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Over the course of several months, beginning in October 2002 and ending in July 2003, a number of different ions (D^{+}, C^{6+}, Si^{5+}, Ti^{18+}, Fe^{10+}, Fe^{20+}, and Au^{32+}) from Tandem were injected and accelerated in Booster. These were injected at several different magnetic rigidities and accelerated to various energies. For each ion the revolution frequency was measured at injection and top energy. The magnetic field at injection was measured with the Booster Hall probe and Gauss clock. The field in TTB transport line dipole 11DH1 was recorded for each ion. (This dipole is the first of two 90° bends in the TTB line. It is used to select the momentum of ions to be transported to Booster. Its field is monitored by an NMR probe.) The C3 electrostatic inflector setpoint was recorded as was the BMM (Booster Main Magnet) field setpoint at extraction.

Assuming a given equilibrium orbit radius, the frequency measurements give the momentum and magnetic rigidity of each ion. (We do not yet have a working orbit acquisition system, so the orbit radius cannot be measured.) These data are used to obtain the relation between the ion momentum (or rigidity) and the measured or set fields. The assumed orbit radius also gives the radius of curvature in the Booster dipoles. This together with the magnetic rigidity gives a value for the field in the dipoles. The ion momentum also gives a value for the voltage required between the cathode and septum of the inflector.

Having the relations between ion momentum and the measured fields is extremely valuable. These allow one to obtain the ion momentum from the measured field in the 11DH1 dipole and then to verify that the measured field in Booster at injection has the correct value for this momentum. This means that one does not have to search for the correct injection field to get beam going around the machine. Similarly the relation between the ion momentum and the inflector setpoint allows one to set the inflector voltage to the correct value for a given ion momentum. The relation between the ion momentum and magnetic field setpoint at extraction allows one to set the extraction field to the correct value for the desired momentum.

In the following, we report on the frequency and field measurements and the resulting relations between ion momentum, measured and set fields, and inflector setpoint. The necessary formulae are given in Sections 1 and 2. The measurements and resulting relations are presented in Sections 3 through 6.

1 Formulae for Rigidity and Field in Terms of Frequency and Radius

Consider a particle of mass m moving along an orbit of radius R with velocity v. In terms of the radius and the revolution frequency, f, the velocity is

$$v = 2\pi R f. \tag{1}$$

This gives momentum, energy, and kinetic energy

$$p = mc\beta\gamma, \quad E = mc^2\gamma = \sqrt{c^2p^2 + m^2c^4}, \quad W = E - mc^2 \tag{2}$$

where

$$\beta = v/c, \quad \gamma = 1/\sqrt{1-\beta^2}. \tag{3}$$

Let df and dR be the errors in the frequency and radius measurements. Then the error in v is given by

$$\frac{dv}{v} = \frac{df}{f} + \frac{dR}{R}. \tag{4}$$

Differentiating p with respect to v we obtain

$$\frac{dp}{dv} = m\gamma^3 = \frac{p}{v}\gamma^2 \tag{5}$$

and therefore

$$\frac{dp}{p} = \gamma^2\frac{dv}{v} = \gamma^2\left\{\frac{df}{f} + \frac{dR}{R}\right\} \tag{6}$$

which gives the error in momentum. Note that the fractional error dp/p is proportional to γ^2 which is large for relativistic particles but is of order one for nonrelativistic particles.

The magnetic rigidity of the particle in units of Tm is

$$B\rho = kp/Q \tag{7}$$

where Q is the particle charge in units of the proton charge, p is the momentum in units of GeV/c, and $k = 10^9/299792458$. The radius of curvature is

$$\rho = \rho_0 (R/R_0)^{1/\alpha} \tag{8}$$

where

$$\rho_0 = 13.8656 \text{ m}, \quad R_0 = 201.78/(2\pi) \text{ m} \tag{9}$$

and

$$1/\alpha = \gamma_t^2, \quad \gamma_t = 4.806. \tag{10}$$

Here ρ_0 is the nominal radius of curvature in the Booster dipoles, R_0 is the nominal radius, and γ_t is the transition gamma. Equation (8) is derived in the Appendix. The errors in $B\rho$ and ρ due to the frequency and radius errors are given by

$$\frac{d(B\rho)}{B\rho} = \frac{dp}{p} = \gamma^2 \left\{ \frac{df}{f} + \frac{dR}{R} \right\} \tag{11}$$

and

$$\frac{d\rho}{\rho} = \gamma_t^2 \frac{dR}{R}. \tag{12}$$

Having obtained $B\rho$ and ρ, the magnetic field is

$$B = (B\rho)/\rho \tag{13}$$

and the error in B is given by

$$\frac{dB}{B} = \frac{d(B\rho)}{B\rho} - \frac{d\rho}{\rho} = \gamma^2 \frac{df}{f} + (\gamma^2 - \gamma_t^2)\frac{dR}{R}. \tag{14}$$

Here we see that for nonrelativistic particles, the contribution of the radial error to dB/B can be much larger than the contribution of the frequency error.

2 Formulae for Inflector Voltage

The voltage V_I required for particles with momentum p and charge Q to follow the nominal trajectory through the inflector is given by

$$eV_I = \frac{G}{R_I}\left(\frac{c^2 p^2}{QE}\right). \tag{15}$$

Here $G = 0.017$ m is the gap between the cathode and septum of the inflector, $R_I = 8.74123$ m is the radius of curvature along the nominal trajectory, and the momentum and energy are given in terms of frequency and radius by equations (1–3). The errors in p and E are given by

$$\frac{dp}{p} = \gamma^2 \left\{\frac{df}{f} + \frac{dR}{R}\right\}, \quad \frac{dE}{E} = \beta^2 \frac{dp}{p} \tag{16}$$

which give

$$\frac{dV_I}{V_I} = 2\,\frac{dp}{p} - \frac{dE}{E} = (2-\beta^2)\,\gamma^2 \left\{\frac{df}{f} + \frac{dR}{R}\right\} \tag{17}$$

for the error in V_I.

