BNL-99443-2013-TECH

C-A/AP/293;BNL-99443-2013-IR

Rest Mass of Fully Stripped Ions in RHIC: Updated Values

K. A. Brown

October 2007

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

DISCLAIMER

C-A/AP/#293
October 2007

Rest Mass of Fully Stripped Ions in RHIC: Updated Values

K.A. Brown, C. Gardner, P. Thieberger

Collider-Accelerator Department
Brookhaven National Laboratory
Upton, NY 11973

Rest Mass of fully stripped Ions in RHIC: Updated Values

K.A. Brown, C. Gardner, P. Thieberger

October 16, 2007

Abstract

The method for calculating the mass of a fully stripped ion was detailed in RHIC AP Note 20, in 1994 [1]. Values for various constants have since been revised. This note gives updated values for various constants and recalculates the rest mass of the various ions used in RHIC. We also explore the possibility of performing ion mass measurements in RHIC.

1 Introduction

The mass of any ion is (note; we use the notation of [2]):

$$m = au - Qm_e + E_b/c^2 \quad (1)$$

Here, for any given ion, a is the relative atomic mass, u is the atomic mass constant, Q is the charge, m_e is the electron mass, E_b is the binding energy of the electrons that have been removed, and c is the speed of light. Table 1 gives values for the constants u, m_e, the proton mass, the deuteron mass, and c, as taken from the National Institute of Standards and Technology (NIST) on-line database [3].

Table 1: Constants

Parameter	**Value**	**Uncertainty**	**Unit**
atomic mass constant (u)	931.494028	0.000023	MeV/c^2
electron mass (m_e)	0.510998910	0.000000013	MeV/c^2
proton mass	938.272013	0.000023	MeV/c^2
deuteron mass	1875.612793	0.000047	MeV/c^2
velocity of light (c)	299792458	(exact)	m/sec

2 Relative Atomic Masses and Atomic Electron Binding Energies

The National Nuclear Data Center (NNDC) on-line database [4] provides a listing of the relative atomic masses. Isotopic compositions of the most common atomic isotopes are taken from the NIST on-line database of relative atomic masses and compositions [5]. The values of the relative atomic masses from the NNDC data base are the same as those found on the NIST database, but are not rounded off. These on-line databases are based on reports from [6, 7, 8]. Table 2 gives the values for ions that are of interest to RHIC. The NIST database for the relative atomic masses does not represent a critical evaluation by the NIST Physics Laboratory, but is given for NIST user's convenience. NNDC values are evaluated by NNDC.

In 2004, new reference tables were published of total atomic energies of ground state configurations [9]. The values published in that report are calculated, but agree well with experimental values. (They state that, even though the outer shells are the most difficult ones to calculate, the standard deviation between theory and experiment is 1.2 eV in cases for ionization of neutral and singly charged atoms.) These values are different

Table 2: Relative Atomic Masses for some Common Isotopes

Isotope	**# nucleon**	**a (amu)**	**Uncertainty**	**Composition %**
H	1	1.0078250321	0.0000000001	99.9885
D	2	2.01410177785	0.00000000036	0.0115
Cu	63	62.929669374	0.000000641	69.17
Au	197	196.966568662	0.000000646	100
Pb	208	207.976652071	0.000001335	52.4
U	238	238.050788247	0.000002044	99.2745

from those obtained by adding individual electron binding energies derived from X-ray emission data. The X-ray derived binding energies are the energies required to lift each individual electron from its bound state to the continuum, leaving all the other electrons intact. To lift electron after electron leaving behind an ion of increasing charge requires more energy than just randomly picking out electrons from different shells. Table 3 gives the values of binding energies for a sample of ions and charge states. These values are taken directly from the tables in ref. [9]. We have included in Appendix A a description of the procedure used to derive the correct values from those tables. Note that the tables in [9] only go down to Li. To get E_b for Helium-like gold we performed a simple extrapolation, shown in figure 1.

Table 3: Binding energies for different isotopes and charge states

Isotope	**charge**	**Binding Energy [eV]**
Cu	11	1,291.0
Cu	29	44,961.7
Au	79	517,015.2
Au*	77	332,391.0
Au	32	14,485.8
Au	31	13,541.0
Pb	82	567,984.8
U	92	761,653.7

[* value is calculated by extrapolation from Li-like gold (Au76)]

3 Ion Masses

From tables 1, 2, and 3 we can now calculate the masses of the ions using Eq. 1. Table 4 gives the various quantities and the final masses. The corresponding uncertainties are listed in Table 5. Number of significant figures are given consistently according to those given in the on-line database tables.

