag{75}$$

The rate of energy loss of a proton passing through the foil with kinetic energy W_p is [26]

$$-\frac{dE_p}{dx} = 5.348 \text{ MeV cm}^2/\text{g}. \tag{76}$$

The rate of energy loss of the Au77+ ion is obtained by scaling the Bethe-Bloch result for protons [27]. Thus

$$-\frac{dE}{dx} = -Z^2 \frac{dE_p}{dx} \text{ cm}^2/\text{g} \tag{77}$$

where $Z = 77$. Multiplying this by the surface density of the aluminum foil (6.45 mg/cm^2) gives

$$\Delta E_a = 1.038 \text{ MeV per nucleon}. \tag{78}$$

This is the energy lost by the Au77+ ion upon passing through the aluminium foil. The kinetic energy of a proton that has the same velocity as the Au77+ ion just downstream of the aluminum foil is then

$$W_p = 107.5 \text{ MeV}. \tag{79}$$

The rate of energy loss of a proton passing through the carbon foil with this kinetic energy is [26]

$$-\frac{dE_p}{dx} = 6.180 \text{ MeV cm}^2/\text{g}. \tag{80}$$

Using this result in (77) with $Z = 77$, and multiplying by the surface density of the carbon foil (8.39 mg/cm^2) gives

$$\Delta E_c = 1.561 \text{ MeV per nucleon}. \tag{81}$$

The total energy lost upon passing through both foils is then

$$\Delta E = \Delta E_a + \Delta E_c = 2.599 \text{ MeV per nucleon}. \tag{82}$$

This agrees reasonably well with the value 2.453 MeV per nucleon obtained in Section 6.

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