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Online nonlinear chromaticity correction using off-momentum tune response matrix

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In this note, we propose a method for the online nonlinear chromaticity correction at store in the Relativistic Heavy Ion Collider (RHIC). With 8 arc sextupole families in each RHIC ring, the nonlinear chromaticities can be minimized online by matching the off-momentum tunes onto the wanted tunes with linear chromaticity only. The Newton method is used for this multi-dimensional nonlinear optimization, where the off-momentum tune response matrix with respect to sextupole strength changes is adopted to simplify and fasten the online optimization process. The off-momentum tune response matrix can be calculated with the online accelerator optics model or directly measured with the real beam. The correction algorithm for the RHIC is given. Simulations are also carried out to verify the method. Issues of its online implementation is discussed.

1 General Description

There are a total of 144 arc sextupole magnets in each RHIC ring. In previous runs, 12 focusing sextupoles (SFs) and 12 defocusing sextupoles (SDs) in each of the 6 arcs were powered by one SF and one SD power supply, respectively. All SF power supplies, and all SD power supplies were operated at the same current. This 2 sextupole family correction scheme does not allow for the nonlinear chromaticity correction.

In the coming run (Run-7), the number of the arc sextupole power supplies will be doubled, that is, from 12 to 24 in each RHIC ring. In Ref. [1], 6 sextupole families in each ring were recommended for the second order chromaticity correction. In 2005, Tepikian proposed a 8-family scheme for the nonlinear chromaticity correction [2]. In this scheme, each outer or inner arc has 4 sextupole families, and all outer or inner arcs have the same sextupole strength patterns. The names for the 8 sextupole families are SFPO, SDPO, SFMO, SDMO, SFPI, SDPI, SFMI, SDMI.

To correct the first and second order chromaticities online, a simple, fast, and robust correction scheme is required. The nonlinear chromaticity matching with MAD [3], or other codes based on the online accelerator optics model may be too slow for an online application.

Here, instead of directly correcting the second order chromaticities online, we propose online matching several off-momentum tunes onto their wanted values. The wanted off-momentum tunes are calculated using the first order chromaticities only. By doing so, all higher order chromaticities will be minimized simultaneously. In addition, the values of all tune deviations used in the matching are of the same order of magnitude, which avoids the need for weights and makes the minimization process more robust.

The off-momentum tune matching is a problem of multidimensional nonlinear optimization [4]. In the following, we will use the Newton method with singular value decomposition to find the correction strengths for the 8 sextupole families. The off-momentum tune response matrix can be calculated with the online accelerator optics model, or directly measured with beam using a high resolution phase-locked-loop (PLL) tune meter.

In the following, we first give the algorithm for the RHIC online off-momentum tune matching. Then, we present simulations carried out to verify the nonlinear chromaticity minimization. Issues of its online implementation are discussed in detail.

2 Correction Algorithm

For simplicity, we give an example of how to match online 8 off-momentum tunes with the above 8 sextupole families. The 8 off-momentum tunes can be taken at 4 relative momentum deviations, say, $\delta_{1,2}$ and $-\delta_{1,2}$,

$$\begin{aligned} \mathbf{q}^T &= (q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8)^T \\ &= (q_x(-\delta_1), q_y(-\delta_1), q_x(-\delta_2), q_y(-\delta_2), q_x(\delta_1), q_y(\delta_1), q_x(\delta_2), q_y(\delta_2))^T. \end{aligned} \quad (1)$$

The measured off-momentum tunes $\mathbf{q}_{meas}$ are

$$\mathbf{q}^T_{meas} = (q_{1,meas}, q_{2,meas}, q_{3,meas}, q_{4,meas}, q_{5,meas}, q_{6,meas}, q_{7,meas}, q_{8,meas})^T. \quad (2)$$

