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The preparation for the next polarized proton run in the Relativistic Heavy Ion Collider (RHIC) includes on-line third order resonance correction schemes. This is a report on simulations to evaluate several proposed schemes. The nonlinear chromaticities and the first order resonance driving terms are calculated before and after each correction. Each correction scheme use different sextupole families with corresponding parameter optimization. The use of the arc sextupole families versus interaction region sextupoles is discussed.

1 Introduction

To increase the tune space available for beam-beam generated tune spread, in RHIC polarized proton operation, both the nonlinear chromaticity and the third order resonance corrections at the current working point are desirable [1]. At the current working point the fractional tunes are constraint by resonances at tunes of 0.666 and 0.7. The resonance at 0.7 affects both the luminosity lifetime and the polarization.

There are a total of 144 sextupole magnets in the 6 arcs of each RHIC ring. In previous runs, only two families, one focusing and one defocussing, were used for the first order chromaticity correction. In an attempt to correct the third order resonance driving term h_{30000} in Run-6, we split the arc sextupoles into 12 sub-families since as there are a total of 12 arc sextupole power supplies in each ring [2]. This correction scheme allowed us to control the 2 first order chromaticities, and 5 complex first order resonance driving terms (5*2) from sextupoles. This attempt was hampered by the unsuccessful measurement of the h_{30000} driving term using AC dipole.

In the next RHIC run, the number of arc sextupole power supplies is doubled from 12 to 24 to correct the nonlinear chromaticity. There are 4 sextupole power supplies in each arc. In Ref. [3], the nonlinear chromaticity correction scheme with six families was recommended. Later on, S. Tepikian proposed a 8-family scheme for the nonlinear chromaticity correction [4]. In this scheme, each outer or inner arc has 4 sextupole families, and all outer or inner arcs have the same sextupole strength patterns. An on-line nonlinear chromaticity correction scheme with these 8 families, based on the off-momentum tune response matrix, was proposed and will be implemented for the next run [5]. This scheme does not take into account any resonance driving term correction.

It is also possible to use the sextupole correctors in the interaction regions (IRs) to reduce the third order resonance driving terms [6]. These IR correctors were designed to locally correct the multipole field errors from the triplets and separation dipoles [7, 8]. Currently, only the 4 sextupole correctors in IR6 and IR8 have power supplies. During the RHIC Run-6, these 4 sextupole correctors were paired into two families, and adjusted to minimize the beam decay as the horizontal tune of the non-colliding beam was approaching the third order resonance line. However, after bringing the beams into collisions with the found best corrector strengths, no clear lifetime improvement was established. This scheme does not consider the compensation of other driving terms and chromaticities. To be able to better control the sextupole's first order driving terms and first order chromaticities, additional sextupole correctors in the IRs other than IR6 and IR8 are needed.

So far, only the resonance driving term h_{30000} could be measured with coherent turn-by-turn (TBT) beam position monitor (BPM) data. The technique to measure h_{30000} with AC dipole excitation is being established at RHIC. The term h_{30000} can be corrected on line by choosing an appropriate correction scheme using arc sextupole families or IR sextupole correctors. Other systematical driving terms contributed by the arc sextupoles can also be corrected.

We present simulations, carried out to evaluate the proposed correction schemes. The nonlinear chromaticities and the first order resonance driving terms are calculated before and after each correction. Calculations of the dynamic aperture after the corrections will be reported in another note. A discussion on using arc sextupole families versus the IR sextupole correctors is presented.

2 Simulation Procedure

Before each nonlinear correction in the simulation, the nonlinear chromaticities and all first order resonance driving terms from sextupoles are calculated. The chromaticities are numerically calculated with the off-momentum tunes through a polynomial fit:

$$\begin{aligned} Q_z(\delta) &= Q_{z,0} + Q'_z\delta + \tfrac{1}{2}Q''_z\delta^2 + \tfrac{1}{6}Q'''_z\delta^3 + ... \\ &= Q_{z,0} + \xi^{(1)}\delta + \xi^{(2)}\delta^2 + \xi^{(3)}\delta^3 + ..., \end{aligned} \quad (1)$$

