

RF Acceleration with Harmonic-Number Jump

A. G. Ruggiero

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Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy

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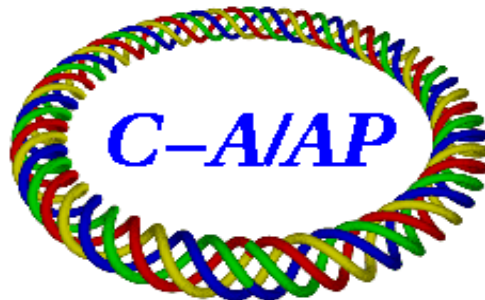
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A. G. Ruggiero



**Collider-Accelerator Department
Brookhaven National Laboratory
Upton, NY 11973**

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Alessandro G. Ruggiero
Brookhaven National Laboratory
May 10, 2006

Abstract

We have recently considered acceleration of protons and heavy ions in a Fixed-Field Alternating-Gradient (FFAG) accelerator with Non-Scaling Lattice and Linear Field Profile [1]. To avoid the problem of frequency modulation for acceleration of low-energy beams over a too short period of time, and to boost the acceleration rate, the method of *Harmonic-Number Jump* (HNJ) was proposed [2]. This method allows the use of constant frequency acceleration, for instance using superconducting cavities, despite the fact that the beam velocity may vary considerably. Of course in this case the accelerating voltage and RF phase will need to be programmed accordingly as we shall show. We shall study first the motion of *Synchronous* particles, and then of those with deviating initial conditions. We estimate the area and height of the RF buckets that are to contain the beam bunches with the added condition of the HNJ. We finally investigate methods to allow the program of energy gain as required by the HNJ method, including the effect of the cavity Transit Time Factor (TTF).

Acceleration of Synchronous Particles

Let us assume that the FFAG ring is made of N RF cavities equally spaced, and that the lattice properties and structure between two consecutive cavities repeat identically around the ring (see Figure 1). The acceleration cycle is then thought as a sequence of pair events: an energy kick at the location of the n -th cavity, followed by an arc A_n that takes the beam from that cavity to the next.

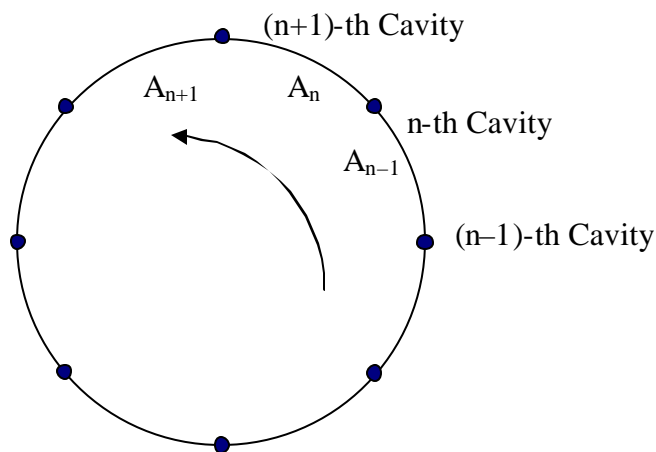


Figure 1. FFAG ring with multiple equally-spaced RF Cavities

Let us start considering a beam made of point-like bunches; that is, all particles have the same energy, and cross the cavities all together at the same instant. Equivalently each point-like bunch can be thought to be a *synchronous* or a *reference* particle around which all other particles oscillate. The total energy of the reference particle in the n -th arc A_n is E_n , and the period of time that it takes to travel the arc A_n is $T_n = h_n T_{RF}$, where T_{RF} is the constant RF period, and h_n is the RF harmonic number local to the arc A_n . Here and in the following bold face parameters apply to the *reference* particle. The same parameters in plain face apply to the other particles.