3 Frequency Measurement

The revolution frequency was measured by observing the turn-by-turn sum signal from a Booster BPM. This signal is captured on an oscilliscope and analyzed by the Turn-by-Turn Program which displays the trace of each turn above the traces of the previous turns. The frequency in the program is adjusted so that the beam bunches on the traces line up vertically. This is the measured frequency. One can get some sense of the error in the measurement by seeing how much the set frequency has to be shifted to see that the bunches no longer line up vertically.

Since the beam coming into Booster from Tandem is not bunched, the revolution frequency at injection normally cannot be seen on a BPM. To put some structure on the incoming beam so that the revolution frequency can be seen, a half-turn is chopped out of the beam before injection as described in Ref. [1]. (Occasionally there is enough structure on the unchopped beam to see the revolution frequency but we do not depend on this to make measurements.)

Turn-by-turn frequency measurements were made by a number of people (L.A. Ahrens, K.A. Brown, C.J. Gardner, and N. Tsoupas) at various times over the course of several months. For most of the measurements a careful estimate of the error was not done. The bunches in the turn-by-turn display were simply lined up vertically as best as could be done by eye.

4 Rigidity and Field from Frequency Measurements at Injection

Tables 1, **2**, and **3** list the measured frequencies for various ions at Booster injection. These were used to calculate the kinetic energy W, magnetic rigidity $B\rho$, field B, and inflector voltage V_I. The orbit radius was assumed to be the nominal radius $R_0 = 201.78/(2\pi)$ m. The estimated fractional errors in the frequency and radius are

$$\frac{df}{f} \approx 3 \times 10^{-4}, \quad \frac{dR}{R} \approx 3 \times 10^{-4} \tag{18}$$

which give

$$\frac{d(B\rho)}{B\rho} = \frac{dp}{p} = \gamma^2 \left\{ \frac{df}{f} + \frac{dR}{R} \right\} \approx 6 \times 10^{-4} \tag{19}$$

and

$$\frac{dB}{B} = \gamma^2 \frac{df}{f} + (\gamma^2 - \gamma_t^2) \frac{dR}{R} \approx 7 \times 10^{-3}. \tag{20}$$

Magnetic field measurements from the 11DH1 NMR probe, Booster Hall probe, and Gauss clock are also listed in the Tables along with the C3 inflector voltage setpoint V_S. The Hall probe reading is the value of the dwell field prior to injection. The Gauss clock gives the change in field from BT0 to the time of injection. (We assume that the field at BT0 is equal to the dwell field.) The measured injection field H is then defined to be the sum of the Hall probe and Gauss clock readings.

Table 4 summarizes the rigidity and field data of Tables 1, 2, and 3. Here B_N is the reading of the NMR probe in TTB dipole 11DH1.

Figure 1 is a plot of the B and H data from Table 4 with fitted line $B = mH + b$. The fitted parameters are $m = 1.00076(125)$ and $b = 4.81(79)$ Gauss. The indicated errors were obtained assuming a statistical error of $\sigma = 0.56$ Gauss for each value of B. This gives a χ^2 per dof (degree of freedom) of 1. **Figure 2** shows the deviation of the Figure 1 data points from the fitted line.

Figure 3 is a plot of the $B\rho$ and B_N data from Table 4 with fitted line $B\rho = mB_N + b$. The fitted parameters are $m = 1.53065(98) \times 10^{-4}$ Tm per

Gauss and $b = -5.44(5.70) \times 10^{-4}$ Tm. The indicated errors were obtained assuming a statistical error of $\sigma = 0.0004$ Tm for each value of $B\rho$. This gives a χ^2 per dof of 1. **Figure 4** shows the deviation of the Figure 3 data points from the fitted line.

Having obtained the values of parameters m and b in the relations $B\rho = mB_N + b$ and $B = mH + b$, we can substitute the measured 11DH1 dipole field B_N into the first relation to obtain the magnetic rigidity of any ion being transported to Booster. Assuming the nominal radius of curvature in the Booster dipoles then gives the field B required for the ions to follow the nominal orbit in Booster. This then can be used in the second relation to obtain the desired injection field H to be measured by the Hall probe and Gauss clock. The rigidity also can be used in (7) to obtain the ion momentum p. This in turn can be used in (15) to calculate the inflector voltage V_I.

5 V_I from Frequency Measurements at Injection

Table 5 summarizes the inflector voltage data from Tables 1, 2, and 3. Here V_I is voltage determined from the frequency measurements and radius. V_S is the voltage setpoint. Using the estimated fractional errors in frequency and radius given by (18), we have

$$\frac{dV_I}{V_I} = (2 - \beta^2)\,\gamma^2 \left\{ \frac{df}{f} + \frac{dR}{R} \right\} \approx 12 \times 10^{-4}. \tag{21}$$

Figure 5 is a plot of the V_I and V_S data from Table 5 with fitted line $V_I = mV_S + b$. The fitted parameters are $m = 0.9858(44)$ and $b = 0.024(199)$ kV. The indicated errors were obtained assuming a statistical error of $\sigma = 0.17$ kV for each value of V_I. This gives a χ^2 per dof of 1. **Figure 6** shows the deviation of the Figure 5 data points from the fitted line.