Table 4: Masses for ions of interest to RHIC

Isotope	Q	au (MeV/c^2)	Qm_e (MeV/c^2)	E_b (MeV)	m (MeV/c^2)
Cu	11	58618.6112059	5.6209880	0.0012910	58612.9915089
Cu	29	58618.6112059	14.8189684	0.0449617	58603.8371992
Au	79	183473.18242431	40.3689139	0.5170152	183433.3305256
Au	77	183473.18242431	39.3469161	0.3323910	183434.1678992
Au	32	183473.18242431	16.3519651	0.0144858	183456.8449450
Au	31	183473.18242431	15.8409662	0.0135410	183457.3549991
Pb	82	193729.00936757	41.9019106	0.5679848	193687.6754418
U	92	221742.88761277	47.0118997	0.7616537	221696.6373668

Table 5: Uncertainties in Masses for ions of interest to RHIC

Isotope	Q	$\sigma_{(au)}$	$\sigma_{(Qm_e)}$	σ_{E_b}	σ_m
Cu	11	0.00156570	0.00000014	0.00000005	0.0015657
Cu	29	0.00156570	0.00000038	0.00000005	0.0015657
Au	79	0.00457002	0.00000103	0.00000005	0.0045700
Au	77	0.00457002	0.00000100	0.00000005	0.0045700
Au	31	0.00457002	0.00000040	0.00000005	0.0045700
Au	32	0.00457002	0.00000042	0.00000005	0.0045700
Pb	82	0.00494246	0.00000107	0.00000005	0.0049425
U	92	0.00579677	0.00000120	0.00000005	0.0057968

As seen in Table 5 the uncertainty in the mass from the lost electrons and the binding energy is insignificant. Uncertainties in reported values represent r.m.s. values, with the assumption that the error distributions are Gaussian.

4 Measuring Masses in RHIC

It is possible to consider measuring the mass of the ions in RHIC, although it is a difficult measurement. There are two approaches one can attempt. First, it would seem best to take the ratio of the revolution frequency of two different ions, circulating in the same magnetic field. That would allow the absolute magnitude of the field to be ignored, which cannot be precisely known. The other approach is to measure just one ion as precisely as possible, in which case we need to evaluate the precision of the field measurement, the accelerator circumference, and the revolution frequency.

The Lorentz force on a particle with velocity v, traveling in a constant magnetic field B, can be expressed as

$$\frac{dP}{dt} = eQ(v \times B) \tag{2}$$

The centrifugal force is

$$F_c = \frac{m\gamma v^2}{\rho}, \tag{3}$$

where ρ is the of the radius-of-curvature of the trajectory. In a circular accelerator, a particle follows a closed circular trajectory and the two forces are in equilibrium.

$$m\gamma v^2/\rho = eQvB \tag{4}$$

Re-expressing this in terms of momentum, p, then we find the familiar expression,

$$p = eQB\rho \tag{5}$$

Expressed in this way, the units of p are $[kg{\cdot}m/s]$ and the units on the right hand side are $[C(N \cdot A^{-1} \cdot m^{-1})m]$. Since $1[T] = 1[N \cdot A^{-1} \cdot m^{-1}]$, $1[A] = 1[C/s]$, and $1[kg] = c^2/e$ $[eV]$, we can re-express again, now in units of $[eV/c]$

$$p = cQB\rho \ \ [\mathrm{eV}/c] \tag{6}$$

This is usually expressed in units of $[GeV/c]$, in which case we write $p = cQB\rho/10^9$. It is convenient here to work in units of $[MeV/c]$, in which case we write,

$$p = \frac{cQB\rho}{10^6} \ \ [MeV/c] \tag{7}$$

For this expression, B is in units of $[T]$, ρ is in units of $[m]$, and c is in $[m/s]$.

