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## Simulation of $3Q_x$ resonance driving term measurement with AC dipole excitation

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To further improve the luminosity in polarized proton operation of the Relativistic Heavy Ion Collider, correction of the horizontal two-third resonance is desirable, to increase the available beam-beam tune space. The long-lasting coherent beam oscillations, produced by the AC dipole, are used to measure  $3Q_x$  resonance driving term  $h_{30000}$ , through the analysis of turn-by-turn beam position data. In this note, we present the results of numerical simulations, carried out to evaluate this measurement technique.

#### **1** Introduction

To increase the tune space available for beam-beam generated tune spread, a third order resonance correction at the current working point ( $Q_x = 28.685, Q_y = 29.695$ ) for polarized proton operation in the Relativistic Heavy Ion Collider (RHIC) is desirable [1]. At the current working point the fractional horizontal tune  $Q_x$ is constrained by the third order resonance  $Q_x = 2/3$ , and the tenth order resonance  $Q_x = 0.7$ . With  $Q_x$ close to 0.7, both luminosity and polarization suffer. A further increase in the bunch intensity requires that particles in the center of the bunch have tunes closer to  $Q_x = 2/3$ .

In the 2006 polarized proton run, the  $3Q_x$  resonance correction at store was tested with different schemes [2]. In Ref. [3], it is suggested to correct the  $3Q_x$  resonance with 12 sextupole correctors in the interaction regions (IRs), and to correct the nonlinear chromaticities with the arc sextupole families. The key point for  $3Q_x$  correction is to measure its resonance driving term  $h_{30000}$  [4, 5, 6, 7]. The beam decay in the vicinity of  $3Q_x$  is used to measure the effectiveness of the correction [8].

In Ref. [4], the basic theory of the sextupole first order driving terms is outlined, and an algorithm is provided to extract first order driving terms from turn-by-turn (TBT) actions  $J_x(N)$ . With a modified Fast Fourier Transformation (FFT) technique, the betatron tune  $Q_x$  in the spectrum of x, and the  $3Q_x$  peak in the spectrum of the action  $J_x(N)$ , can be precisely calculated [9, 10, 11]. According to Ref. [4], the peak at  $3Q_x$  in the spectrum of  $J_x(N)$  is linked to the  $3Q_x$  resonance.

To be able to extract  $h_{30000}$  from TBT beam position monitor (BPM) data, a long-lasting, coherent, and large amplitude betatron oscillation is required. At 100 GeV, after 5 consecutive kicks with the tune meter kicker, the maximum oscillation amplitude of the beam center is about 1.5 mm at BPM rbpm.bo6-bh4. A disadvantage of the kicked BPM data is its fast decoherence and small amplitude. In addition, too many kicks increase the emittance significantly.

The AC dipole has been proven to be a powerful tool to produce long-lasting coherent beam oscillations with large amplitude. If the driving strength of AC dipole is ramped up and down adiabatically, no significant emittance increase is observed [12]. The TBT BPM data taken at the flattop of AC dipole excitation can be used for linear as well as nonlinear lattice measurements [13, 6, 7].

In the following, we first review the sextupole driving term theory and the data processing procedure to extract  $h_{30000}$  from TBT BPM data. Then, we simulate the AC dipole excitation to measure the  $h_{30000}$  under various conditions.

## 2 Background

#### 2.1 First order resonance driving terms

Sextupoles give rise to the following first order geometric driving terms,

$$h_{21000} = -\frac{1}{8} \sum_{i=1}^{N} (k_2 dl)_i \beta_{x,i}^{3/2} e^{i\mu_{x,i}}, \qquad (1)$$

$$h_{30000} = -\frac{1}{24} \sum_{i=1}^{N} (k_2 dl)_i \beta_{x,i}^{3/2} e^{i3\mu_{x,i}}, \qquad (2)$$