Let Q and A denote respectively the charge state and the mass number of the ion particle. The energy gain when crossing the n -th cavity is

$$\begin{aligned} \Delta E_n &= (Q e V_n / A) \sin(\omega_{RF} t_n) \\ &= (Q e V_n / A) \sin(f_n) \end{aligned} \quad (1)$$

where V_n is the peak voltage, $\omega_{RF} / 2\pi$ the RF frequency, t_n the instant of traversal of the cavity, and $f_n = \omega_{RF} t_n$ the RF *synchronous* phase. Both V_n and f_n can vary from cavity to cavity, and, in the same cavity, from turn to turn.

The HNJ method requires that the energy gain is adjusted to cause a change in the travel period T_n in the following arc A_n , so that the *reference* particle is pushed forward or back exactly by Δh RF harmonics and appears in an exactly identical bucket ahead or trailing by Δh RF wavelengths (see Figure 2). Thus, by denoting

$$T_n = h_n T_{RF} \quad (2a)$$

$$T_{n-1} = h_{n-1} T_{RF} \quad (2b)$$

$$h_n - h_{n-1} = -\Delta h \quad (2c)$$

this is accomplished by requiring, in linear approximation,

$$\Delta E_n = \beta_n^2 \gamma_n^3 E_0 \Delta h / h_n (1 - \alpha_{pn} \gamma_n^2) \quad (3)$$

where E_0 is the ion particle rest energy, β_n and γ_n the usual relativistic velocity and energy factors, and α_{pn} the momentum compaction factor local to the n -th arc A_n . The HNJ is executed by combining Eq.s (1, 2 and 3).

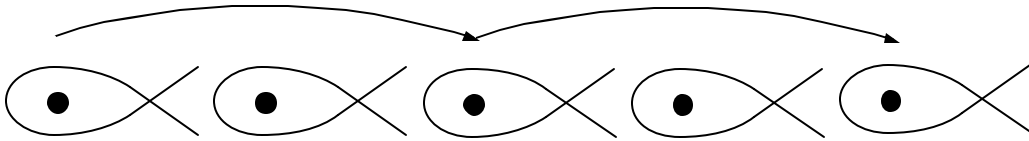


Figure 2. RF Bucket Jump at Cavity Crossings by Δh RF wavelengths (harmonic number)

Acceleration of Non-Synchronous Particles

Let us consider now an off-momentum particle following (or heading) the *reference* particle with a time difference τ_n . It will cross the n -th cavity at the instant $t_n = t_n + \tau_n$, and the corresponding energy gain is

$$\Delta E_n = (Q e V_n / A) \sin(\omega_{RF} t_n) \quad (4)$$

Subtracting side by side Eq. (1) from Eq. (4) gives the change in the energy difference $\epsilon_n = E_n - \bar{E}_n$ after crossing the n -th cavity

$$\Delta \epsilon_n = (Q e V_n / A) [\sin(f_n + \omega_{RF} \tau_n) - \sin(f_n)] \quad (5)$$

In the limit of small delay τ_n

$$\Delta \epsilon_n = (Q e V_n / A) (\cos f_n) \omega_{RF} \tau_n \quad (6)$$

The change of the time delay, in linear approximation, is given by

$$\begin{aligned} \Delta \tau_n &= \tau_n - \tau_{n-1} \\ &= (t_n - t_n) - (t_{n-1} - t_{n-1}) \\ &= T_n - T_n \\ &= -(1 - \alpha_{pn} \gamma_n^2) T_n \epsilon_n / \beta_n^2 \gamma_n^3 E_0 \end{aligned} \quad (7)$$

Combining Eq.s (6 and 7) gives the phase (or energy) oscillation equation in the limit of small amplitude

$$\Delta^2 \tau_n / \Delta n^2 + \Omega_n^2 \tau_n = 0 \quad (8)$$

where the angular oscillation frequency

$$\Omega_n^2 = 2 p \Delta h / t g f_n \quad (9)$$

Motion within RF Buckets

The two canonical Eq.s (5 and 7) can be derived from the following Hamiltonian

$$\begin{aligned} H &= (Q e V_n / A \omega_{RF}) [\cos(f_n + \omega_{RF} \tau_n) + \omega_{RF} \tau_n \sin(f_n)] + \\ &- (1 - \alpha_{pn} \gamma_n^2) T_n \epsilon_n^2 / (2 \beta_n^2 \gamma_n^3 E_0) \end{aligned} \quad (10)$$

The motion of all particles is to occur in RF buckets as the one shown in Figure 3.