$Q_{z,0}$, $z = x, y$, is the on-momentum tune, $\xi_z^{(n)}$ is nth order chromaticity. The first order geometric driving terms are [9]

$$h_{21000} = -\frac{1}{8}\sum_{i=1}^{N}(k_2 dl)_i \beta_{x,i}^{3/2} e^{i\mu_{x,i}}, \quad (2)$$

$$h_{30000} = -\frac{1}{24}\sum_{i=1}^{N}(k_2 dl)_i \beta_{x,i}^{3/2} e^{i3\mu_{x,i}}, \quad (3)$$

$$h_{10110} = +\frac{1}{4}\sum_{i=1}^{N}(k_2 dl)_i \beta_{x,i}^{1/2} \beta_{y,i} e^{i\mu_{x,i}}, \quad (4)$$

$$h_{10020} = +\frac{1}{8}\sum_{i=1}^{N}(k_2 dl)_i \beta_{x,i}^{1/2} \beta_{y,i} e^{i(\mu_{x,i}-2\mu_{y,i})}, \quad (5)$$

$$h_{10200} = +\frac{1}{8}\sum_{i=1}^{N}(k_2 dl)_i \beta_{x,i}^{1/2} \beta_{y,i} e^{i(\mu_{x,i}+2\mu_{y,i})}. \quad (6)$$

They will drive betatron resonances with the following frequencies:

$$Q_x, \quad 3Q_x, \quad Q_x - 2Q_y, \quad Q_x + 2Q_y. \quad (7)$$

Each driving term is a complex number. Therefore, to control all the 5 first order resonance driving terms, at least 10 sextupole families or sextupole correctors are needed.

In the following simulation corrections, the Newton method with singular value decomposition (SVD) [10] is adopted to fit the resonance driving terms and/or nonlinear chromaticities. The response matrix with respect to each sextupole family or corrector is numerically calculated with Tracy-II [11]. Several matching iterations may be needed to converge to the wanted values.

The optics for the simulations is the design for the next polarized proton run. The sextupole strengths are calculated with MAD-X [12] using 2 families only. All other nonlinear corrections start from the 2-family scheme. The multipole field errors in the IRs are not included in the simulation.

Tab. 1 lists the optics parameters for the simulation. Tab. 2 gives chromaticities, first order driving terms, and the strengths of the SFs and SDs without any further nonlinear correction.

Table 1: Beam and optics parameters used for the correction simulation.

quantity	unit	value
energy	GeV	100
$(Q_{x,0}, Q_{y,0})$		(28.685, 29.695)
$(\xi_x^{(1)}, \xi_y^{(1)})$		(2.0, 2.0)
$\beta^*_{x,y}$ at IP6 and IP8	m	0.9
$\beta_{x,y}$ at IP10,IP12,IP2,IP4	m	5.0

Table 2: Calculated first order driving terms, chromaticities, and the arc sextupole strengths before any further nonlinear correction.

quantity	value
Driving terms in (real part, imaginary part):	
h_{21000}	(3.73, 1.88)
h_{30000}	(4.15, 4.96)
h_{10110}	(1.38, 9.27)
h_{10020}	(6.83, -1.84)
h_{10200}	(9.73, 0.09)
Chromaticities:	
$(\xi_x^{(1)},\ \ \xi_y^{(1)})$	(1.85, 1.94)
$(\xi_x^{(2)},\ \ \xi_y^{(2)})$	(222, 2498)
$(\xi_x^{(3)},\ \ \xi_y^{(3)})$	(352090, 16140)
Sextupole Strengths [m^{-2}] :	
SF1	0.240
SD1	-0.449

3 Using arc sextupole families

For a standard FODO cell with 90° phase advance, it is recommended to have four sextupole families [13, 14]. The third order resonance driving term h_{30000} from the arc sextupoles is canceled in the arc due to the phase difference between the sextupoles. The corrections of linear and nonlinear chromaticities is easy. The four sextupoles are powered like (SF+dsf, SD+dsd, SF-dsf, SD-dsd). SF and SD are used for the first order chromaticity correction, dsf and dsd are used to correct the second order chromaticities. However, for the current polarized proton optics, the phase advance per FODO cell is about 80°, and the third order resonance terms are not canceled. In previous runs, RHIC had only 2 sextupole families. The 2-family correction scheme only allows for the first order chromaticity correction. No third order resonance or nonlinear chromaticity corrections are possible.