$$h_{10110} = \frac{1}{4} \sum_{i=1}^{N} (k_2 dl)_i \beta_{x,i}^{1/2} \beta_{y,i} e^{i\mu_{x,i}}, \qquad (3)$$

$$h_{10020} = \frac{1}{8} \sum_{i=1}^{N} (k_2 dl)_i \beta_{x,i}^{1/2} \beta_{y,i} e^{i(\mu_{x,i} - 2\mu_{y,i})}, \qquad (4)$$

$$h_{10200} = \frac{1}{8} \sum_{i=1}^{N} (k_2 dl)_i \beta_{x,i}^{1/2} \beta_{y,i} e^{i(\mu_{x,i} + 2\mu_{y,i})}.$$
 (5)

They will drive the resonances  $Q_x$ ,  $3Q_x$ ,  $Q_x$ ,  $Q_x - 2Q_y$ , and  $Q_x + 2Q_y$ , respectively, and are therefore called resonance driving terms. From Eqs. (1)-(5), each resonance driving term is a complex number,

$$h_{ijkl0} = A_{ijkl0}e^{i\phi_{ijkl0}}.$$
(6)

 $A_{ijkl0}$  and  $\phi_{ijkl0}$  are the amplitude and phase of the driving term  $h_{ijkl0}$ , respectively.

According to Refs. [4], the perturbed betatron motion with first order resonance driving terms are:

$$J_{x}(N) = J_{x} + \frac{A_{21000}(2J_{x})^{3/2}}{\sin(\pi Q_{x})} \cos\left(\hat{\phi}_{21000} + \phi_{x} + 2\pi NQ_{x}\right) \\ + \frac{A_{10110}(2J_{x})^{1/2}(2J_{y})}{\sin(\pi Q_{x})} \cos\left(\hat{\phi}_{10110} + \phi_{x} + 2\pi NQ_{x}\right) \right) \\ + \frac{3A_{30000}(2J_{x})^{3/2}}{\sin(3\pi Q_{x})} \cos\left[\hat{\phi}_{30000} + 3\phi_{x} + 2\pi N \cdot 3Q_{x}\right] \\ + \frac{A_{10200}(2J_{x})^{1/2}(2J_{y})}{\sin[\pi(Q_{x} - 2Q_{y})]} \cos\left[\hat{\phi}_{10020} + \phi_{x} - 2\phi_{y} + 2\pi N(Q_{x} - 2Q_{y})\right] \\ + \frac{A_{10200}(2J_{x})^{1/2}(2J_{y})}{\sin[\pi(Q_{x} + 2Q_{y})]} \cos\left[\hat{\phi}_{10200} + \phi_{x} - 2\phi_{y} + 2\pi N(Q_{x} + 2Q_{y})\right] \\ + O(k_{2}^{2}).$$

$$J_{y}(N) = J_{y} - \frac{A_{10020}(2J_{x})^{1/2}(2J_{y})}{\sin[\pi(Q_{x} - 2Q_{y})]} \cos\left[\hat{\phi}_{10020} + \phi_{x} - 2\phi_{y} + 2\pi N(Q_{x} - 2Q_{y})\right] \\ + \frac{A_{10200}(2J_{x})^{1/2}(2J_{y})}{\sin[\pi(Q_{x} + 2Q_{y})]} \cos\left[\hat{\phi}_{10020} + \phi_{x} - 2\phi_{y} + 2\pi N(Q_{x} - 2Q_{y})\right] \\ + O(k_{2}^{2}).$$

$$(8) \\ + O(k_{2}^{2}),$$

where  $J_x(N)$  and  $J_y(N)$  are the horizontal and vertical actions of the Nth turn. The  $\phi_x$  and  $\phi_y$  are phases of the horizontal and vertical betatron oscillations. In Eqs. (7) and (8),  $J_x$  and  $J_y$  are the average actions.