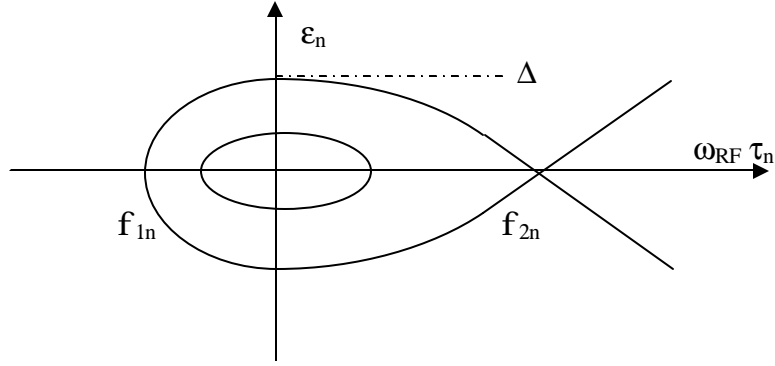


Figure 3. Separatrix of RF Bucket

From Eq. (5) we see that there are two fixed points at

$$\omega_{RF} \tau_n = 0 \quad (\text{stable})$$

$$\omega_{RF} \tau_n = p - 2 f_n = f_{2n} \quad (\text{unstable})$$

The value of the bucket separatrix is given by letting $\epsilon_n = 0$ and $\omega_{RF} \tau_n = p - 2 f_n$ in Eq. (10), that is

$$H_n = -(Q_e V_n / A \omega_{RF}) [\cos(f_n) - (p - 2 f_n) \sin(f_n)] \quad (11)$$

The RF bucket extends over the phase range between $\omega_{RF} \tau_n = f_{1n}$ and $\omega_{RF} \tau_n = f_{2n}$, that are the roots of the following equation

$$\cos(f_n + \omega_{RF} \tau_n) + \cos(f_n) + (\omega_{RF} \tau_n - p + 2 f_n) \sin(f_n) = 0 \quad (12)$$

The two phase angles f_{1n} and f_{2n} are plotted versus f_n in Figure 4.

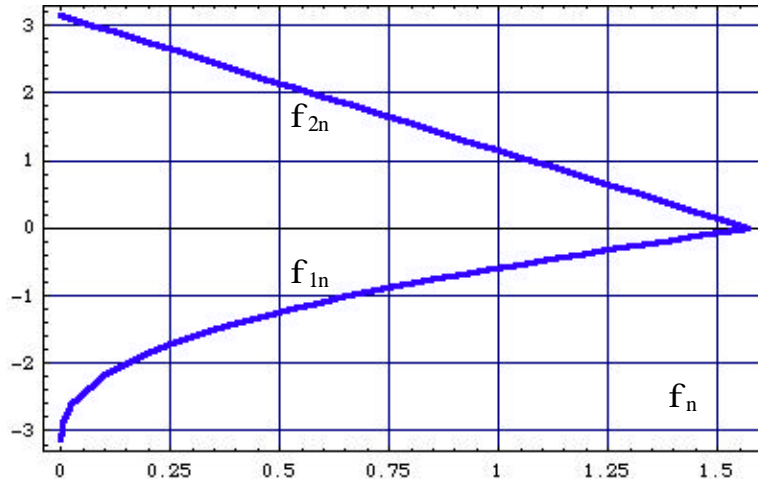


Figure 4. Plot of f_{1n} and f_{2n} versus f_n

We can also find the bucket half height

$$\Delta^2 = 2 Q_e V_n \beta_n^2 \gamma_n^3 E_0 F(f_n) / A \pi (1 - \alpha_{pn} \gamma_n^2) h_n \quad (13)$$

where

$$F(f_n) = \cos(f_n) - (p/2 - f_n) \sin(f_n) \quad (14)$$

which is plotted in Figure 5. Similarly we can calculate the area B_n surrounded by the RF bucket. This can be expressed as