Given the wanted first order chromaticities $Q'_{x,y}$, the wanted off-momentum tunes $\mathbf{q}_{want}$ are

$$\begin{aligned} \mathbf{q}^T_{want} &= (q_{1,want}, q_{2,want}, q_{3,want}, q_{4,want}, q_{5,want}, q_{6,want}, q_{7,want}, q_{8,want})^T \\ &= (-Q'_x\delta_1, -Q'_y\delta_1, -Q'_x\delta_2, -Q'_y\delta_2, Q'_x\delta_1, Q'_y\delta_1, Q'_x\delta_2, Q'_y\delta_2)^T. \end{aligned} \quad (3)$$

As discussed above, to minimize nonlinear chromaticities, we match online the measured and uncorrected off-momentum tunes $\mathbf{q}_{meas}$ onto the wanted tunes $\mathbf{q}_{want}$. The optimization error function is defined as

$$\chi^2 = |\mathbf{\Delta q}^T|^2 = |\mathbf{q}^T_{meas} - \mathbf{q}^T_{want}|^2 = \sum_{i=1}^{8}(q_{i,meas} - q_{i,want})^2, \quad (4)$$

where we have defined the distance of the measured off-momentum tunes to the wanted ones,

$$\Delta\mathbf{q}^T = (dq_1, dq_2, dq_3, dq_4, dq_5, dq_6, dq_7, dq_8)^T = \mathbf{q}^T_{meas} - \mathbf{q}^T_{want}. \quad (5)$$

The above off-momentum tune matching is a problem of multidimensional nonlinear optimization. In the following, we will use the Newton method to minimize the error function χ^2. In each iteration, we solve the following linear equation with the singular value decomposition (SVD) technique,

$$\mathbf{A}\Delta\mathbf{x} = \Delta\mathbf{q}, \quad (6)$$

where $\Delta\mathbf{x}$ is the correction strengths,

$$\Delta\mathbf{x}^T = (\Delta(k_2l)_1, \Delta(k_2l)_2, \Delta(k_2l)_3, \Delta(k_2l)_4, \Delta(k_2l)_5, \Delta(k_2l)_6, \Delta(k_2l)_7, \Delta(k_2l)_8)^T. \quad (7)$$

$\mathbf{A}$ is the off-momentum tune response matrix with respect to sextupole family strength changes,

$$\mathbf{A}_{i,j} = \frac{\partial q_i}{\partial(k_2l)_j}, i = 1, 2, ..., nconstr, j = 1, 2, ..., nvari. \quad (8)$$

Note that the tunes q_i are the off-momentum horizontal or vertical tune as defined in Eq. (1). Therefore, $\mathbf{A}$ is momentum dependent. Here the number of matching constraints is $nconstr = 8$, the number of matching variables is $nvari = 8$. We will see in the following that it is not required to have $nconstr = nvari$. $\mathbf{A}$ can be numerically calculated with the offline accelerator optics model, or directly measured with the beam.

3 Offline Simulation

Simulations are carried out to confirm the above nonlinear chromaticity correction algorithm. The store optics for the next polarized proton run at 100 GeV is used. The on-momentum tunes are set to $(Q_{x,0}, Q_{y,0}) = (28.685, 29.695)$. The β-functions at the collision points IP6 and IP8 are 0.9 m. At the other interaction points, the β-functions are set to 5m.

First, with a 2-family correction scheme, we match the first order chromaticities to +2. The red lines in Fig. 1 and Fig. 2 show the off-momentum tunes, respectively. Tab. 1 shows the chromaticities up to third order. With the 2-family correction scheme at least the vertical second order chromaticity is not acceptable.

Then, we use the correction algorithm outlined in Sec. 2 to match the 8 off-momentum tunes to their wanted values, which are calculated with the wanted first order chromaticities $Q'_{x,y} = 2$. In the simulation, we choose $\delta_1 = 0.0007$ and $\delta_2 = 0.0003$. The off-momentum tune response matrix is calculated numerically with the optics model and the Tracy-II [5] code.

The green lines in Fig. 1 and Fig. 2 show the off-momentum horizontal and vertical tunes after the first correction iteration, respectively. From The vertical second order chromaticity is reduced by factor of 6 (Tab. 1). The horizontal second order chromaticity, however, becomes larger by a factor of 3. After the second iteration, both the horizontal and vertical second order charomaticities are below 60. The third order chromaticities are also reduced by a factor of 20, comparing with those of the 2-family correction scheme. The blue lines in Fig. 1 and Fig. 2 show the off-momentum tunes after the second iteration. Tab. 1 summarizes the linear and nonlinear chromaticities in the above simulation.