#### 2.2 Data Processing

According to Eq. (7), we can determine the amplitude and phase of the  $3Q_x$  resonance driving term  $h_{30000}$  with an FFT of the TBT action  $J_x(N)$ . The amplitude and phase of the  $3Q_x$  peak in the  $J_x(N)$  spectrum are  $A_{3Q_x}$  and  $\Phi_{3Q_x}$ , respectively, then:

$$A_{30000} = \kappa * \sin(3\pi Q_x)/3,$$
(9)

$$\phi_{30000} = \Phi_{3Q_x} - 3\phi_x,\tag{10}$$

where  $\kappa$  is the amplitude ratio of the  $Q_x$  and  $3Q_x$  peaks in the spectrum of  $J_x(N)$ . Note that the  $Q_x$  peak in the spectrum of  $J_x(N)$  is located at zero frequency, or at the average of  $J_x(N)$ . For Eq. (10), we have assumed that  $3Q_x \approx p$ , with p being an integer. On either side of the RHIC IRs, there are two dual-plane BPMs, located between quadrupoles Q3 and Q4. These two BPMs are separated by a 36.2984 m long drift. A construction of the  $(x, p_x)$  and  $J_x(N)$  from the 1024 turn-by-turn data at the horizontal 'x-position readings' of the BPM rbpm.bo6-bh3 was done in both  $h_{30000}$  measurements and in the following simulations. The horizontal Twiss parameters can be derived with an ellipse fitting, or a harmonic analysis with 1024 TBT  $(x, p_x)$  data.

Without betatron coupling, the coordinates  $(x, p_x)$  and the normalized coordinates  $(x_n, p_{xn})$  are linked by a matrix **A**:

$$\begin{pmatrix} x \\ p_x \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_n \\ p_{x_n} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \sqrt{2J_x} \cos \Phi_x \\ -\sqrt{2J_x} \sin \Phi_x \end{pmatrix}, \tag{11}$$

where

$$\mathbf{A} = \begin{pmatrix} \sqrt{\beta_x} & 0\\ -\alpha_x/\sqrt{\beta_x} & 1/\sqrt{\beta_x} \end{pmatrix} \quad \text{and} \quad \mathbf{A}^{-1} = \begin{pmatrix} 1/\sqrt{\beta_x} & 0\\ \alpha_x/\sqrt{\beta_x} & \sqrt{\beta_x} \end{pmatrix}.$$
(12)

In Eq. (12),  $\Phi_x = 2\pi(N-1) + \phi_x$ . After transforming the coordinates  $(x, p_x)$  into the normalized coordinates  $(x_n, p_{xn})$ , the TBT action  $J_x(N)$  can be constructed. The frequencies of  $Q_x$  in spectrum of x and  $3Q_x$  in spectrum of  $J_x(N)$  are precisely calculated with a modified FFT.

#### 2.3 AC dipole excitation

The AC dipole has become a powerful diagnostic tool in the linear optics measurement. By exciting the beam with an AC dipole at a frequency in the vicinity of the betatron frequency, a coherent beam oscillation with large amplitude can be generated at the drive frequency. If the amplitude of the AC dipole strength is ramped up and down adiabatically, there is no significant emittance increase.

In RHIC, the AC dipole drive tune is typically  $Q_d = Qx \pm 0.01$ . The AC dipole strength ramps from zero to its maximum value in about 6000 turns, or 76 ms. The TBT BPM data are taken at the flattop of the AC dipole excitation. The AC dipole kick strength  $\Delta\theta(N)$  at the Nth turn is given by

$$\Delta\theta(N) = \begin{cases} A_d * \cos(2\pi NQ_d + \phi_d) * N/N_{ramp}, & N \le N_{ramp} \\ A_d * \cos(2\pi NQ_d + \phi_d), & N > N_{ramp} \end{cases}$$
(13)

 $N_{ramp}$  is the number of turns for the AC dipole strength to ramp from zero to its maximum strength  $A_d$ .  $Q_d$  and  $\phi_d$  are the AC dipole drive tune and initial phase.