$$B_n = B_{0n} I(f_{1n}, f_{2n}) \quad (15)$$

with

$$B_{0n} = (8 / \omega_{RF}) [2 Q_e V_n \beta_n^2 \gamma_n^3 E_0 / A \pi h_n (1 - \alpha_{pn} \gamma_n^2)]^{1/2} \quad (16)$$

and

$$I(f_{1n}, f_{2n}) = \int_{f_{1n}}^{f_{2n}} [\cos(f_n + \phi) + \phi \sin(f_n) + G(f_n)]^{1/2} / 4 \sqrt{2} d\phi \quad (17)$$

where

$$G(f_n) = \cos(f_n) - (p - 2 f_n) \sin(f_n) \quad (18)$$

This integral is plotted in Figure 6 versus the RF phase . As a curiosity observe that the two functions $I(f_{1n}, f_{2n})$ and $F(f_n)$ are almost equal within a few percent.

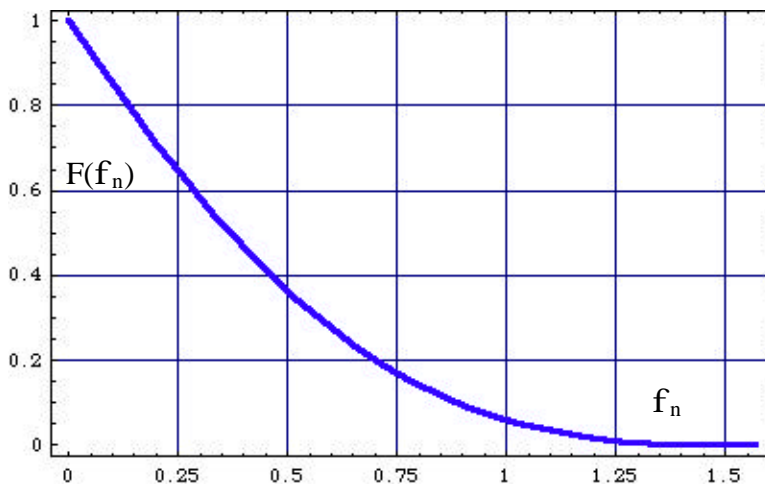


Figure 5. Plot of $F(f_n)$ versus f_n

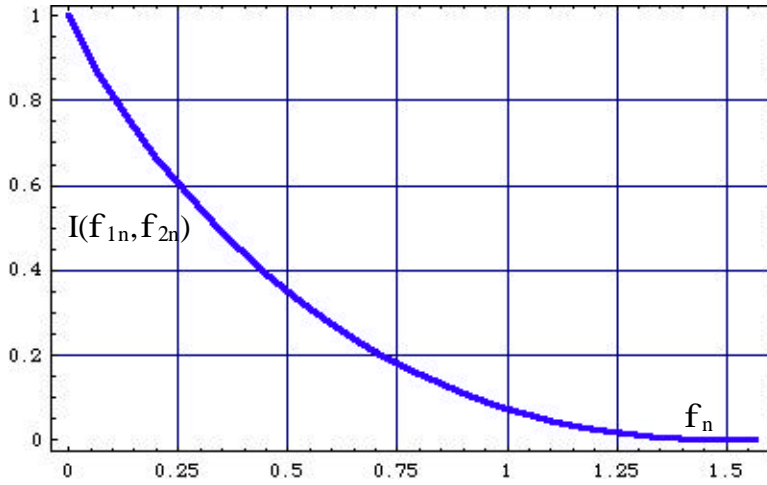


Figure 6. Plot of $I(f_{1n}, f_{2n})$ versus RF phase f_n

In conclusion we have determined that the longitudinal equations of motion for the HNJ method are similar to the conventional ones for constant harmonic acceleration, except that the energy gain per cavity is programmed according to the combination of Eq.s (3 and 4). The rate of acceleration cannot be given a-priori, but is determined by the required energy gain program. This is most advantageous indeed in FFAG accelerators where the guiding field does not change with time.