The momentum of any particle, of mass m, is (note: normal convention is for c to be held in the units)

$$p = m\beta\gamma \ \ [MeV/c] \tag{8}$$

The mass of the ion is then equal to the ratio of the beam rigidity to the average velocity factor.

$$m = \frac{cQB\rho}{10^6\beta\gamma} \;\; [MeV/c^2] \tag{9}$$

To calculate β we need to measure radius and revolution frequency.

$$\beta = 2\pi Rf/c \tag{10}$$

$$\gamma = (1-\beta^2)^{-1/2} \tag{11}$$

The value of ρ depends on the radius and the lattice γ_{tr}. This is important since the first method of measuring the mass ratios requires we set the radius to R_0 as best as possible for the same optics and magnetic field.

$$\rho = \rho_0 \left(R/R_0\right)^{\gamma_{tr}^2} \tag{12}$$

The observables are radius, frequency and field, but we factor out the field by measuring the ratio of frequencies for two different ions, one of which has a mass known to a higher degree of precision than the other (e.g., carbon would be ideal, since the value of u is defined as 1/12 of a neutral carbon atom). The ratio of the masses we denote as M, is then,

$$M = \frac{m}{m_r} = \frac{Q}{Q_r}\frac{\beta_r\gamma_r}{\beta\gamma} \tag{13}$$

where Q_r, β_r, γ_r, and m_r correspond to the charge state, velocity factors, and mass of the reference ion. The technique requires a method be developed to determine that the magnetic field is the same for both ions and the radius is set as close to R_0 as possible. In this case the observables would be the radius of the two ions, R and R_r, revolution frequency of the two ions, f and f_r, the uncertainty in the difference in the field for the two ions, and the uncertainty in the difference in the radius of the two ions. But we need not evaluate all of these variables into the uncertainty in the measured mass, since the best we can do in measuring the absolute mass is dependent on the uncertainty in the mass of the reference particle.

$$\sigma_m^2 = m_r^2\sigma_M^2 + M^2\sigma_{m_r}^2 \tag{14}$$

As seen in tables 1 and 5 the absolute uncertainties in the masses of gold ions and deuterons are 0.00475 and 0.000047, respectively. The ratio of the two masses is approximately 100. From this we can see that it is not possible to measure the absolute mass in units of [MeV] to higher precision than already exists. But we do not need to measure the absolute mass, since we can measure the relative atomic masses directly. If we replace the deuteron with a carbon beam, then we can re-express Eq. 13 as,

$$M = \frac{au - Qm_e + E_b/c^2}{12u - 6m_e + E_{bc}/c^2} \tag{15}$$

The relative atomic mass of m can be obtained from this ratio. Of course we then need to consider the fact that we will be comparing two atomic nuclei, and not two neutral atoms. So the uncertainty in the binding energies and in the electron mass will add to the uncertainty in our measurement. To fully evaluate the uncertainty in the measurement, we need to consider that Eq. 13 is only valid if both beams are in exactly the same field and at exactly the same radius. More precisely,

$$M = \frac{Q}{Q_r}\left(1 \pm \frac{\delta(\beta\gamma)}{\beta_r\gamma_r}\right)^{-1}\left(1 \pm \frac{\delta B}{B_r}\right)\left(1 \pm \frac{\delta R}{R_r}\right)^{\gamma_{tr}^2} \tag{16}$$

Where $\delta(\beta\gamma)$ is the difference in $\beta\gamma$ from the measured difference in the revolution frequency of the two beams and δR and δB are the measured differences in the radii and fields of the two beams.

To determine how well M needs to be measured we need to evaluate the final uncertainty in a in Eq. 15. Approximating, and leaving only the more significant terms, then

$$\sigma_a^2 \approx 144\sigma_M^2 + \left(\frac{Q-6M}{u}\right)^2\sigma_{m_e}^2 + \left(\frac{m_e(Q-6M)}{u^2}\right)^2\sigma_u^2 \tag{17}$$

The values for the uncertainties in u, m_e, and a for gold are given in tables 1 and 2. From these we can estimate how good the measurement of M needs to be to make any improvement in the value of a for gold. The value of the last two terms in Eq. 17 are both on the order of 10^{-19} and are insignificant compared to $\sigma_a^2 \approx 4\times10^{-13}$. So to improve in the value of a, we need to measure M to the level of $\sigma_M < 5\times10^{-8}$.