#### **3** Simulation Results

The driving term  $h_{30000}$  simulation of the measurements with the AC dipole excitation is obtained by the simulation code Tracy-II [14]. The used optics model is the same as the one prepared for the next polarized proton run [15]. In the model, the multipole errors in the interaction regions are not included. The first order chromaticities are calculated with a two family correction scheme only. Tab. 1 lists the beam and optics parameters used in the simulation.

Table 1: The beam and optics parameters used in the simulations.

quantity	unit	value
energy	${\rm GeV}$	100
normalized emittances $\epsilon_n$ , 95%	mm.mrad	15
tunes $(Q_{x,0}, Q_{y,0})$		(28.685, 29.695)
chromaticities $(\xi_x^{(0)}, \xi_y^{(0)})$		(1.0, 1.0)
lattice functions $\beta^*_{x,y}$ at IP6 and IP8	m	0.9
lattice functions $\beta_{x,y}^{*,v}$ at IP10, IP12, IP2, IP4	m	5.0
rms beam size $\sigma_x$ at IP6 and IP8	$\mathbf{m}\mathbf{m}$	0.15
lattice fuction $\beta_x$ at rbpm.bo6-bh3	m	503.94
lattice function $\alpha_x$ at rbpm.bo6-bh3		16.65
rms beam size $\sigma_x$ at rbpm.bo6-bh3	$\mathbf{m}\mathbf{m}$	3.45
lattice function $\beta_x$ at AC dipole	m	10.95
lattice function $\alpha_x$ at AC dipole		1.10

#### 3.1 An example

In this example, we choose  $Q_d = Q_x + 0.01 = 0.695$ ,  $N_{ramp} = 6000$  turns, and  $A_d = 5 \mu$ rad. At the flattop of the AC dipole excitation, we obtain a maximum oscillation amplitude of about 3 mm at rbpm.bo6-bh3, equal to approximately one rms beam size  $\sigma_x$  there. Tab. 2 summarizes the AC dipole excitation parameters for this case.

Table 2: AC dipole excitation parameters in the simulation

quantity	$\operatorname{unit}$	value
drive tune $Q_d$		0.695
ramp-up time $N_{ramp}$	$\operatorname{turns}$	6000
maximum strength $A_d$	$\mu \mathrm{rad}$	5
initial phase $\phi_d$	degrees	0

Fig. 1 shows the TBT x data at rbpm.bo6-bh3. In the data processing, we only take 1024 turns of  $(x, p_x)$  data at the flattop of AC dipole excitation. Fig. 2 shows the TBT  $(x, p_x)$  data at rbpm.bo6-bh3. After fitting the TBT  $(x, p_x)$  points to an ellipse, we obtain the Twiss parameters at rbpm.bo6-bh3:  $\beta_x(meas.) = 476.077$  m,  $\alpha_x(meas.) = 15.5784$ . Then, according to Eq. (11), we transfer the coordinates  $(x, p_x)$  into normalized coordinates  $(x_n, p_{xn})$ . Fig. 3 shows the TBT normalized coordinates  $(x_n, p_{xn})$ .

The spectrum of  $x_n$  obtained with a FFT is shown in Fig. (4). The two largest peaks, at tunes 0.305 and 0.39, correspond to  $Q_d$  and  $2Q_d$ , respectively. From the modified FFT, we get  $Q_d(meas.) = 0.694999$ , which is exactly the given AC dipole drive tune. A peak at the betatron tune 1 - 0.685 = 0.315 in Fig. (4) is not observed since the obtained spectrum comes from a driven oscillation at tune  $Q_d$ , after ramp-up to the full strength.