Consequences of the Harmonic Number Jump

The procedure we have followed applies correctly to the case when beam energy is constantly below the transition energy, namely when $\alpha_{pn} \gamma_n^2 < 1$. We have indeed set Eq.s (1 to 3) explicitly for this case, that requires a decreasing harmonic number as acceleration proceeds. In the opposite case when the beam energy is above the transition energy, namely that $\alpha_{pn} \gamma_n^2 > 1$, the harmonic number will increase monotonically. Thus Eq. (2c) is to be replaced with $h_n - h_{n-1} = + \Delta h$, and a sign minus is to be placed in front of the r.h. side of Eq. (3).

Obviously, to avoid unnecessary beam losses, the number of bunches, that is a constant during acceleration, ought to be always less than the harmonic number at all time. On the other end, because of the change of the revolution period due to the beam velocity variation, the number of RF buckets will vary. The actual distribution of the beam bunches with respect to the available buckets during acceleration is shown in Figure 7a for the case of acceleration below the transition energy, and in Figure 7b for the case above. It is seen that in the former case the beam longitudinal extension at injection ought to be shorter than the revolution period and at most equal to the revolution period at extraction. That is, the number of injected bunches cannot be larger than the RF harmonic number at extraction. The situation is different when the beam is injected at an energy above the transition energy value. In this case the revolution period decreases and the harmonic number increases during acceleration.

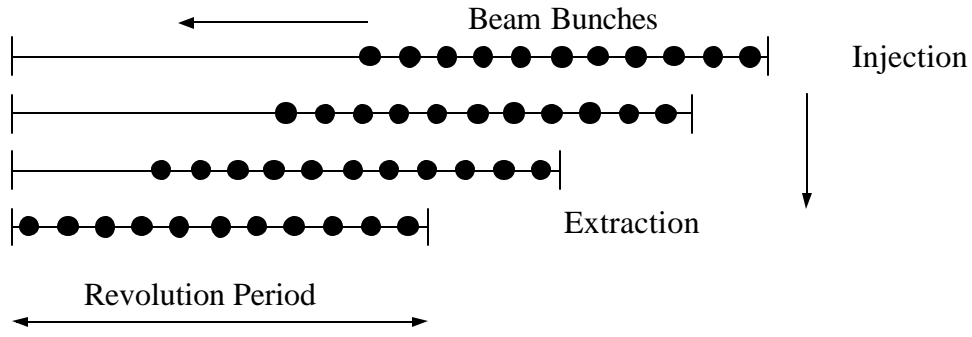


Figure 7a. Bunch to Bucket Configuration *below* Transition Energy

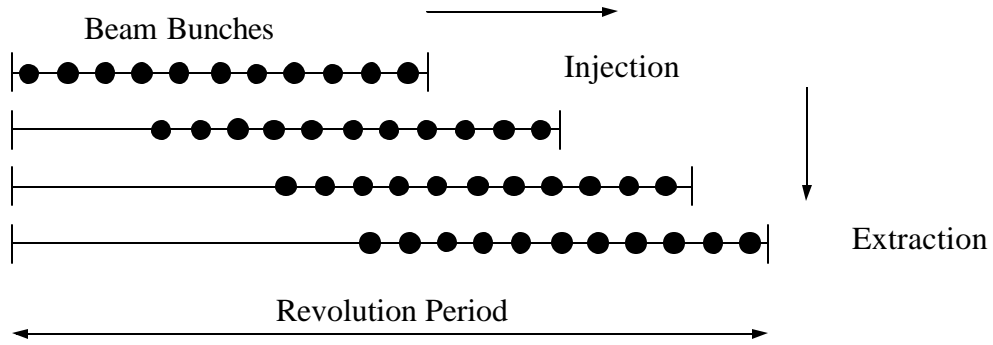


Figure 7b. Bunch to Bucket Configuration *above* Transition Energy

Energy Gain Programming

The acceleration method by HNJ requires that the energy gain at the n -th cavity is given by the combination of Eq.s (1 and 3), that is

$$eV_n \sin(f_n) = A \beta_n^2 \gamma_n^3 E_0 \Delta h / Q h_n (1 - \alpha_{pn} \gamma_n^2) \quad (19)$$