3.1 Using 12 arc sextupole families

In a Run-6 beam experiment, we had split the 2 sextupole families into 12 sextupole families using the 12 arc sextupole power supplies. In the simulation, we name these sextupole families SF1, SD1, SF2, SD2, SF3, SD3, SF4, SD4, SF5, SD5, SF6, SD6. The number after “SF” and “SD” indicates the arc number counted clockwise from IP6 for the Blue ring. The 12 constraints come from the 2 first order chromaticities $\xi_{x,y}^{(1)}$ and the 5 complex first order driving terms.

We first use these 12 arc sextupole families only to correct the driving term h_{30000}, while keeping the first order chromaticities and other first order driving terms unchanged. Tab. 3 shows the resonance driving terms, chromaticities and sextupole strengths after correction.

Then, we use these 12 arc sextupole families to correct all the first order driving terms while keeping the first order chromaticity unchanged. Tab. 4 shows the resonance driving terms, chromaticities and sextupole strengths after this correction.

In the above corrections, only the first order chromaticities are considered in the correction. If only the driving term h_{30000} is corrected, a comparison of Tab. 3 and Tab. 3 shows that the horizontal second order chromaticity increases by a factor of 8, while the vertical second order chromaticity increases by 57%. Correcting all the first order driving terms, while maintaining the first order chromaticities only, the second and third order chromaticities increase to an unacceptable level. Therefore, to correct the first order driving terms with arc sextupole families, the nonlinear chromaticities must be taken into account.

Since the correction strengths are calculated with the SVD technique, they must be checked after each correction to ensure that they are within their strength limitations. According to Tab. 3, the maximum strength for correcting only the driving term h_{30000} is about 1.0 m^{-2}. From Tab. 4, after all the first order driving terms from arc sextupoles are corrected, most of the sextupole strengths are one order of magnitude larger with the maximum strength of 3.9 m^{-2}. This might reduce the long-term dynamic aperture. From Tab. 3 and 4, both above corrections require sextupole polarity changes.

Table 3: Using 12 arc sextupole families to correct only the h_{30000} driving term, while keeping the first order chromaticities and other first driving terms unchanged.

quantity	value
Driving terms in (real part, imaginary part):	
h_{21000}	(3.73, 1.88)
h_{30000}	(0.00, 0.00)
h_{10110}	(1.38, 9.27)
h_{10020}	(6.83, -1.84)
h_{10200}	(9.73, 0.09)
Chromaticities:	
$(\xi_x^{(1)}, \quad \xi_y^{(1)})$	(1.85, 1.94)
$(\xi_x^{(2)}, \quad \xi_y^{(2)})$	(-1875, 3928)
$(\xi_x^{(3)}, \quad \xi_y^{(3)})$	(839379, 376513)
Strengths after correction [m^{-2}]:	
SF1	-0.169
SD1	-0.656
SF2	0.839
SD2	-0.190
SF3	-0.393
SD3	-1.028
SF4	0.464
SD4	0.251
SF5	0.391
SD5	-0.961
SF6	0.312
SD6	-0.128

Table 4: Using 12 arc sextupole families to correct all first order driving terms while keeping the first order chromaticities unchanged.

quantity	value
Driving terms (real part, imaginary part):	
h_{21000}	(0.00, 0.00)
h_{30000}	(0.00, 0.00)
h_{10110}	(0.00, 0.00)
h_{10020}	(0.00, 0.00)
h_{10200}	(0.00, 0.00)
Chromaticities:	
$(\xi_x^{(1)}, \quad \xi_y^{(1)})$	(1.84, 1.94)
$(\xi_x^{(2)}, \quad \xi_y^{(2)})$	(-117820, 45686)
$(\xi_x^{(3)}, \quad \xi_y^{(3)})$	(-2.062e+07, 278777)
Strengths after correction [m^{-2}]:	
SF1	-2.671
SD1	-0.986
SF2	3.267
SD2	0.457
SF3	-3.871
SD3	-3.015
SF4	3.201
SD4	2.710
SF5	-0.245
SD5	-3.080
SF6	1.785
SD6	1.149

Table 5: Using 8 sextupole families to only correct the nonlinear chromaticities.