Having obtained the values of parameters m and b in the relation $V_I = mV_S + b$, we can obtain the desired setpoint V_S once V_I has been determined from the 11DH1 dipole field B_N as discussed above.

6 Rigidity and Field from Frequency Measurements at Extraction

Tables 6, **7**, and **8** list the measured frequencies for various ions at Booster extraction. These were used to calculate the kinetic energy W, magnetic rigidity $B\rho$, and field B. The orbit radius was assumed to be the nominal radius $R_0 = 201.78/(2\pi)$ m. The setpoint of the field at extraction is also listed. This is the programmed value of the field in the BMM program. The calculated MM current is equal to the calculated field B (in Gauss) divided by 2.43 Gauss per Amp [2]. The MM current setpoint is the programmed value of the current in the BMM program.

Table 9 summarizes the rigidity and field data of Tables 6, 7, and 8. Here B_0 is the field setpoint at extraction.

Figure 7 is a plot of the B and B_0 data from Table 9 with fitted line $B = mB_0 + b$. The fitted parameters are $m = 0.98710(81)$ and $b = -24.9(7.4)$ Gauss. The indicated errors were obtained assuming a statistical error of $\sigma = 6.7$ Gauss for each value of B. This gives a χ^2 per dof of 1. **Figure 8** shows the deviation of the Figure 7 data points from the fitted line.

Having obtained the values of parameters m and b in the relation $B = mB_0 + b$, we can obtain the setpoint B_0 required for a given ion rigidity at extraction. Here we assume the nominal radius of curvature in the Booster dipoles. The given rigidity $B\rho$ then gives the field B required for the ions to follow the nominal orbit in Booster at extraction. Substituting B into the relation $B = mB_0 + b$ then gives the required setpoint B_0.

7 Appendix

The radius, R, of an equilibrium orbit in a synchrotron is defined to be the circumference of the orbit divided by 2π. Let R_0 be the radius of the design orbit, and let p_0 be the momentum of a charged particle traveling along this orbit when the field at a reference point in one of the synchrotron dipoles is b_0. We shall assume that when the field at the reference point is changed to b, the field at every other point in the synchrotron aperture is scaled by the same factor b/b_0. We assume further that the radius of the equilibrium orbit is a unique function of b and the particle momentum p. Thus we can write

$$R = R(p, b), \quad R_0 = R(p_0, b_0). \tag{22}$$

Now, if we change the momentum from p to $p + h$ and scale the field b by the same factor $(p + h)/p$, then the particle will follow the same orbit it did when the momentum and field were p and b. The function $R(p, b)$ therefore has the property

$$R(p + h,\ b + bh/p) = R(p, b) \tag{23}$$

and it follows that

$$\frac{\partial R}{\partial p} + \frac{b}{p}\frac{\partial R}{\partial b} = 0. \tag{24}$$

Thus

$$dR = \frac{\partial R}{\partial p}\,dp + \frac{\partial R}{\partial b}\,db = p\frac{\partial R}{\partial p}\left(\frac{dp}{p} - \frac{db}{b}\right) \tag{25}$$

and defining momentum compaction factor

$$\alpha = \frac{p}{R}\frac{\partial R}{\partial p} = -\frac{b}{R}\frac{\partial R}{\partial b} \tag{26}$$

we have

$$\frac{dR}{R} = \alpha\left(\frac{dp}{p} - \frac{db}{b}\right) = \frac{1}{\gamma_t^2}\left(\frac{dp}{p} - \frac{db}{b}\right) \tag{27}$$

where

$$\gamma_t^2 = 1/\alpha. \tag{28}$$

For sufficiently small regions of the p, b space we shall assume that α is constant. Then writing the first of equations (26) as

$$\frac{1}{R}\frac{\partial R}{\partial p} = \frac{\alpha}{p} \tag{29}$$

and integrating from p_0 to p we have

$$\ln R(p, b) - \ln R(p_0, b) = \ln p^\alpha - \ln p_0^\alpha \tag{30}$$

and therefore

$$R(p, b) = R(p_0, b)(p/p_0)^\alpha. \tag{31}$$

Similarly, writing the second of equations (26) as

$$\frac{1}{R}\frac{\partial R}{\partial b} = -\frac{\alpha}{b} \tag{32}$$

and integrating from b_0 to b we have

$$\ln R(p_0, b) - \ln R(p_0, b_0) = -\ln b^\alpha + \ln b_0^\alpha \tag{33}$$

and therefore

$$R(p_0, b) = R(p_0, b_0)(b_0/b)^\alpha. \tag{34}$$

Finally, putting (34) into (31) we have

$$R(p, b) = R_0 \left(\frac{b_0}{b} \frac{p}{p_0} \right)^\alpha. \tag{35}$$

This is the fundamental relation between field, momentum, and radius. In terms of the radii of curvature

$$\rho_0 = \frac{cp_0}{eb_0}, \quad \rho = \frac{cp}{eb} \tag{36}$$

we then have

$$R = R_0(\rho/\rho_0)^\alpha, \quad \rho = \rho_0(R/R_0)^{1/\alpha}. \tag{37}$$

References

[1] C.J. Gardner, L.A. Ahrens, and N. Williams, “Turn-by-Turn Analysis of Proton and Gold Beams at Injection in the AGS Booster”, Proceedings of the 1999 Particle Accelerator Conference, New York, 2063–2065 (1999)

[2] R. Thern, “Booster Dipole Production Measurements”, Booster Tech. Note 190, March 13, 1991.