As an example we show in Figure 8 the energy gain per cavity required for the case of one of the FFAG rings used for Radio-Isotopes Acceleration (RIA) [3]. What is shown is $eV_n \sin(f_n)$ versus the actual radial position of the beam during acceleration as directly given by Eq. (19) for $\Delta h = 1$. The ring accelerates a beam of ions Uranium +28 from 6 to 50 MeV/u, and has a circumference of 807.091 m. Acceleration occurs at the constant RF frequency of 201.25 MHz with 8 equally-spaced superconducting cavities. Let ξ_n denote the average axial field in the cavity that we take to be made of one single elliptical shape cell with gap g operating in half-wavelength mode. Let also λ denote the RF wavelength. The cavity voltage can then be expressed as

$$V_n = g \xi_n \text{TTF}(\beta_0/\beta) \quad (20)$$

where

$$g = \lambda \beta_0 / 2 \tag{21}$$

and

$$\text{TTF}(\beta_0/\beta) = \sin(p \beta_0 / 2 \beta) / (p \beta_0 / 2 \beta) \tag{22}$$

is the Transit Time Factor (TTF) that is a function of the beam velocity β , and β_0 is a reference value that corresponds to the gap g for which the TTF has been optimized. Typically β_0 is adjusted to the value of the beam velocity in the middle of the acceleration cycle.

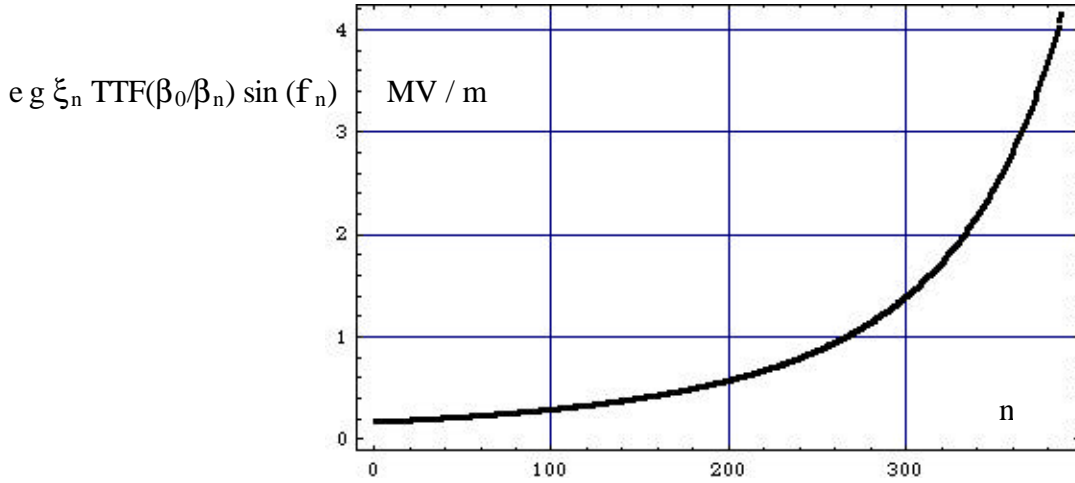


Figure 8. Energy Gain ΔE_n Program vs. number of Cavity Crossings

The profile of Figure 8 can be realized in two different modes of operation:

1. Constant RF phase. The average axial accelerating field ξ_n is shown in Figure 9 across the radial aperture for $f_n = 60^\circ$.