quantity	value
Driving terms (real part, imaginary part):	
h_{21000}	(-1.77, 8.98)
h_{30000}	(7.27, 2.70)
h_{10110}	(3.44, 1.51)
h_{10020}	(4.66, -3.23)
h_{10200}	(10.05, -1.13)
Chromaticities:	
$(\xi_x^{(1)}, \ \xi_y^{(1)})$	(2.00, 2.00)
$(\xi_x^{(2)}, \ \xi_y^{(2)})$	(-52, 23)
$(\xi_x^{(3)}, \ \xi_y^{(3)})$	(-18165, 907)
Strengths after correction:	
SFPO	0.418
SDPO	-0.198
SFMO	0.422
SDMO	-0.507
SFPI	0.097
SDPI	-0.547
SFMI	0.023
SDMI	-0.562

3.2 Using 8 arc sextupole families

To correct nonlinear chromaticities with 8 sextupole families, several off-momentum tunes are matched onto their requested values. These values of the off-momentum tunes are calculated with the required first order chromaticities. In the correction scheme, the resonance driving terms are not included in the optimization.

Tab. 5 shows the resonance driving terms, chromaticities and sextupole strengths after the nonlinear chromaticity correction. After the correction, the amplitude of h_{30000} increases by about 20%. The sextupole strengths after correction are within their limitations. From Tab. 5, there is no polarity change due to the nonlinear chromaticity correction. The impact of the nonlinear chromaticity correction on the dynamic apertures will be calculated in another paper.

3.3 Using 24 arc sextupole families

The upgrade of the RHIC sextupole circuits has already been provided. There are 24 arc sextupole power supplies. They allow to correct the nonlinear chromaticities and the first order driving terms simultaneously. In the simulation, we name these arc sextupole families as $\mathrm{SF}_{i,j}$ or $\mathrm{SD}_{i,j}$, $i = 1, 2..., 5, 6$, $j = 1, 2$. $\mathrm{SF}_{i,j}$ or $\mathrm{SD}_{i,j}$ is the jth SF or SD in the ith arc.

Simply combining the above optimizer for the first order driving terms in section 3.1 with the one for the nonlinear chromaticities in section 3.2, there are a total of 18 constraints with 24 variables. Among these 18 constraints, 10 are from the first order sextupole resonance driving terms, and 8 are from off-momentum tunes to control the nonlinear chromaticities.

By carefully adjusting the weights for the nonlinear chromaticities and for the driving terms, we could zero all driving terms and obtain small nonlinear chromaticities simultaneously after 3 or 4 iterations. Tab. 6 shows the nonlinear chromaticities, driving terms, and sextupole strengths after this correction. According to Tab. 6, the maximum sextupole strength is about 1.1 m^{-2}. Note that some sextupole families change their polarities.

Table 6: Using 24 sextupole families to correct both nonlinear chromaticities and all first order resonance driving terms.

quantity	value
Driving terms (real part, imaginary part):	
h_{21000}	(0.00, 0.00)
h_{30000}	(0.00, 0.00)
h_{10110}	(0.00, 0.00)
h_{10020}	(0.00, 0.00)
h_{10200}	(0.00, 0.00)
Chromaticities:	
$(\xi_x^{(1)},\ \ \xi_y^{(1)})$	(2.00, 2.00)
$(\xi_x^{(2)},\ \ \xi_y^{(2)})$	(-10.3, 42.7)
$(\xi_x^{(3)},\ \ \xi_y^{(3)})$	(-2788, 2819)
Strengths after correction:	
SF11	0.664
SD11	-0.088
SF12	0.557
SD12	-0.453
SF21	0.399
SD21	-1.096
SF22	0.182
SD22	-0.988
SF31	-0.046
SD31	0.209
SF32	0.273
SD32	-0.221
SF41	-0.103
SD41	-0.311
SF42	0.347
SD42	-0.784
SF51	0.327
SD51	-0.653
SF52	0.396
SD52	-0.803
SF61	0.145
SD61	-0.176
SF62	-0.239
SD62	-0.083

Table 7: Optical parameters for the 12 IR sextupole correctors.