Table 1: Gold and Deuteron Parameters at Booster Injection

Parameter	Au^{32+}	Deuteron	Deuteron	Unit
mc^2	183.456812	1.875612762	1.875612762	GeV
Date	10 Oct 02	12 Oct 02	21 Nov 02	
11DH1 NMR Probe	5580	4640	5580	Gauss
hf	397.74(06)	502.60(15)	401.922	kHz
h	6	3	2	
$T = 1/f$	15.0852	5.9690	4.9761	μs
Kinetic Energy W	182.8790	12.0392	17.3965	MeV
$B\rho$	0.854085	0.710002	0.854085	Tm
$B\rho/\rho$	615.974	512.060	615.974	Gauss
Booster Hall Probe	593.7	492.6	593.7	Gauss
Booster Gauss Clock	16.7	13.3	16.7	Gauss
Injection Field H	610.4	505.9	610.4	Gauss
Inflector Setpoint V_S	22.538	47.396	68.070	kV
Calculated Voltage V_I	22.218	46.678	67.355	kV

Table 2: Iron and Silicon Parameters at Booster Injection

Parameter	Fe^{10+}	Si^{5+}	Unit
mc^2	52.097954	26.057785	GeV
Date	5 Nov 02	15 Nov 02	
11DH1 NMR Probe	7958.15	7958	Gauss
hf	622.98(12)	622.80(12)	kHz
h	6	6	
$T = 1/f$	9.6311	9.6339	μs
Kinetic Energy W	127.6867	63.8279	MeV
$B\rho$	1.217428	1.217486	Tm
$B\rho/\rho$	878.021	878.062	Gauss
Booster Hall Probe	824.4	820.1	Gauss
Booster Gauss Clock	48.0	52.8	Gauss
Injection Field H	872.4	872.9	Gauss
Inflector Setpoint V_S	50.507	50.455	kV
Calculated Voltage V_I	49.604	49.588	kV

Table 3: Iron, Titanium, and Carbon Parameters at Booster Injection

Parameter	Ti^{18+}	Fe^{20+}	C^{6+}	Unit
mc^2	44.6540277	52.0928437	11.1748622	GeV
Date	24 June 03	2 June 03	27 May 03	
11DH1 NMR Probe	4364.1	4364.1	4364.1	Gauss
hf	358.45(10)	341.15(10)	475.89(10)	kHz
h	3	3	3	
$T = 1/f$	8.3694	8.7938	6.3040	μs
Kinetic Energy W	145.101856	153.258446	64.24366	MeV
$B\rho$	0.667638100	0.666936878	0.66711783	Tm
$B\rho/\rho$	481.507	481.001	481.132	Gauss
Booster Hall Probe	451.2	451.2	451.2	Gauss
Booster Gauss Clock	25.2	25.2	25.2	Gauss
Injection Field H	476.4	476.4	476.4	Gauss
Inflector Setpoint V_S	31.556	30.079	42.122	kV
Calculated Voltage V_I	31.304	29.762	41.528	kV

Table 4: Summary of Rigidities and Fields. Units are Tm and Gauss. $B\rho$ and B are determined from frequency measurements and radius. H is the injection field obtained from the Booster Hall probe and Gauss clock. B_N is the reading of the NMR probe in TTB dipole 11DH1.

Date	$B\rho$	B	H	B_N
10 Oct 02	0.854085	615.974	610.4	5580
12 Oct 02	0.710002	512.060	505.9	4640
5 Nov 02	1.217428	878.021	872.4	7958.15
15 Nov 02	1.217486	878.062	872.9	7958
27 May 03	0.667118	481.132	476.4	4364.1
2 Jun 03	0.666937	481.001	476.4	4364.1
24 Jun 03	0.667638	481.507	476.4	4364.1

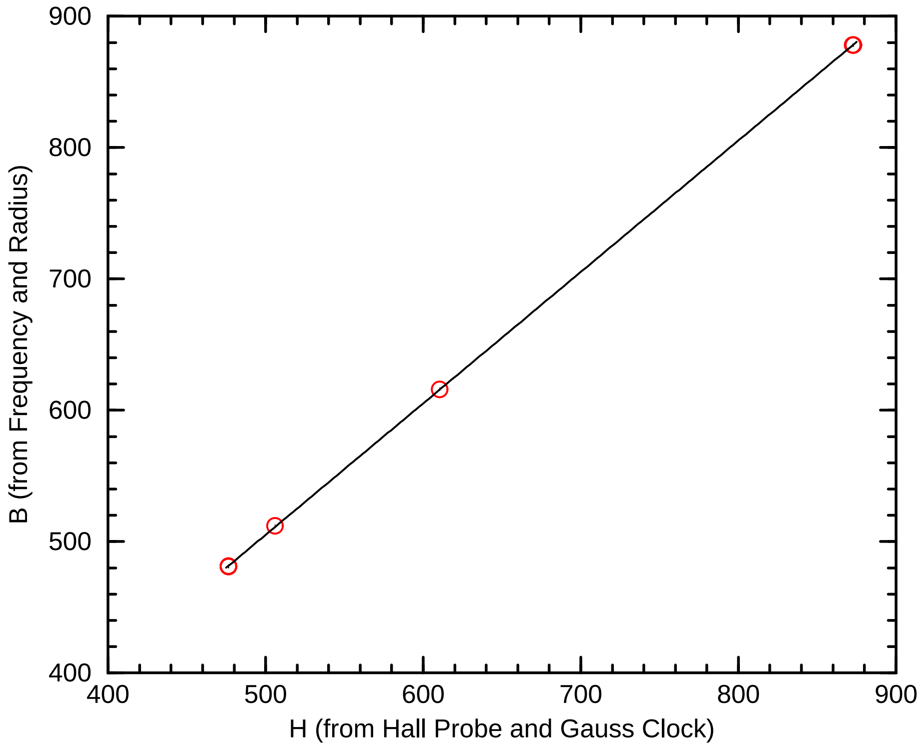

Figure 1: Plot of B and H data from Table 4 with fitted line. Units are Gauss. B is the field determined from frequency and radius. H is the injection field determined from the Booster Hall probe and Gauss clock. The line $B = mH + b$ is fitted to the data points. This gives fitted parameters $m = 1.00076(125)$ and $b = 4.81(79)$ Gauss. The indicated errors were obtained assuming a statistical error of $\sigma = 0.56$ Gauss for each value of B. This gives a χ^2 per dof of 1.