The uncertainty in M can be approximated if we assume that δB and δR are insignificantly small (in which case we can drop all terms of the form $(1 \pm \delta B/B_r)$ and $(1 \pm \delta R/R_r)$).

$$\begin{aligned}\sigma_M^2 \approx{}& \left(\frac{Q\delta B}{Q_r B_r^2}\right)^2\sigma_{B_r}^2 + \left(\gamma_{tr}^2\frac{Q\delta R}{Q_r R_r^2}\right)^2\sigma_{R_r}^2 + \left(\frac{Q\delta(\beta\gamma)}{Q_r(\beta_r\gamma_r)^2}\left(1 \pm \frac{\delta(\beta\gamma)}{\beta_r\gamma_r}\right)^{-2}\right)^2\sigma_{\beta_r\gamma_r}^2 + \\ & \left(\frac{Q}{Q_r B_r}\left(1 \pm \frac{\delta(\beta\gamma)}{\beta_r\gamma_r}\right)^{-1}\right)^2\sigma_{\delta B}^2 + \left(\gamma_{tr}^2\frac{Q}{Q_r R_r}\left(1 \pm \frac{\delta(\beta\gamma)}{\beta_r\gamma_r}\right)^{-1}\right)^2\sigma_{\delta R}^2 + \\ & \left(\frac{Q}{Q_r(\beta_r\gamma_r)}\left(1 \pm \frac{\delta(\beta\gamma)}{\beta_r\gamma_r}\right)^{-2}\right)^2\sigma_{\delta(\beta\gamma)}^2\end{aligned} \tag{18}$$

The first three terms of Eq. 18 turn out to be insignificantly small. From the last three terms we find the uncertainty in $\sigma_{\beta_r \gamma_r}$ needs to be less than 4×10^{-7}, the uncertainty in the difference between the two magnetic fields needs to be less than 1×10^{-2} [G], and the uncertainty in the difference between the two radii of the two beams needs to in the range of nano-meters.

The second method of measuring the mass is extremely difficult, since it is obvious we need to know to high precision the magnetic field. The uncertainty in the mass being measured is

$$\sigma_m^2 = \left(\frac{\partial m}{\partial B}\right)^2 \sigma_B^2 + \left(\frac{\partial m}{\partial \rho}\right)^2 \sigma_\rho^2 + \left(\frac{\partial m}{\partial \beta\gamma}\right)^2 \sigma_{\beta\gamma}^2 \tag{19}$$

Working in units of $[MeV]$ (and one must be careful how the units are carried, thus the c^2 factor), then

$$\sigma_m^2 = \frac{c^2 Q^2}{10^{12}(\beta\gamma)^2}\left[\rho^2\sigma_B^2 + B^2\sigma_\rho^2 + \left(\frac{B\rho}{\beta\gamma}\right)^2 \sigma_{\beta\gamma}^2\right] \tag{20}$$

where,

$$\sigma_{\beta\gamma}^2 = (\beta\gamma)^2\left[\left(\frac{\gamma^2}{C}\right)^2 \sigma_C^2 + \left(\frac{\gamma^2}{f}\right)^2 \sigma_f^2\right] \tag{21}$$

If we want to match the current precision in the mass of the gold ion, making the measurement at RHIC injection at a $B\rho = 70$ $[Tm]$, then the factor $c^2Q^2/(10^{12}\beta\gamma)$ is approximately 7×10^6. In this case, ignoring how well we can measure frequency and the equilibrium orbit, we would need to measure field to better than 1×10^{-4} $[G]$ and the value of ρ to better than 5×10^{-3} $[mm]$.

5 Summary

The value for the mass of fully stripped gold in RHIC, published in [1] was 183,433.18 MeV/c^2. The new value, given more recent values for the various constants is now 183,433.33053$\pm$0.00457 MeV/c^2. Methods of measuring the mass of the ions in RHIC prove to be extremely difficult, due to the extremely high precision required in measuring radius and field. Current technology makes these kinds of measurements impracticable.

A Determination of Binding Energies

To obtain the energy required to remove a certain number of electrons from a given atom, we use the tables given in Ref. [9]. Here the table numbered N gives the energy required to remove all electrons from atoms consisting of N electrons and Z protons, with Z running from N to 118. Tables are given for $N = 3$ (Lithium-like atoms) through $N = 105$ (Dubnium-like atoms).

Let E_Q be the energy required to remove the outer Q electrons from a neutral atom containing Z protons, and let $\mathcal{E}_{Z-Q}$ be the energy required to remove the remaining $Z - Q$ electrons. Then we have

$$E_Q = E_Z - \mathcal{E}_{Z-Q} \tag{22}$$

where E_Z is the energy required to remove all Z electrons. Here E_Z is obtained from the first entry of Table Z and $\mathcal{E}_{Z-Q}$ is obtained from entry Z of Table $Z - Q$.

A.1 Some Examples

The use of the tables is best illustrated with a few examples.

