

Results for electron cooling rates using electron cooling as an intrabeam scattering process

G. Parzen

August 2006

Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

C-A/AP/#232
August 2006

Results for Electron Cooling Rates Using Electron Cooling as an Intrabeam Scattering Process

George Parzen



Collider-Accelerator Department
Brookhaven National Laboratory
Upton, NY 11973

Results for electron cooling rates using electron cooling as an intrabeam scattering process

George Parzen

August 18, 2006

Abstract

Electron cooling that results when a bunch of electrons overlaps a bunch of ions, with both bunches moving at the same velocity, may be considered to be an intrabeam scattering process. This paper lists the results found for the emittance cooling rates using the methods developed in intrabeam scattering theory. Only the results are given here. Derivations of these results will be given in a future paper. It is assumed that the dispersion is zero in the cooling section, and that the ion bunch and the electron bunch have gaussian distributions.

1 Introduction

Electron cooling that results when a bunch of electrons overlaps a bunch of ions, with both bunches moving at the same velocity, may be considered to be an intrabeam scattering process. The process is similar to the usual intrabeam scattering where the ions scatter from each other and usually results in beam growth. An important difference is that in electron cooling the mass of the ion is different from and much larger than the mass of the electron. This difference considerably complicates the intrabeam scattering theory. It introduces a new term in the emittance growth rate, which vanishes when the particles are identical and their masses are equal, and can give rise to emittance cooling of the heavier particles. The term that gives rise to beam growth for the usual intrabeam scattering is also present but is much smaller than the cooling term when one particle is much heavier than the other.

This paper lists the results found for the emittance cooling rates due to the scattering of the ions in the ion bunch by the electrons in the electron bunch. Only the results are given here. Derivations of these results will be given in a future paper, Ref.[3]. It is assumed that the dispersion is zero in the cooling section, that the ion bunch and the electron bunch have gaussian distributions, and that there are no magnetic fields in the cooling section.

2 Results

Ions are indicated by the subscript a and the electrons by the subscript b. The cooling rate for $\langle p_{ia}^2/p_{0a}^2 \rangle$, $i = x, s, y$ in the Rest CS is due to the scattering of the ions from the electrons. This cooling rate has to be added to the growth rate due to the scattering of the ions from each other to find the actual growth rate of the ions. p_{0a} is the central momentum of the ion bunch. The assumed gaussian distributions of the bunches are described by the parameters

$$\bar{\epsilon}_{ia}, \beta_{ia}, \alpha_{ia}, \quad \bar{\epsilon}_{ib}, \beta_{ib}, \alpha_{ib}, \quad i = x, s, y$$

The gaussian distribution is further defined using these parameters in Ref.[1].

2.1 $d \langle p_{ia} p_{ja} \rangle / dt$, $i, j = x, s, y$ in the Rest CS.

$$\begin{aligned} \delta \langle (\bar{p}_{ia} \bar{p}_{ja}) \rangle &= \frac{N_b}{\Gamma_a \Gamma_b} \frac{1}{\bar{A}_p^{1/2}} \pi^3 r_{ab}^2 c \frac{\mu}{m_a} \hat{W}_{ij} \\ &\int d^3 \Delta \frac{\exp[-(\lambda_x \Delta_x^2 + \lambda_y \Delta_y^2 + \lambda_s \Delta_s^2)]}{\beta_{ab}^3} \Delta_i \Delta_j \\ &\ln \left[1 + \left(\frac{\beta_{ab}^2 b_{maxab}}{r_{ab}} \right)^2 \right] dt \\ \hat{W}_{ij} &= 2\pi \left[(W_\eta \frac{B_{10}}{2} + W_{p_\eta} \frac{B_{01}}{2})_i + (W_\eta \frac{B_{10}}{2} + W_{p_\eta} \frac{B_{01}}{2})_j \right] \\ \beta_{ab} &= \gamma_0 \beta_0 (\Delta_x^2 + \Delta_y^2 + \Delta_s^2)^{1/2} \\ \lambda_i &= \left[A_{00} - \left(\frac{B_{10}}{2} \right)^2 - \left(\frac{B_{01}}{2} \right)^2 \right]_i \\ \bar{A}_i &= [A_{11} A_{22} - A_{12}^2]_i \end{aligned}$$

$$\begin{aligned}
\bar{A}_p^{1/2} &= \bar{A}_x^{1/2} \bar{A}_y^{1/2} \bar{A}_s^{1/2} \\
x_{\eta i} &= \left[\frac{A_{22}^{1/2}}{\bar{A}^{1/2}} \right]_i & W_{\eta i} &= \left[-\frac{A_{12}}{\bar{A}^{1/2}} \right]_i & W_{p_{\eta i}} &= \left[\frac{1}{A_{22}^{1/2}} \right]_i \\
B_{10i} &= [A_{10} x_{\eta} + A_{01} W_{\eta}]_i & B_{01i} &= [A_{01} W_{p_{\eta}}]_i \\
B_{10i} &= \left[A_{10} \frac{A_{22}^{1/2}}{\bar{A}^{1/2}} - A_{01} \frac{A_{12}}{\bar{A}^{1/2}} \right]_i & B_{01i} &= \left[A_{01} \frac{1}{A_{22}^{1/2}} \right]_i \\
A_{11i} &= \left[\frac{1 + \alpha_i^2}{\beta_i \bar{\epsilon}_i} \right]_+ & A_{22i} &= \left[\frac{\beta_i}{\bar{\epsilon}_i} \right]_+ \\
A_{12i} &= \left[\frac{\alpha_i}{\bar{\epsilon}_i} \right]_+ & A_{10i} &= \left[2 \frac{\mu}{m} \frac{\alpha_i}{\bar{\epsilon}_i} \right]_- \\
A_{01i} &= \left[2 \frac{\mu}{m} \frac{\beta_i}{\bar{\epsilon}_i} \right]_- & A_{00i} &= \left[\left(\frac{\mu}{m} \right)^2 \frac{\beta_i}{\bar{\epsilon}_i} \right]_+
\end{aligned} \tag{1}$$