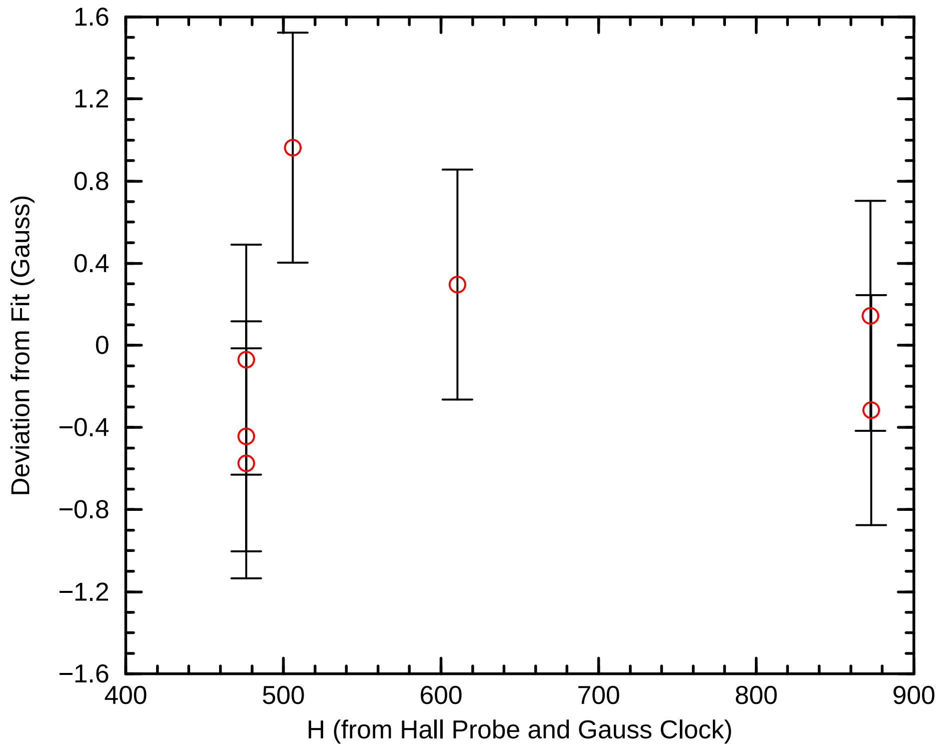

Figure 2: Deviation of Figure 1 data points from fitted line. Units are Gauss. The indicated errors are ± 0.56 Gauss.

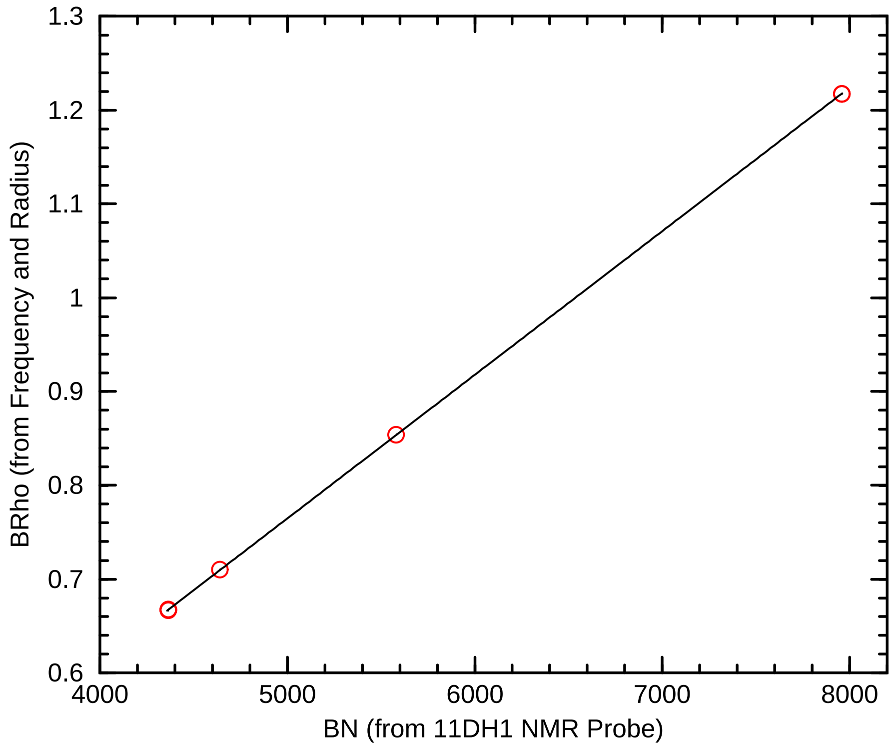

Figure 3: Plot of $B\rho$ and B_N data from Table 4 with fitted line. Units are Tm and Gauss. $B\rho$ is the rigidity determined from frequency and radius. B_N is the NMR probe reading in TTB dipole 11DH1. The line $B\rho = mB_N + b$ is fitted to the data points. This gives fitted parameters $m = 1.53065(98) \times 10^{-4}$ Tm per Gauss and $b = -5.44(5.70) \times 10^{-4}$ Tm. The indicated errors were obtained assuming a statistical error of $\sigma = 0.0004$ Tm for each value of $B\rho$. This gives a χ^2 per dof of 1.