Name	S[m]	β_x[m]	β_y[m]	Φ_x [2π]	Φ_y [2π]	D_x[m]
B2M06C3B	36.42	524.57	1364.38	0.2416	0.2499	-0.02226
B2M07C3B	603.01	524.57	1364.38	5.0985	4.0894	-0.02226
B2M08C3B	675.87	1414.56	505.63	5.5912	4.5819	0.03167
B2M09C3B	1241.52	266.08	91.77	9.3128	9.6174	0.62640
B2M10C3B	1314.37	98.08	255.10	9.7735	10.0759	-0.37933
B2M11C3B	1880.96	97.00	257.16	14.6684	14.0238	-0.37853
B2M12C3B	1953.82	261.73	93.50	15.1286	14.4829	0.62417
B2M01C3B	2519.46	261.73	93.50	18.8965	19.5514	0.62417
B2M02C3B	2592.32	97.00	257.16	19.3566	20.0105	-0.37853
B2M03C3B	3158.91	98.08	255.10	24.2515	23.9583	-0.37933
B2M04C3B	3231.77	266.08	91.77	24.7122	24.4169	0.62640
B2M05C3B	3797.41	1414.56	505.63	28.4338	29.4524	0.03167

Table 8: Using 4 IR sextupole correctors in IR6 and IR8 to compensate h_{30000} only.

quantity	value
Driving terms (real part, imaginary part):	
h_{21000}	(-34.73, 58.65)
h_{30000}	(-0.00, -0.00)
h_{10110}	(188.44, -119.17)
h_{10020}	(120.97, 69.24)
h_{10200}	(3.21, 215.21)
Chromaticities:	
$(\xi_x^{(1)}, \xi_y^{(1)})$	(1.85, 1.94)
$(\xi_x^{(2)}, \xi_y^{(2)})$	(221, 2513)
$(\xi_x^{(3)}, \xi_y^{(3)})$	(352173, 12236)
Strengths after correction:	
B2M05C3B	-0.005903
B2M06C3B	-0.031119
B2M07C3B	0.031119
B2M08C3B	0.005903

4 Using IR sextupole correctors

There are three multipole corrector packages in each triplet in the RHIC rings to correct locally the multipole field errors from the triplet quadrupoles and separation dipoles. On either side of the triplet, there is one sextupole corrector, two in each IR. In the simulation these correctors are named B2M06C3B, B2M07C3B, ..., B2M04C3B, B2M05C3B. Currently, only the sextupole correctors in the interaction regions IR6 and IR8 are connected to the (bi-polar) power supplies. Tab. 7 lists the β-functions and phase advances from IP6. From Tab. 7, the horizontal phase advances between the two sextupole correctors in one IR is almost π.

4.1 Using 4 IR sextupole correctors in IR6 and IR8

Previously in RHIC the 4 sextupole correctors in IR6 and IR8 to attempt the compensation of the h_{30000} driving term. B2M05C3B and B2M06C3B, B2M07C3B and B2M08C3B were paired into two knobs. The two sextupole correctors in each pair were knobed with the same value but different sign, to reduce the beam decay, while moving the horizontal tune of the non-colliding beam to the third order resonance line. From Tab. 7, the dispersion at the two correctors in each pair are different from each other. And the dispersion in IRs other than IR6 and IR8 are relatively large.

In the simulation, we use the 4 sextupole correctors in IR6 and IR8 to correct the h_{30000} resonance driving term while keeping the first order chromaticities unchanged. The 4 constraints are two first order chromaticities and the real and the imaginary parts of h_{30000}. Tab. 8 shows the nonlinear chromaticities, driving terms, and sextupole strengths after this correction.

Comparing Tab. 2 and Tab. 8, this correction does not affect the nonlinear chromaticities. However, only h_{30000} can be corrected with these 4 correctors, and the other first order driving terms increase significantly.

From Tab. 8, the correction strengths for B2M05C3B and B2M08C3B have the same value and opposite sign. The same is true for B2M06C3B and B2M07C3B. This can be explained with their dispersion as shown in Tab. 7. Since the dispersion at B2M05C3B and B2M08C3B, and that at B2M06C3B and B2M07C3B are same, to keep the first order chromaticities unchanged, the correction strengths for B2M05C3B and B2M08C3B, and that for B2M06C3B and B2M07C3B should have the same value and opposite sign. Therefore, to use these 4 correctors, it is better to pair B2M05C3B and B2M08C3B, and B2M06C3B and B2M07C3B.

4.2 Using 12 IR sextupole correctors

We now use all 12 IR sextupole correctors to correct all first order driving terms while keeping the first order chromaticities unchanged. In this correction, the 12 constraints are the real and imaginary parts of the 5 first order driving terms and the two first order chromaticities. The first order chromaticities must be included to keep the nonlinear chromaticity changes smaller.