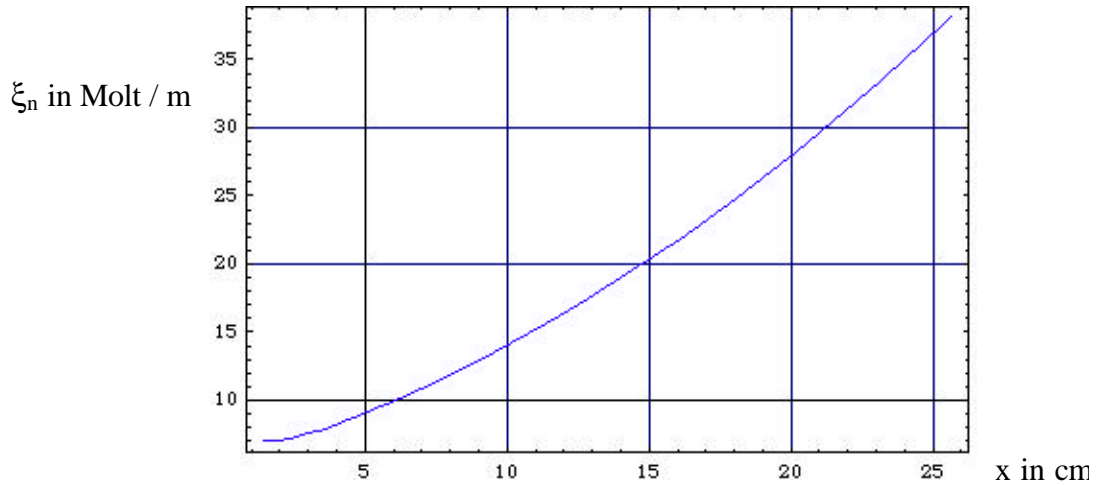


Figure 9. Average Axial Field versus Radial Beam Position with $f_n = 60^\circ$

The realization of such field profile across the radial aperture is problematic but not impossible. It could be made with ordinary cavity cells resonating in TM010 mode but displaced horizontally [2].

2. Constant average axial field ξ_n . This is obtained by programming the RF phase during the acceleration cycle. The result is shown in Figure 10 that corresponds to the same axial field in all cavities of 38.7 MVolt/m. That includes also the transit time factor TTF. This mode of operation requires that the RF cavities are properly phased with respect to each, and that the phase in all cavities varies from turn to turn. In both cases the axial field may be exceedingly too large. It could be lowered with multi-cell cavities, though at the same time the transit time factor behavior may get more irregular due to the large beam velocity change.

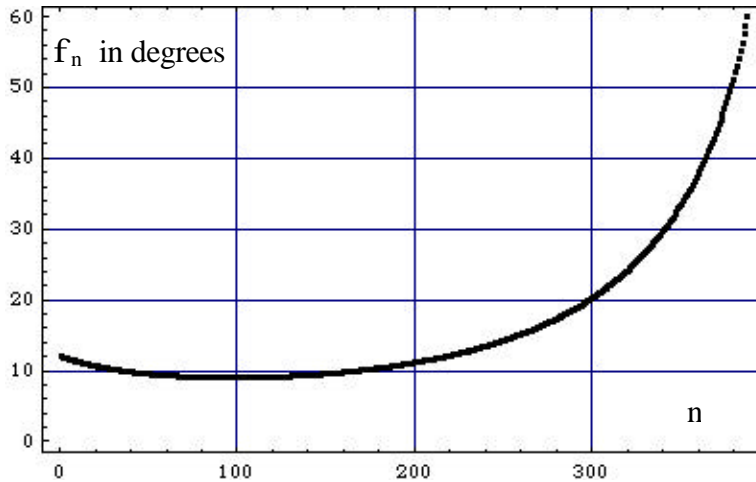


Figure 10. RF Phase f_n Program versus no. of Cavity Crossings

Conclusions

We have investigated a method of low-energy hadron beam acceleration in FFAG accelerators with constant frequency RF superconducting cavities. The method requires an energy gain at each cavity adjusted to allow the beam to skip one or more RF wavelengths between cavities. The energy gain program can be obtained either by requiring a radial profile of the accelerating electric field with the proper design of the cavity, or by setting a constant accelerating field throughout the cycle together to a prescribed program of RF phase. In both approaches transit times factors are evaluated and taken into account as the beam changes velocity over an appreciable range. The analysis performed shows that the longitudinal motion occurs in RF buckets of calculable height and area.

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