Table 1: Chromaticities in the chromaticity correction simulations

Condition	Q'_x	Q'_y	$\frac{1}{2}Q''_x$	$\frac{1}{2}Q''_y$	$\frac{1}{6}Q'''_x$	$\frac{1}{6}Q'''_y$
2 families	1.85	1.93	206	2440	355730	19557
8 families: 1 iteration	2.00	2.00	-641	467	-103455	14453
8 families: 2 iterations	2.00	2.00	-55	24	-18660	934

4 Online implementation

To apply the above nonlinear chromaticity minimization online, the off-momentum tunes can be obtained either from the model or from beam measurements using a high resolution tune measurement system. Tab. 2 shows the distances of 8 off-momentum tunes to their wanted values in the above simulation. Tab. 3 lists the off-momentum tune changes during the numerical calculation of $\mathbf{A}$. To numerically calculate $\mathbf{A}$, we change each sextupole family's strength by $\Delta(k_2 dl) = 0.03\text{m}^{-2}$, about $5-10\%$ of its typical strength. According to Tab. 2 and Tab. 3, the tune measurement resolution should be better than 10^{-4}.

To mitigate the requirements on the tune measurement system, we can select to match off-momentum tunes at higher δs. For example, we can only match the off-momentum tunes at $\delta = \pm 0.0007$, which reduces the number of matching constraints from 8 to 4. Simulations show that this does not change the matching result significantly. Fig. 3 and Fig. 4 show the off-momentum tunes after one correction iteration with 8 and 4 matching constraints.

Table 2: Distances of off-momentum tunes to their wanted values in the above simulation (in unit of $\times 10^{-3}$).

Condition	dq_1	dq_2	dq_3	dq_4	dq_5	dq_6	dq_7	dq_8
2 families	-0.074	- 1.234	-0.052	-0.240	0.015	-0.198	-0.114	-1.147
8 families: 1 iteration	0.276	-0.216	0.054	-0.041	0.060	-0.042	0.348	-0.226
8 families: 2 iterations	0.018	-0.003	0.004	-0.001	0.005	-0.001	0.031	-0.004

Table 3: Changes of off-momentum tunes during numerical calculation of $\mathbf{A}$ (in unit of $\times 10^{-3}$). The change in each sextupole family strength is $\Delta(k_2 dl) = 0.03\text{m}^{-2}$.

Family	dq_1	dq_2	dq_3	dq_4	dq_5	dq_6	dq_7	dq_8
1	-2.3140	0.4888	-0.9886	0.2073	0.9908	-0.2057	2.3240	-0.4803
2	-0.2876	1.1248	-0.1183	0.5139	0.1124	-0.5647	0.2557	-1.4011
3	-2.3616	0.4765	-0.9954	0.2072	0.9788	-0.2134	2.2697	-0.5102
4	-0.2705	1.2724	-0.1181	0.5090	0.1215	-0.4606	0.2887	-1.0078
5	-2.2084	0.4559	-0.9960	0.2044	1.0733	-0.2181	2.6264	-0.5299
6	-0.2828	1.2169	-0.1198	0.5185	0.1190	-0.5161	0.2785	-1.2025
7	-2.4223	0.5067	-0.9734	0.2091	0.8972	-0.1986	2.0023	-0.4496
8	-0.2825	1.2103	-0.1211	0.5176	0.1223	-0.5184	0.2888	-1.2132

Since Eq. (5) is solved using the SVD technique, one must carefully choose the cut to zero the small singular values, which would yield large correction strengths. Before sending the correction strength to the real machine, it is required to check all the correction strengths to make sure they do not exceed the sextupole strength limitation. The maximum sextupole integral strength at 100 GeV is $(k_2dl)_{max} = 6.33\text{m}^{-2}$ [6]. Tab. 4 shows the sextupole strengths in the above simulation. We find that the polarity of $(k_2dl)_7$ changed in the first correction iteration. Fortunately, its value is small.