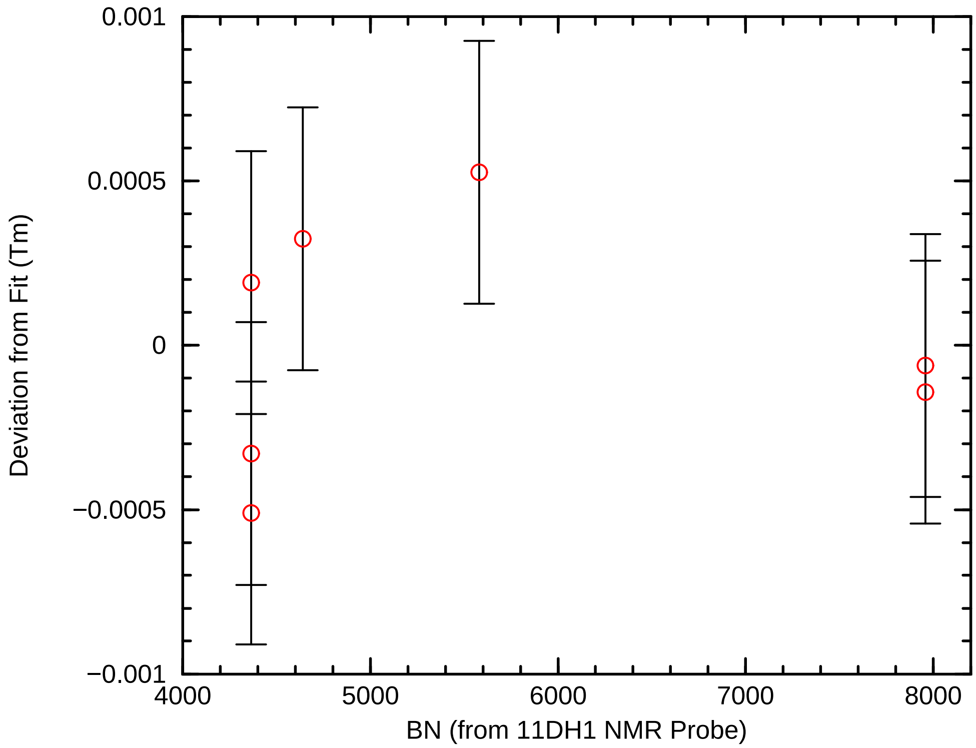

Figure 4: Deviation of Figure 3 data points from fitted line. Units are Tm and Gauss. The indicated errors are ± 0.0004 Tm.

Table 5: Summary of Inflector Voltages. Units are kilovolts. V_I is determined from frequency measurements and radius. V_S is the inflector voltage setpoint.

Date	V_I	V_S
10 Oct 02	22.218	22.538
12 Oct 02	46.678	47.396
5 Nov 02	49.604	50.507
15 Nov 02	49.588	50.455
21 Nov 02	67.355	68.070
27 May 03	41.528	42.122
2 Jun 03	29.762	30.079
24 Jun 03	31.304	31.556

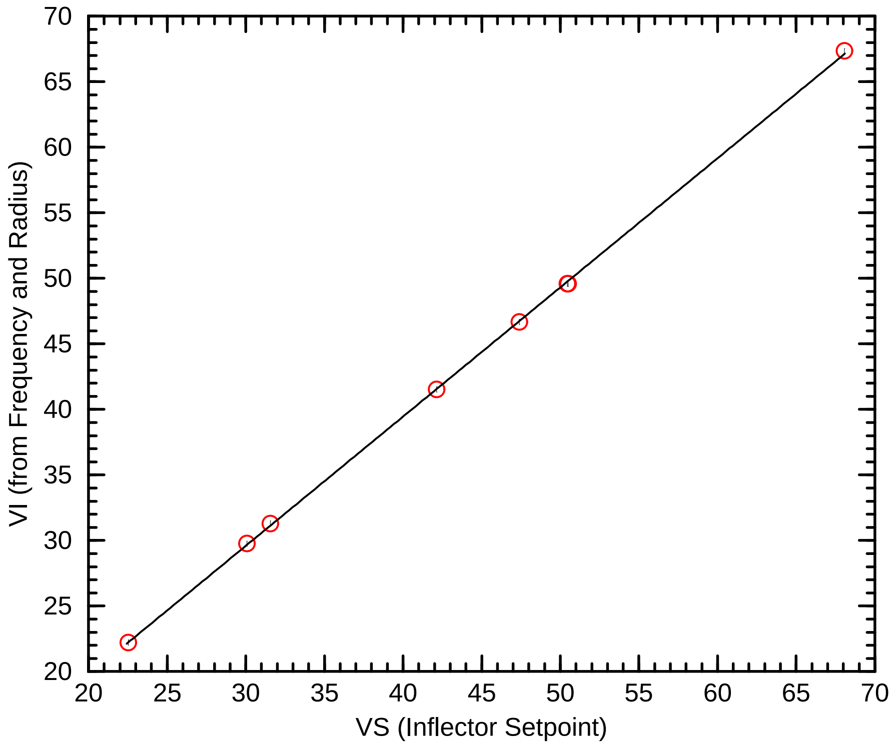

Figure 5: Plot of V_I and V_S data from Table 5 with fitted line. Units are kilovolts. V_I is the inflector voltage determined from frequency and radius. V_S is the inflector voltage setpoint. The line $V_I = mV_S + b$ is fitted to the data points. This gives fitted parameters $m = 0.9858(44)$ and $b = 0.024(199)$ kV. The indicated errors were obtained assuming a statistical error of $\sigma = 0.17$ kV for each value of V_I. This gives a χ^2 per dof of 1.