Fig. (5) shows  $2J_x(N)$  turn-by-turn. The small modulations of  $J_x$  are a consequence of the lattice nonlinearities. The spectrum of 2Jx(N) obtained with a FFT is shown in Fig. 6. The peak of  $3Q_d$  is located at tune  $3 \times 0.695 - 3 = 0.085$ , which corresponds to  $h_{30000}$ . Its amplitude is 2 order of magnitude smaller than the amplitude of the average  $J_x$ . Other peaks in Fig. 6 are related to other resonance driving terms.

The measurement results from the simulation are summarized in Tab. 3. With the optics model, according to Eq. (2), The analytical calculation of the driving term and phase from the model are:  $A_{30000}$ (analy.)=6.40 and phase  $\phi_{30000}$ (analy.)= 149.3°, respectively. Comparison with the measurement from the predication gives an relative error for  $A_{30000}$  of 22.5%, and 2.7% for  $\phi_{30000}$ .

quantity	unit	value
time of data analysis	turns	1024
measured $\beta_x$	m	476.077
measured $\alpha_x$		15.5784
measured drive tune $Q_d$ from $x_n$		0.694999
measured tune $3Q_d$ from $2Jx(N)$		0.0849981
average $2J_x$	m.rad	$1.73 \times 10^{-8}$
amplitude $A_{3Q_d}$ from $2J_x(N)$	m.rad	$2.02732 \times 10^{-10}$
amplitude $A_{30000}$		7.84
phase $\phi_{30000}$	degrees	145.32

Table 3: Measurement results from the simulation.

#### 3.2 Measurement with different AC dipole excitation strengths

In the following, we present simulations to evaluate impacts of a number of the AC dipole excitation parameters. In beam experiments, a ramp-up period of 6000 turns was shown to be slow enough to avoid measurable emittance growth. This value was kept constant in the following simulations.

The maximum oscillation amplitude of the beam center is determined by the drive tune  $Q_d$  and kicking strength  $A_d$ . In the beam experiment,  $A_d$  has to be chosen properly. If  $A_d$  is too small, the  $h_{30000}$  measurement will have a large error. If  $A_d$  is too large, the beam lifetime will deteriorate. The maximum kicking strength  $A_d$  is given by the physical and dynamic apertures. The simulation measurement results of  $h_{30000}$  with different AC dipole kick strengths are shown in Tab. 4. In the simulation,  $Q_d = 0.695$ .

$A_d$	$x_{max}$	Average $J_r$	$A_{3O}$ ,	$A_{30000}$	Ø30000
u	mux		from $J_x(N)$ spectrum	measured	measured
$[\mu rad]$	[mm]	[m.rad]	[m.rad]	[]	[degrees]
2.5	1.38	$4.33 \times 10^{-9}$	$2.54 \times 10^{-11}$	7.83	150.6
5	2.87	$1.73 \times 10^{-8}$	$2.03 \times 10^{-10}$	7.84	145.3
10	5.76	$6.89\times10^{-8}$	$1.62\times10^{-9}$	7.87	142.7

Table 4: Measurement results with different AC dipole kick strengths.

#### 3.3 Measurement with different AC dipole drive tunes

The optimum drive tune of the AC dipole was found experimentally to be  $Q_d = Q_x \pm 0.01$ . The maximum oscillation amplitude of the beam center is proportional to  $1/|Q_d - Q_x|$ . The simulation measurement results of  $h_{30000}$  with different AC dipole drive tunes are shown in Tab. 5. In the simulation,  $A_d = 5 \mu$ rad.

From Tab. 5, the measured phase of  $h_{30000}$  has not changed much when the drive tune is chosen between  $Q_x - 0.015$  and  $Q_x + 0.015$ . However, the measured amplitude  $A_{30000}$  at  $Q_d = Q_x + 0.015 = 0.700$  is rather different from that measured with  $Q_d = Q_x \pm 0.01$ .

Table 5: Measurement results with different AC dipole drive tunes.