Let E_{31} be the energy required to remove the outer 31 electrons from a neutral gold atom, and let $\mathcal{E}_{48}$ be the energy required to remove the remaining 48 electrons. The sum of these energies is the energy E_{79} required to remove all 79 electrons from the atom. Thus we have

$$E_{31} = E_{79} - \mathcal{E}_{48}. \tag{23}$$

From the first entry ($Z = 79$) of Table 79 of Ref. [9] we obtain

$$E_{79} = 517015 \text{ eV} \tag{24}$$

and from the $Z = 79$ entry of Table 48 we obtain

$$\mathcal{E}_{48} = 503474 \text{ eV}. \tag{25}$$

This gives

$$E_{31} = 13541 \text{ eV}. \tag{26}$$

Similarly, the energy required to remove the outer 32 electrons from a neutral gold atom is given by

$$E_{32} = E_{79} - \mathcal{E}_{47} \tag{27}$$

where

$$\mathcal{E}_{47} = 502529 \text{ eV} \tag{28}$$

is obtained from the $Z = 79$ entry of the Table 47. This gives

$$E_{32} = 14486 \text{ eV}. \tag{29}$$

Finally, the energy required to remove the outer 11 electrons from a neutral copper atom is

$$E_{11} = E_{29} - \mathcal{E}_{18} \tag{30}$$

where E_{29} is the energy required to remove all 29 electrons from the neutral atom, and $\mathcal{E}_{18}$ is the energy required to remove all electrons from a copper ion with 18 electrons. Here

$$E_{29} = 44962 \text{ eV} \tag{31}$$

is obtained from the first entry ($Z = 29$) of Table 29, and

$$\mathcal{E}_{18} = 43671 \text{ eV} \tag{32}$$

is obtained from the $Z = 29$ entry of Table 18. This gives

$$E_{11} = 1291 \text{ eV}. \tag{33}$$

A.2 Binding Energy for Helium-Like Gold

Because there are no tables in Ref. [9] for atoms with fewer than 3 electrons, we can not obtain directly the energy

$$E_{77} = E_{79} - \mathcal{E}_2 \tag{34}$$

required to remove the outer 77 electrons from a neutral gold atom. However, we can obtain

$$E_{76} = E_{79} - \mathcal{E}_3 = 517015 - 207635 = 309380 \text{ eV} \tag{35}$$

from Tables 79 and 3. Then if we know the ionization energy W required to remove the outer electron from the Lithium-like atom consisting of three electrons and 79 protons, we can calculate

$$E_{77} = E_{76} + W. \tag{36}$$

We can obtain a reasonable estimate of W by extrapolation. Let W_M be the energy required to remove one electron from a gold ion with M electrons. Then we have

$$W_M = \mathcal{E}_M - \mathcal{E}_{M-1} \tag{37}$$

where $\mathcal{E}_M$ is the energy required to remove all electrons from a gold ion with M electrons. Here $\mathcal{E}_M$ is given by the $Z = 79$ entry of Table M. Thus we can obtain W_4 through W_{79} from the tables. The ionization energy