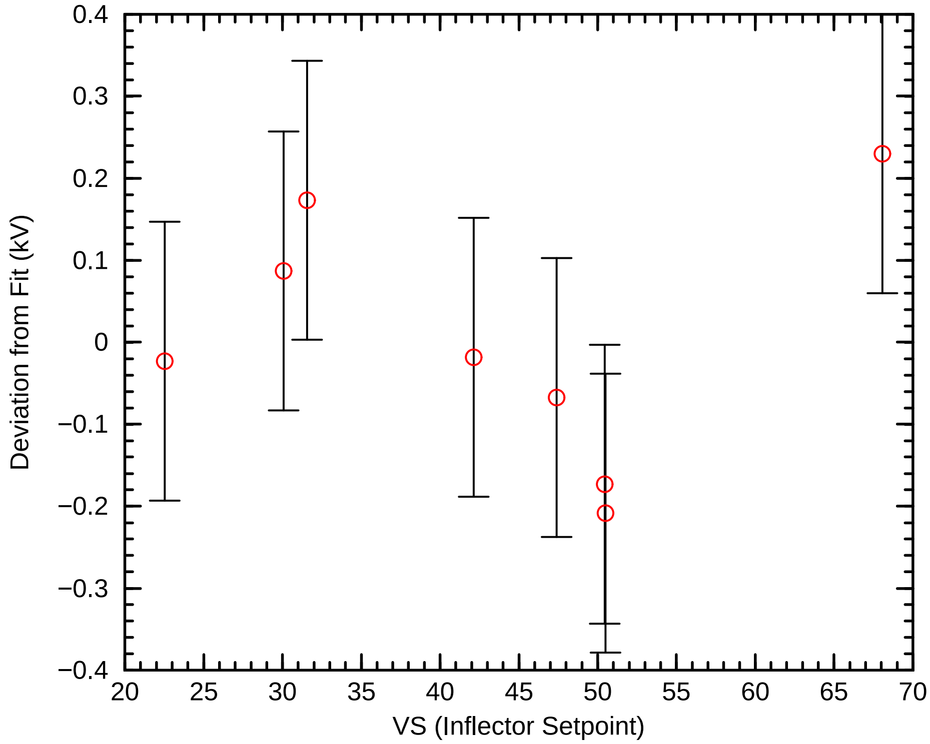

Figure 6: Deviation of Figure 5 data points from fitted line. Units are kV. The indicated errors are ± 0.17 kV.

Table 6: Carbon and Iron Parameters at Extraction

Parameter	C^{6+}	Fe^{20+}	C^{6+}	Unit
mc^2	11.1748622	52.0928437	11.1748622	GeV
Date	27 May 03	30 May 03	9 June 03	
hf	2.8503(6)	3.68724(60)	2.9208(6)	MHz
h	3	3	3	
$T = 1/f$	1.05252	0.81362	1.02712	μs
Kinetic E per Nucleon	280.037	725.4786	301.5832	MeV
$B\rho$	5.1674829	12.7925945	5.3894886	Tm
$B\rho/\rho$	3726.84	9226.14	3886.95	Gauss
Magnetic Field Setpoint	3800	9376	3962	Gauss
Calculated MM Current	1534	3797	1600	Amps
MM Current Setpoint	1563	3878	1629	Amps

Table 7: Iron Parameters at Extraction

Parameter	Fe^{20+}	Fe^{20+}	Fe^{20+}	Unit
mc^2	52.0928437	52.0928437	52.0928437	GeV
Date	30 May 03	18 June 03	19 June 03	
hf	3.68724(60)	3.89245(60)	3.90872(60)	MHz
h	3	3	3	
$T = 1/f$	0.81362	0.77072	0.76751	μs
Kinetic E per Nucleon	725.4786	979.1155	1005.3420	MeV
$B\rho$	12.7925945	15.5733095	15.85321093	Tm
$B\rho/\rho$	9226.14	11231.62	11433.48	Gauss
Magnetic Field Setpoint	9376	11400	11600	Gauss
Calculated MM Current	3797	4622	4705	Amps
MM Current Setpoint	3878	4822	4934	Amps

Table 8: Carbon, Iron, and Titanium Parameters at Extraction

Parameter	C^{6+}	Fe^{20+}	Ti^{18+}	Unit
mc^2	11.1748622	52.0928437	44.6540277	GeV
Date	9 June 03	19 June 03	23 June 03	
hf	2.9208(6)	3.90872(60)	3.91144(60)	MHz
h	3	3	3	
$T = 1/f$	1.02712	0.76751	0.76698	μs
Kinetic E per Nucleon	301.5832	1005.3420	1009.9121	MeV
$B\rho$	5.3894886	15.85321093	15.14496712	Tm
$B\rho/\rho$	3886.95	11433.48	10922.69	Gauss
Magnetic Field Setpoint	3962	11600	11100	Gauss
Calculated MM Current	1600	4705	4495	Amps
MM Current Setpoint	1692	4934	4663	Amps