$Q_d$	$A_{30000}$	$\phi_{30000}$
	measured	measured
[]	[]	[degrees]
0.700	14.31	148.9
0.695	7.84	145.3
0.675	8.72	151.8
0.300	14.31	149.0
0.305	7.89	145.1
0.325	8.72	151.9

#### 3.4 Measurement with different first order chromaticities

The  $h_{30000}$  measurement results, under different first order chromaticity settings, are shown in Tab. 6. The analytical results of  $h_{30000}$  are also listed. Except for the first order chromaticities, other optics and the AC dipole excitation parameters are same as that in Tab. 1 and Tab. 2. From Tab. 6, when slightly changing the first order chromaticities, the phase of  $h_{30000}$  is almost constant while the amplitude of  $h_{30000}$  exhibits tolerable changes.

If the first order chromaticities is varied by several units, the strengths of all the chromatic sextupoles are scaled up or down simultaneously. Since the betatron phase advances between the sextupoles are unchanged, the phase of  $h_{30000}$  doesn't change and the amplitude of  $h_{30000}$  scales up or down.

$(Q_x, Q_y)$	$(\xi_x^{(0)},\!\xi_y^{(0)})$	$A_{30000}$	$\phi_{30000}$	$A_{30000}$	$\phi_{30000}$
		measured	measured	analytical	analytical
[]	[]	[]	[degrees]	[]	[degrees]
(28.685, 29.695)	(0.5, 0.5)	7.79	145.4	6.36	149.3
(28.685, 29.695)	(1.0, 1.0)	7.84	145.3	6.40	149.3
(28.685, 29.695)	(2.0, 2.0)	7.93	145.3	6.48	149.3
(28.685, 29.695)	(3.0, 3.0)	8.03	145.2	6.56	149.3

Table 6: Measurements with different chromaticities.

#### **3.5** Measurement in the vicinity of the $3Q_x$ resonance

The  $h_{30000}$  measurement results in the vicinity of third order resonance line  $Q_x = 2/3$  are shown in Tab. 7. For each case, we only match  $Q_x$  while keeping the sextupole strengths unchanged. These sextupole strengths are calculated at tunes (28.685, 29.695). Therefore the first order chromaticities presented in Tab. 7, at different  $Q_x$  are slightly different from 1. The AC dipole drive tune is always  $Q_x + 0.01$ .

The variations of predicated  $\phi_{30000}$  (obtained from the analytical calculation based on Eq. (2)) for different  $Q_x$  in the vicinity of  $3Q_x$  resonance line, are small, as shown in Tab. 7. And the measured  $\phi_{30000}$ agrees well with its prections, except at  $Q_x = 28.675$  and 28.6725. At these two working points, the difference in  $\phi_{30000}$  between the measurement and the prediction is larger than  $20^{\circ}$ .

The measurements of  $h_{30000}$  in the vicinity of the third order resonance line  $Q_x = 2/3$ , with adjustment of the first order chromaticities are shown in Tab. 8. At each  $Q_x$ , the first order chromaticities are set to  $(\xi_x^{(0)}, \xi_y^{(0)}) = (1, 1)$ . A comparison between the measured and analytically calculated  $h_{30000}$  for the same values of  $Q_x$  with and without the first order chromaticity adjustment, presented in Tab. 7 and Tab. 8, shows a close agreement.

The analytical estimate for  $h_{300000}$  in Eq.(2) is obtained by the first order perturbation theory. However, when  $Q_x$  is close to the  $3Q_x$  resonance and the higher order terms will contribute. Therefore, in the vicinity of the  $3Q_x$  resonance there is a relatively large discrepancy in  $\phi_{30000}$  between the measurement and the prediction as shown in Tab. 7 and Tab. 8.

Table 7: Measurements in the vicinity of  $3Q_x$  without adjustment of first order chromaticities.