Table 4: The strengths of the 8 sextupole families in the above simulation.

Condition	$(k_2dl)_1$	$(k_2dl)_2$	$(k_2dl)_3$	$(k_2dl)_4$	$(k_2dl)_5$	$(k_2dl)_6$	$(k_2dl)_7$	$(k_2dl)_8$
2 families	0.241	-0.448	0.241	-0.448	0.241	-0.448	0.241	-0.448
8 families: 1 iteration	0.441	-0.315	0.507	-0.552	0.034	-0.471	-0.018	-0.468
8 families: 2 iterations	0.418	-0.193	0.420	-0.497	0.099	-0.553	0.026	-0.564

In operation, it is not required to calculate or to measure the off-momentum tune response matrix each time when a nonlinear chromaticity correction is required. We need to measure **A** only when the correction using the online optics model does not work well. Even during in the correction iterations, the same matrix **A** also can be used.

5 Conclusion

We have proposed a method for the online nonlinear chromaticity correction using the off-momentum tune response matrix with respect to the sextupole familiy strengths. The sextupole family correction changes are calculated with the Newton method. Simulations verified the correction algorithm. Issues of the method's online implementation were discussed.

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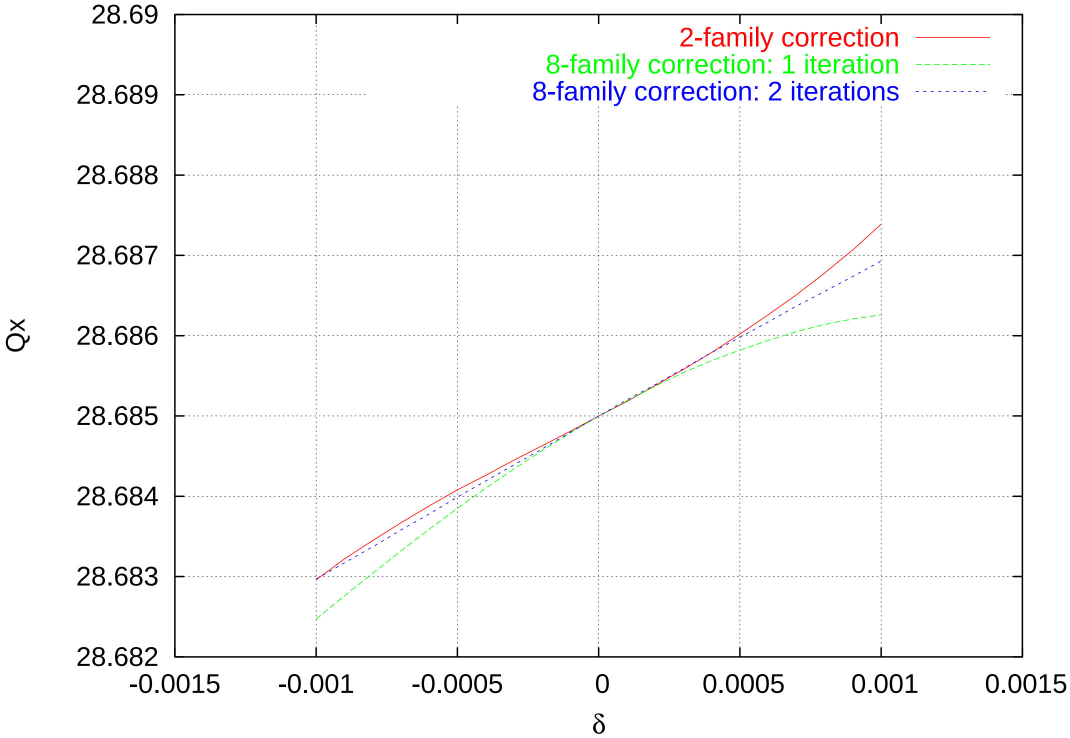

Figure 1: The horizontal off-momentum tunes in the chromaticity correction simulations.