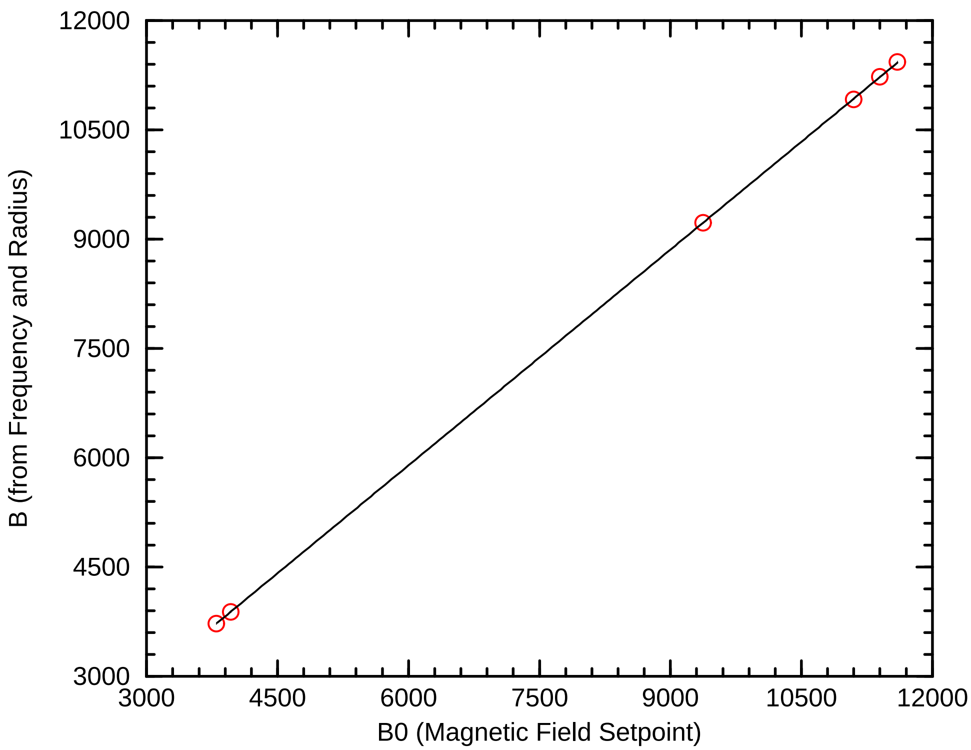

Figure 7: Plot of B and B_0 data from Table 9 with fitted line. Units are Gauss. B is the field at extraction determined from frequency and radius. B_0 is field setpoint in the Booster Main Magnet program. The line $B = mB_0 + b$ is fitted to the data points. This gives fitted parameters $m = 0.98710(81)$ and $b = -24.9(7.4)$ Gauss. The indicated errors were obtained assuming a statistical error of $\sigma = 6.7$ Gauss for each value of B. This gives a χ^2 per dof of 1.

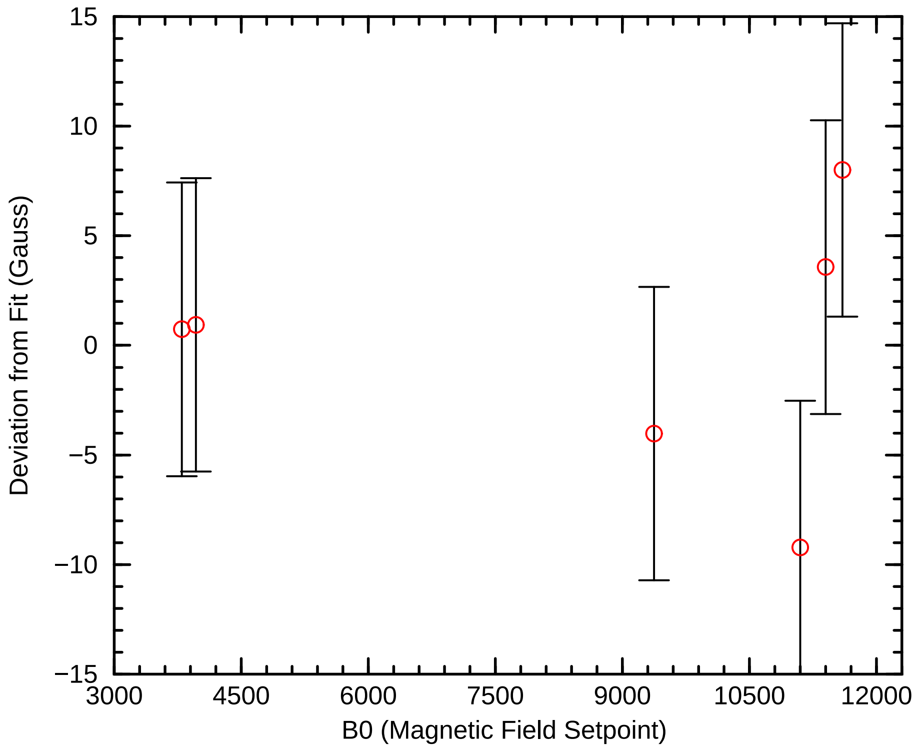

Figure 8: Deviation of Figure 7 data points from fitted line. Units are Gauss. The indicated errors are ±6.7 Gauss.

Table 9: Summary of Rigidities and Fields at Extraction. Units are Tm and Gauss. $B\rho$ and B are determined from frequency measurements and radius. B_0 is the field setpoint in the Booster Main Magnet program.

Date	$B\rho$	B	B_0
27 May 03	5.1674829	3726.84	3800
30 May 03	12.7925945	9226.14	9376
9 Jun 03	5.3894886	3886.95	3962
18 Jun 03	15.5733095	11231.62	11400
19 Jun 03	15.85321093	11433.48	11600
23 Jun 03	15.14496712	10922.69	11100