Table 9: Using 12 IR sextupole correctors in IR6 and IR8 to zero h_{30000} only, while keeping first order chromaticities unchanged.

quantity	value
Driving terms in (real part, imaginary part):	
h_{21000}	(3.73, 1.88)
h_{30000}	(0.00, 0.00)
h_{10110}	(1.38, 9.27)
h_{10020}	(6.83, -1.84)
h_{10200}	(9.73, 0.09)
Chromaticities:	
$(\xi_x^{(1)},\ \xi_y^{(1)})$	(1.85, 1.94)
$(\xi_x^{(2)},\ \xi_y^{(2)})$	(246, 2494)
$(\xi_x^{(3)},\ \xi_y^{(3)})$	(355613, 15981)
Strengths after correction:	
B2M06C3B	0.002674
B2M07C3B	-0.002473
B2M08C3B	-0.000865
B2M09C3B	0.021625
B2M10C3B	0.14055
B2M11C3B	-0.154867
B2M12C3B	-0.020450
B2M01C3B	-0.006598
B2M02C3B	0.120365
B2M03C3B	-0.106483
B2M04C3B	0.005596
B2M05C3B	-0.001755

Table 10: Using 12 IR sextupole correctors in all IRs to zero all first order driving terms while keeping first order chromaticities unchanged.

quantity	value
Driving terms in (real part, imaginary part):	
h_{21000}	(0.00, 0.00)
h_{30000}	(0.00, 0.00)
h_{10110}	(0.00, 0.00)
h_{10020}	(0.00, 0.00)
h_{10200}	(0.00, 0.00)
Chromaticities:	
$(\xi_x^{(1)},\ \xi_y^{(1)})$	(1.85, 1.94)
$(\xi_x^{(2)},\ \xi_y^{(2)})$	(111, 2336)
$(\xi_x^{(3)},\ \xi_y^{(3)})$	(365151, 55943)
Strengths after correction:	
B2M06C3B	0.015036
B2M07C3B	-0.007325
B2M08C3B	0.005747
B2M09C3B	0.099879
B2M10C3B	0.826607
B2M11C3B	-0.891083
B2M12C3B	-0.052841
B2M01C3B	0.011937
B2M02C3B	0.820689
B2M03C3B	-0.759048
B2M04C3B	-0.058393
B2M05C3B	-0.010571

First, only the driving term h_{30000} is corrected while keeping first order chromaticities and other driving terms unchanged. Tab. 9 shows the nonlinear chromaticities, driving terms, and sextupole strengths after this correction. Comparing Tab. 2 and Tab. 9, the nonlinear chromaticities change only slightly after the correction. The maximum correction strength is $(k_2L) = 0.16$ m^{-2}.

Second, all the first order driving terms are corrected while keeping the first order chromaticities unchanged. Tab. 10 shows the nonlinear chromaticities, driving terms, and sextupole strengths after this correction. From Tab. 10, the nonlinear chromaticities changes due to this correction are acceptable. The maximum correction strength is $(k_2L) = 0.90$ m^{-2}. This is about six times larger than the value obtained by correcting only the h_{30000} driving term.

5 Discussion

The above third order resonance correction schemes are sorted into two categories, one using arc sextupole families, the other using the IR sextupole correctors.

With 12 or 24 arc sextupole families for the third order resonance driving term correction and/or the nonlinear chromaticity correction, we find that the sextupole strengths increase significantly and the polarities of some sextupole families are inverted. The large correction strengths come from the driving term contributions from individual sextupoles in one family canceling each other. Therefore, the net contribution from one family to the global driving term is small. The irregular contribution angles of the sextupole families to the third order resonance driving term may be another reason for the large correction strengths. Using arc sextupole families with large strengths to correct a modest third order resonance driving term is not efficient. Fig. 1 shows the contributions to h_{30000} from the 144 arc sextupoles. Fig. 2 shows the contributions to h_{30000} from the 24 arc sextupoles in the first arc. Fig. 3 shows the contributions to h_{30000} from the 24 arc sextupole families.

To correct both first order resonance driving terms and the nonlinear chromaticities, we further split the sextupole families into 24. However, too many knobs will make the on-line correction procedure complicated and therefore less robust. With the 24 arc sextupole families, even in our simulations more than 3 iterations were needed to obtain the wanted first order driving terms and nonlinear chromaticities. In addition, so far we are only able to measure the third order resonance driving term h_{30000}. To measure other driving terms is difficult at present.