$$W = W_3 = 23011 \tag{38}$$

is then obtained by extrapolation. This gives

$$E_{77} = 309380 + 23011 = 332391 \text{ eV}. \tag{39}$$

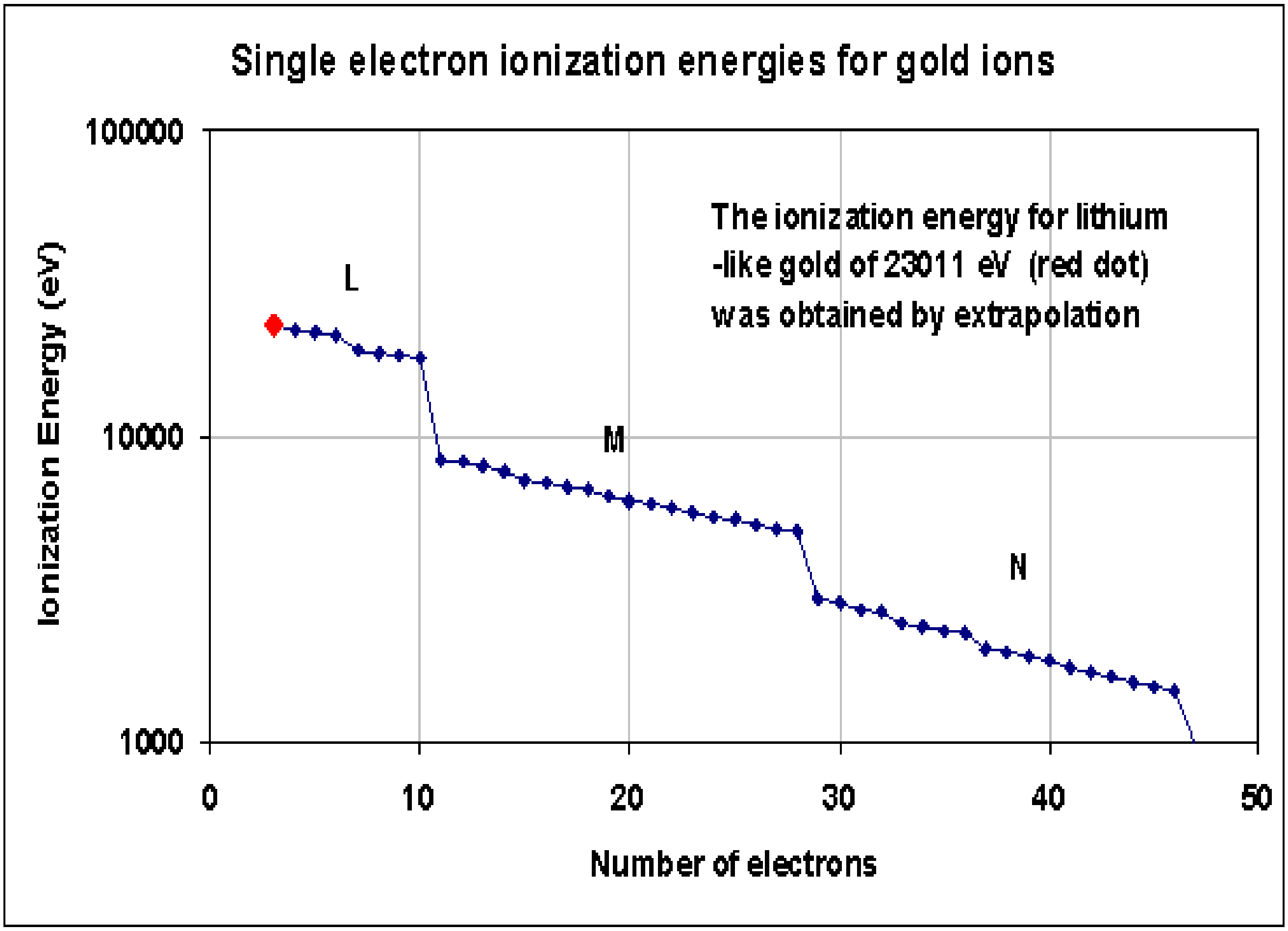

Figure 1: Extrapolation of the single electron ionization (the energy to remove the last L electron from the Li-like ion)

References

[1] D. Trbojevic, "Rest Mass of the Fully Stripped Gold Ions" RHIC/AP Note 20, January 1994.

[2] C.J. Gardner, "FY08 Parameters for the Injection, Acceleration, and Extraction of Gold Ions and Deuterons in the Booster, AGS, and RHIC" from Gardner Notes web page for CAD Operations.
http://www.cadops.bnl.gov/AGS/Operations/GardnerNotes/RhicRunParameters/

[3] National Institute of Standards and Technology Fundamental Physical Constants (Based on all data up to 31 December 2006)
http://physics.nist.gov/constants

[4] National Nuclear Data Center Atomic Mass Table (last updated Dec. 22, 2003)
http://www.nndc.bnl.gov/masses/

[5] National Institute of Standards and Technology Atomic Weights and Isotopic Compositions Elements
http://physics.nist.gov/PhysRefData/Compositions/index.html

[6] T.B. Coplen, Atomic Weights of the Elements 1999, (changes reported from the 2001 review in Chem. Int., 23, 179 (2001))

[7] K.J.R. Rosman and P.D.P. Taylor, "Isotopic Compositions of the Elements 2001", Journal of Physical and Chemical Reference Data, March 2005, Volume 34, Issue 1, pp. 57-67

[8] G. Audi and A.H. Wapstra, "The 1995 Update To The Atomic Mass Evaluation", Nuclear Physics A, Volume 595, Number 4, 25 December 1995 , pp. 409-480(72)

[9] G.C. Rodrigues, P. Indelicato, J.P. Santos, P. Patte, and F. Parente, "Systematic Calculation of Total Atomic Energies of Ground State Configurations", Atomic Data and Nuclear Data Tables 86 (2004) 117–233.