$(Q_x, Q_y)$	$(\xi_x^{(0)},\!\xi_y^{(0)})$	$A_{30000}$	$\phi_{30000}$	$A_{30000}$	$\phi_{30000}$
		measured	measured	analytical	analytical
[]	[]	[]	[degrees]	[]	[degrees]
(28.690, 29.695)	(0.66, 0.87)	11.69	147.7	6.46	153.7
(28.685, 29.695)	(1.00, 1.00)	7.84	145.3	6.40	149.3
(28.680, 29.695)	(1.35, 1.12)	12.57	133.0	6.34	146.2
(28.675, 29.695)	(1.72, 1.24)	6.74	113.6	6.29	143.1
(28.6725, 29.695)	(1.90, 1.30)	10.20	165.5	6.26	142.5
(28.670, 29.695)	(2.08, 1.36)	9.73	131.5	6.24	140.1

Table 8: Measurements in the vicinity of  $3Q_x$  with adjustment of first order chromaticities.

$(Q_x, Q_y)$	$(\xi_x^{(0)},\!\xi_y^{(0)})$	$A_{30000}$	$\phi_{30000}$	$A_{30000}$	$\phi_{30000}$
		measured	measured	analytical	analytical
[]	[]	[]	[degrees]	[]	[degrees]
(28.690, 29.695)	(1.00, 1.00)	11.72	147.6	6.48	152.2
(28.685, 29.695)	(1.00, 1.00)	7.84	145.3	6.40	149.3
(28.680, 29.695)	(1.00, 1.00)	12.54	133.0	6.32	146.1
(28.675, 29.695)	(1.00, 1.00)	6.69	113.3	6.24	143.1
(28.6725, 29.695)	(1.00, 1.00)	10.12	165.7	6.21	141.5
(28.670, 29.695)	(1.00, 1.00)	9.63	131.4	6.17	140.1

#### 3.6 Measurement after $h_{30000}$ correction with 12 IR sextupoles

The  $h_{30000}$  correction with the 12 sextupoles in the interaction regions is applied. The correction scheme is given in Ref. [3]. Tab. 9 lists all the first order resonance driving terms before and after the  $h_{30000}$ correction. The correction strengths for these 12 sextupoles are also listed. After correction of the  $h_{30000}$ term, the simulation measurement shows that the amplitude and phase of  $h_{30000}$  are 0.702 and 202.0°, respectively. The spectrum of  $2J_x(N)$  before and after  $h_{30000}$  correction is shown in Fig. 7. The peak of  $3Q_x$  after correction is about one order of magnitude smaller than that without correction. The amplitudes of other peaks have no significant change before and after  $h_{30000}$  correction.

Before correction: (real part, imaginary part)	
$h_{21000}$	(2.07, -3.59)
$h_{30000}$	(-5.50, 3.27)
$h_{10110}$	(9.22, -0.87)
$h_{10020}$	(1.45, 6.85)
$h_{10200}$	(-0.59, 9.61)
After correction: (real part, imaginary part)	
$h_{21000}$	(2.07, -3.59)
$h_{30000}$	(0.00, 0.00)
$h_{10110}$	(9.22, -0.87)
$h_{10020}$	(1.45, 6.85)
$h_{10200}$	(-0.59, 9.61)
Strengths after correction:	
B2M06C3B	0.00051
B2M07C3B	-0.00065
B2M08C3B	-0.00182
B2M09C3B	0.00578
B2M10C3B	0.00389
B2M11C3B	0.00811
B2M12C3B	-0.00833
B2M01C3B	-0.00754
B2M02C3B	-0.00704
B2M03C3B	0.02376
B2M04C3B	0.01704
B2M05C3B	9.5e-05

Table 9: Analytical first order sextupole resonance driving terms before and after correction.