The merit of using IR sextupole correctors is to separate the nonlinear chromaticity and third order resonance driving term corrections. The dispersion in the IRs is much smaller than that in the arcs, and simulations show that with 4 or 12 IR sextupoles h_{30000} can be corrected while maintaining the linear chromaticity, and without large changes in the nonlinear chromaticity. Using single IR correctors instead of arc sextupole families to correct the driving term is more efficient.

Therefore, in operation, we could first correct the nonlinear chromaticities with the 8-family correction scheme. Then, we use the IR sextupole correctors to compensate the third order resonance driving term h_{30000}, based on its measurement with the AC dipole, or by scanning the sextupole strength while observing the beam decay.

Using the existing 4 IR sextupole correctors to correct only the h_{30000} driving term, the simulation shows the other resonance driving terms will increase. To fully control all driving terms and the linear and the nonlinear chromaticities, all 12 IR sextupole correctors should be used. More detailed studies have to be carried out to check the robustness of this correction, before adding power supplies to the 8 IR sextupole correctors currently not powered.

We used only the first order resonance driving terms and nonlinear chromaticities in the proposed schemes. Dynamic aperture calculation should be carried out to assess the full impact of these correction schemes.

6 Conclusion

Simulations were carried out to test several proposed on-line third order resonance correction schemes at store, for the next polarized proton run of RHIC. The nonlinear chromaticities and the first order resonance driving terms are calculated before and after each correction. Based on the simulations, we suggest to use 12 IR sextupole correctors for the third order resonance correction, and to leave the arc sextupole families for the nonlinear chromaticity correction. More detailed studies to calculate the dynamic apertures after the nonlinear corrections, and to check the robustness of correction schemes are under way.

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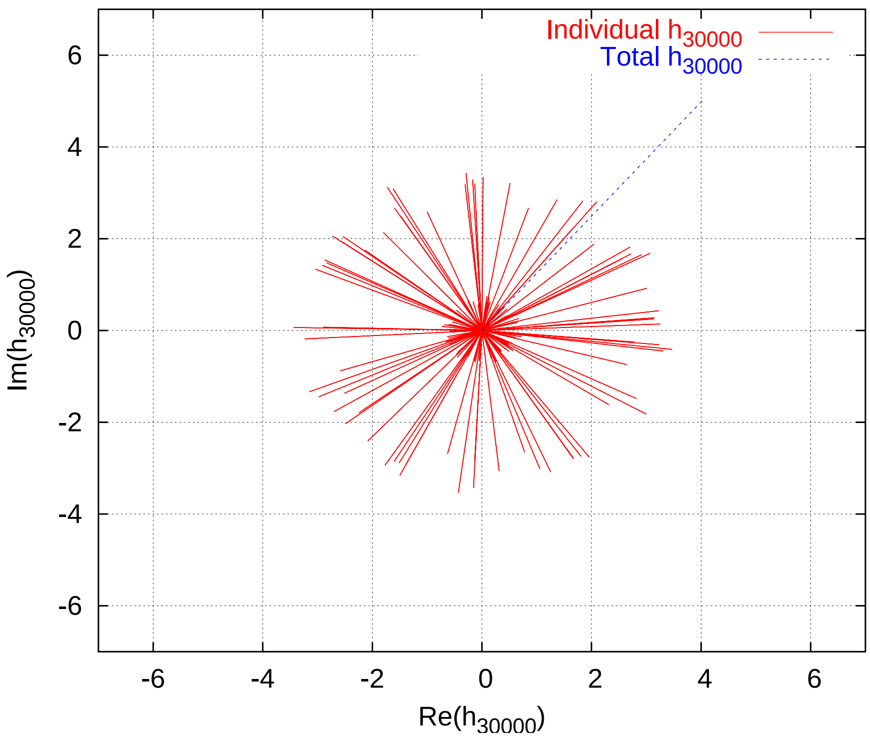

Figure 1: Contributions to h_{30000} from individual arc sextupoles in one ring. The blue line is the total h_{30000} calculated at IP6.