b_{maxab} is the largest allowed impact parameter for the scattering of the ions by the electrons. β_s in the Rest CS is larger than β_s in the Laboratory CS by the factor γ_0^2 . s, p_s are the particle longitudinal position and momentum in the Rest CS.

The symbols $[()]_+$ and $[()]_-$ are defined by

$$[()]_+ = ()_a + ()_b$$

$$[()]_- = ()_a - ()_b$$

The integral over $d^3\Delta$ is an integral over all possible values of the relative velocity of any two particles in a bunch. β_0, γ_0 are the beta and gamma of the center of the bunches in the Laboratory Coordinate System. p_{0a} is the momentum of the central particle in the ion bunch. With no dispersion in the cooling section, one gets zero results when $i \neq j$.

2.2 $d \langle x_i p_{ja} \rangle / dt$, $i, j = x, s, y$ in the Rest CS.

Growth rates for the emittances require the growth rates for $\langle x_i p_{ja} \rangle$, $i = x, s, y$ in the Rest CS.

$$\begin{aligned}
\delta \langle (x_i \bar{p}_{ja}) \rangle &= \frac{N_b}{\Gamma_a \Gamma_b} \frac{1}{\bar{A}_p^{1/2}} \pi^3 r_{ab}^2 c \frac{\mu}{m_a} \hat{x}_{ij} \\
&\int d^3 \Delta \frac{\exp[-(\lambda_x \Delta_x^2 + \lambda_y \Delta_y^2 + \lambda_s \Delta_s^2)]}{\beta_{ab}^3} \Delta_i \Delta_j \\
&\ln \left[1 + \left(\frac{\beta_{ab}^2 b_{maxab}}{r_{ab}} \right)^2 \right] dt \\
\hat{x}_{ij} &= 2\pi \left[x_\eta \frac{B_{10}}{2} \right]_i \\
x_{\eta i} &= \left[\frac{A_{22}^{1/2}}{A^{1/2}} \right]_i \quad W_{\eta i} = \left[-\frac{A_{12}}{A^{1/2}} \right]_i \quad W_{p\eta i} = \left[\frac{1}{A_{22}^{1/2}} \right]_i \\
B_{10i} &= [A_{10} x_\eta + A_{01} W_\eta]_i \quad B_{01i} = [A_{01} W_{p\eta}]_i \\
B_{10i} &= \left[A_{10} \frac{A_{22}^{1/2}}{A^{1/2}} - A_{01} \frac{A_{12}}{A^{1/2}} \right]_i \quad B_{01i} = \left[A_{01} \frac{1}{A_{22}^{1/2}} \right]_i
\end{aligned} \tag{2}$$

This expression is the same as that given for $\langle p_{ia} p_{ja} \rangle$ except that \hat{W}_{ij} is replaced by \hat{x}_{ij} .

2.3 $d \langle \epsilon_{ia} \rangle / dt$ in the Laboratory CS $i = x, s, y$

In the following, $d\tilde{t}$ is the time interval in the Laboratory System and dt is the time interval in the Rest System. $d\tilde{t} = \gamma dt$. β_s is the longitudinal beta function in the Laboratory System. The dispersion is assumed to be zero in the cooling section.

$$\begin{aligned}
\frac{d}{d\tilde{t}} \langle \epsilon_{xa} \rangle &= \frac{\beta_x}{\gamma_0} \frac{d}{dt} \langle p_{xa}^2 / p_{0a}^2 \rangle + 2 \frac{\alpha_x}{\gamma_0} \frac{d}{dt} \langle x_a p_{xa} / p_{0a} \rangle \\
\frac{d}{d\tilde{t}} \langle \epsilon_{ya} \rangle &= \frac{\beta_y}{\gamma_0} \frac{d}{dt} \langle p_{ya}^2 / p_{0a}^2 \rangle + 2 \frac{\alpha_y}{\gamma_0} \frac{d}{dt} \langle y_a p_{ya} / p_{0a} \rangle \\
\frac{d}{d\tilde{t}} \langle \epsilon_{sa} \rangle &= \beta_s \gamma_0 \frac{d}{dt} \langle p_{sa}^2 / p_{0a}^2 \rangle
\end{aligned} \tag{3}$$

A computer program called BIG, Ref.[2], has been written that evaluates the results given in this paper.

References

1. G.Parzen, BNL report C-A/AP/N0.169 (2004)
2. G.Parzen, BNL report C-A/AP/N0.213 (2005)
3. G.Parzen, BNL report C-A/AP/N0.243 (2006)