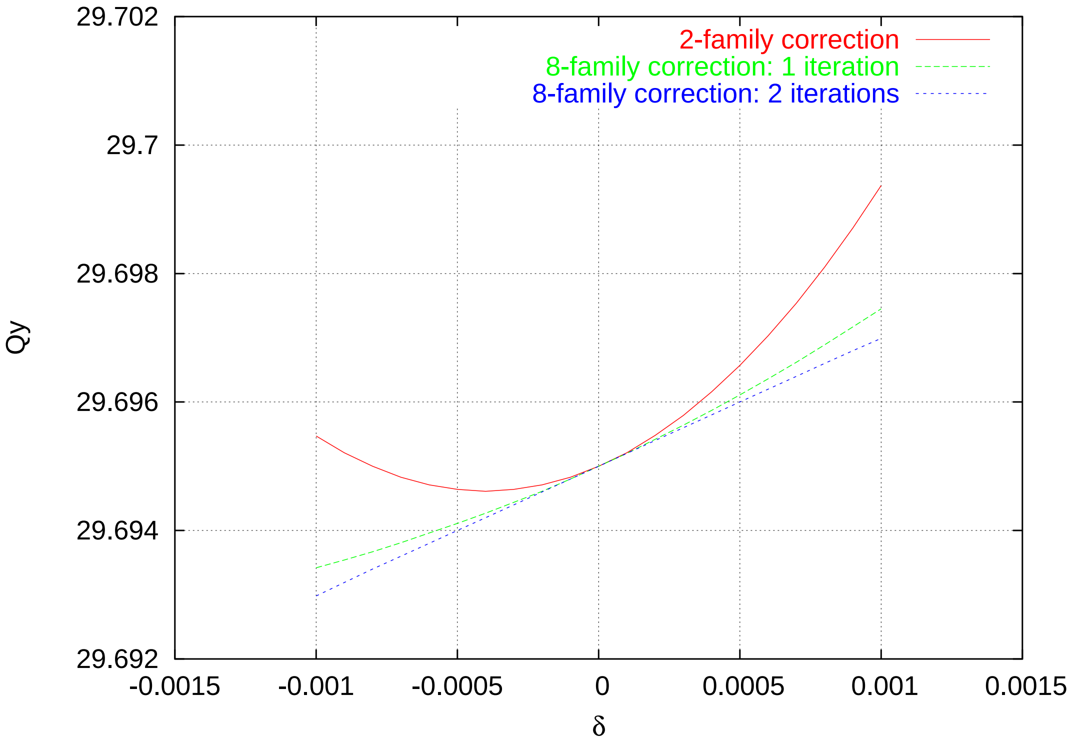

Figure 2: The vertical off-momentum tunes in the chromaticity correction simulations.