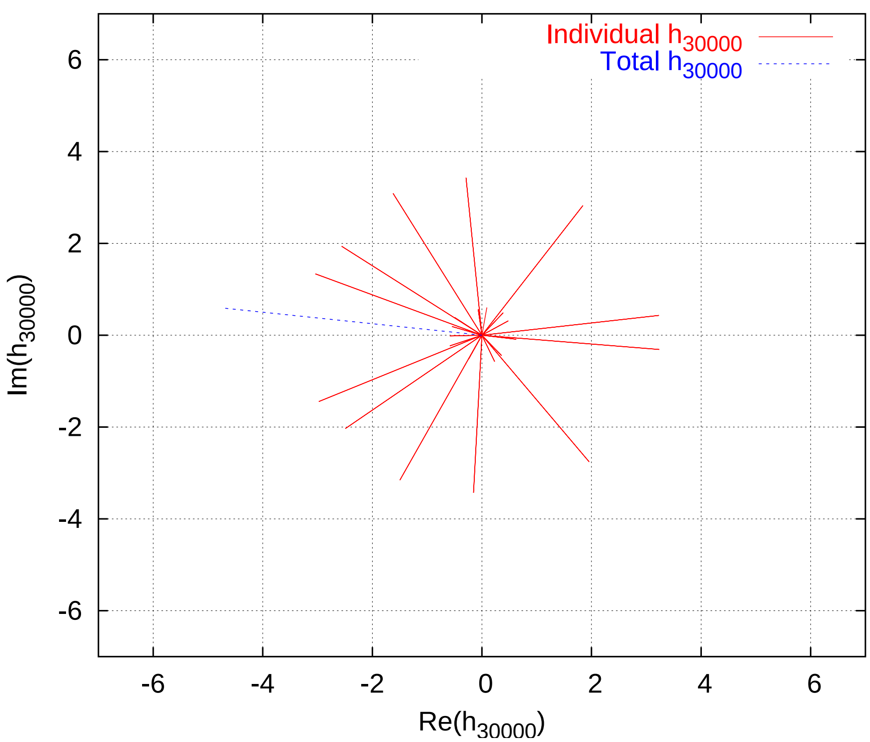

Figure 2: Contributions to h_{30000} from individual arc sextupoles in the first arc. The blue line is the total contribution to h_{30000} from all sextupoles in the first arc.

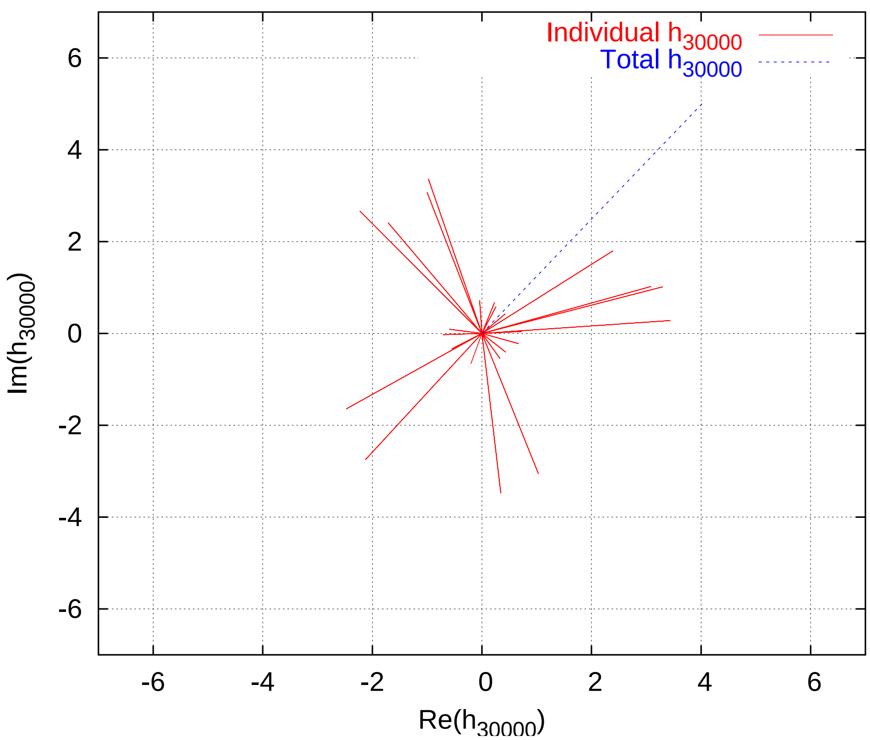

Figure 3: Contributions to h_{30000} from the 24 families in one ring. The blue line is total h_{30000} calculated at IP6.