#### 3.7 Local and global observations

The measurement simulation and correction of the  $h_{30000}$  as presented above, was performed only at BPM rbpm.bo6-bh3. But  $h_{30000}$  may vary along the ring. The values of the  $A_{30000}$  obtained by analytical calculation along the ring before and after the correction arre shown in Fig. 8. The  $A_{30000}$  is almost constant in each interaction region (Fig. 8). In the arcs, regular fluctuations in  $A_{30000}$  are seen. These fluctuations indicate that perhaps contributions from arc sextupoles to  $h_{30000}$  do not cancel very well. After the correction,  $A_{30000}$  is smaller in most parts of the ring, especially in the region between the interaction regions IP4 and IP8.

#### 4 Conclusion

We have simulated the measurement of third order resonance driving term  $h_{30000}$  with the AC dipole excitation. The simulations confirm that the AC dipole is a promising tool for the  $h_{30000}$  measurement. The theory of third order resonance driving terms and the BPM data processing algorithms were reviewed. Influence of AC dipole parameters variations, first order chromaticities, and horizontal set tunes  $Q_x$  on the  $h_{30000}$ measurement result has been evaluated. In addition the  $h_{30000}$  driving term was estimated both globally and locally, after correction with 12 IR sextupole correctors.

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## References

- W. Fischer, Beam-beam and BTF, 2006 RHIC Accelerator Physics Experiments Workshop, November 2-3, 2005, BNL.
- [2] Y. Luo, Nonlinear chromaticities and 3Q<sub>x</sub> resonance corrections, 2006 RHIC Accelerator Physics Experiments Workshop, November 2-3, 2005, BNL.
- [3] Y. Luo, J. Bentsson, W. Fischer, and D. Trbojevic, Simulation of proposed on-line third order resonance correction schemes, BNL C-AD AP Note-264, Dec., 2006.
- [4] J. Bengtsson, The sextupole scheme for the Swiss Light Source(SLS): an analytical approach, SLS Note 9/97, March 7, 1997.
- [5] R. Bartolini and F. Schmidt, Normal form via tracking or beam data, Part. Accel. 59, 93(1998).
- [6] F. Schmidt, R. Tomas, *Measurement of drving terms*, Proceedings of the 2001 Particle Accelerator Conference, Chicago, 2001.
- [7] R. Tomas, et al., Measurement of global and local resonance terms, Phys. Rev. ST Accel. Beams, 8, 024001(2005).
- [8] R. Calaga and W. Fischer, private communications, 2006.
- [9] E. Asseo, J. Bengtsson and M. Chanel, Proceedings of 4th European Signal Processing Conference, North Holland, Amsterdam, pp.1317-1320, 1988.
- [10] J. Laskar, Physica D 67, 257 (1993).
- [11] R. Bartolini, A. Bazzani, M. Giovannozzi, W. Scandale and E. Todesco, Precise measurement of the betatron tune, Part. Acc. 55, 247(1996).
- [12] M. Bai, et al., Experimental test of coherent betatron resonance excitation, Phys. Rev. Lett. 56, 6002, 1999.
- [13] M. Bai, et al., RHIC vertical AC dipole commissioning, Proceedings of EPAC 2002, Paris, France.
- [14] J. Bengtsson, private communications, 2006.
- [15] S. Tepikian, private communications, 2006.



Figure 1: Turn-by-turn x data at rbpm.bo6-bh3 during ramp-up and flattop of AC dipole excitation.



Figure 2: Turn-by-turn data  $(x, p_x)$  at rbpm.bo6-bh3 taken at the flattop of AC dipole excitation.



Figure 3: Turn-by-turn normalized data  $(x_n, p_{xn})$  at rbpm.bo6-bh3 taken at the flattop of AC dipole excitation.



Figure 4: Spectrum of  $x_n(N)$  obtained with a FFT.



Figure 5: Turn-by-turn data  $2J_x(N)$  with AC dipole excitation.



Figure 6: Spectrum of  $2J_x(N)$  obtain with a FFT.



Figure 7: Spectrum of  $2J_x(N)$  before and after  $h_{30000}$  correction.



Figure 8: Analytically calculated  $A_{30000}$  along the ring before and after  $h_{30000}$  correction.