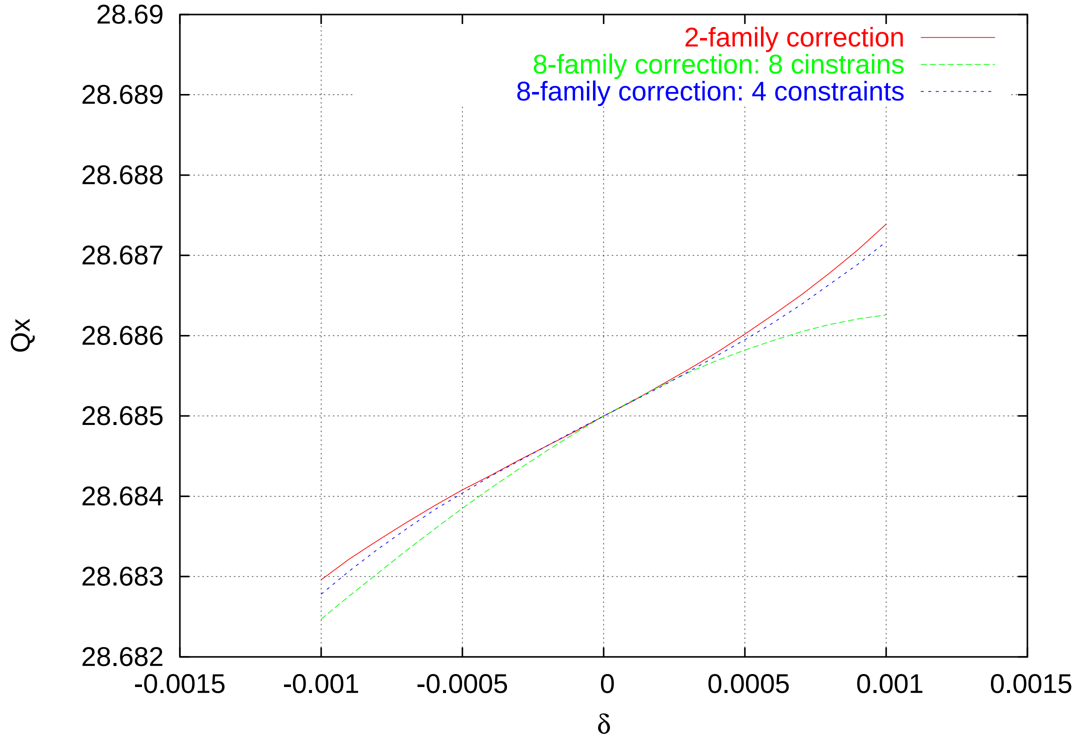

Figure 3: The horizontal off momentum tunes after one correction iteration with 8 and 4 matching constraints.

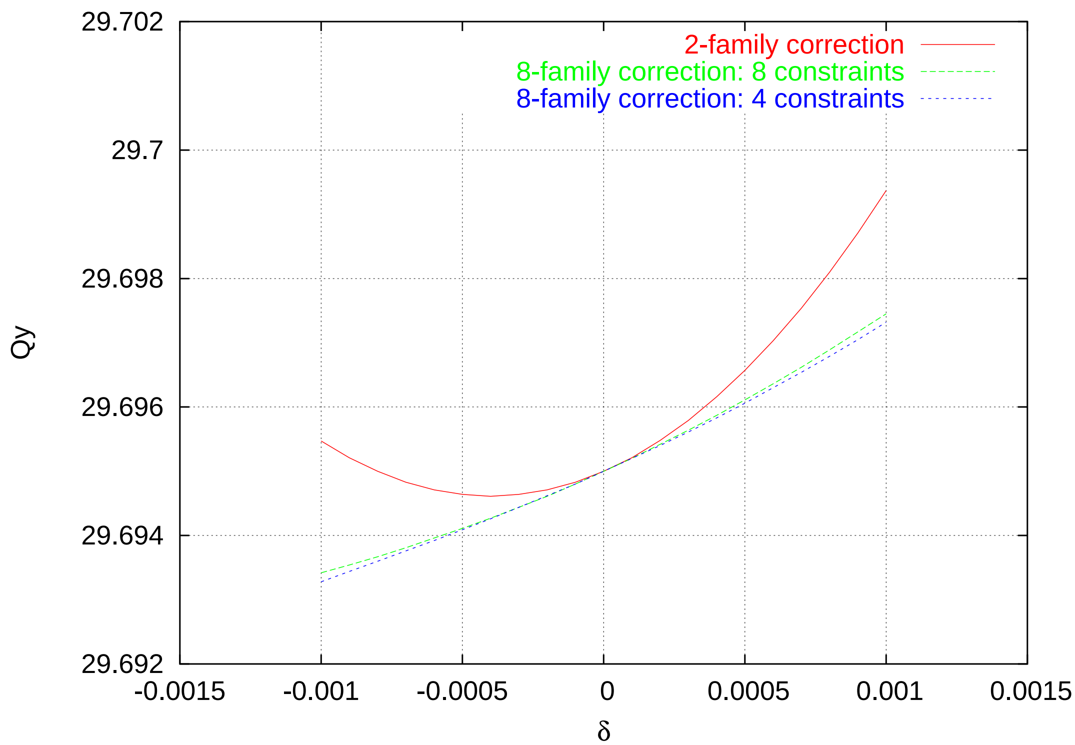

Figure 4: The vertical off momentum tunes after one correction iteration with 8 and 4 matching constraints.